10 Magnetic fields due to Electric Current

Syllabus

10.1 Introduction

Magnetic Field :

i. *A magnet attracts small pieces of iron, cobalt,nickel etc.*

ii. *The Space around a magnet within which the influence can be experienced is called its magnetic field.*

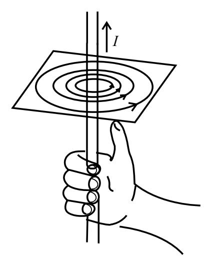
iii. *Hans Christian Oersted first discovered the magnetic fieldis produced by an electric current passing through a wire.*

iv. *Later, Gauss, Henry, Faraday and others showed that magnetic field is an important partner of electric field*

v. *Maxwell's theoretical work highlighted the close relation between electric and magnetic field.*

Q.1 State right hand thumb rule.

Ans: Imagine that a current carrying wire is grabbed with your right hand with the thumb pointing in the direction of the current, then your fingers curl around in the direction of the magnetic field.



Right hand thumb rule.

10.2 Magnetic Force

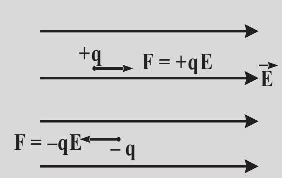
Note:

i Force acting on charge in uniform electric field

$$\boxed{\vec{F} = q\vec{E}}$$

a. Positive charge moves in the direction of electric field

b. Negative charge moves against the direction of field.



ii. Force acting on charge in uniform magnetic field

$$\boxed{\vec{F} = q(\vec{v} \times \vec{B})}$$

$$F = qvB\sin\theta$$

Case 1: When charge is stationary (Rest) in magnetic field ,v = 0

$\therefore$ $\boxed{F = 0}$

Case 2 : When charge is moving parallel to magnetic field , $\theta = 0$

$\therefore$ $\boxed{F = 0}$

Case 3: When a charge is moving perpendicular to magnetic field , $\theta = 90^0$

$\therefore$ $\boxed{F_{max} = qvB}$

This force provides necessary centripetal force due to this charge trace circular path in uniform magnetic field

Case 4: When a charge is moving making some angle θ ***in uniform magnetic field***

$$\boxed{F = qvB\sin\theta}$$

Due to this force, charge trace helical path

Q.2 What is Lorentz force? Obtain equation of Lorentz force law and Discuss consequences of Lorentz force law.

Ans:

i. When a charged particle moves through a region in which both electric and magnetic field are present, then the net force experienced by that charged particle is sum of electrostatic force and magnetic force.This force is called as Lorentz force.

ii. Equation of Lorentz Force law:

a. Force acting on charge in uniform electric field

$$\vec{F}_e = q\vec{E}$$

b. Force acting on charge in uniform magnetic field

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

c. Magnetic force on charge q in both electric and magnetic field

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

$$\vec{F} = q\left[\vec{E} + (\vec{v} \times \vec{B})\right]$$

This is the required equation of Lorentz force.

Specified cases:

a. If the velocity $\vec{v}$ of a charged particle is parallel to the magnetic field $\vec{B}$, the magnetic force is zero

b. If the charge is stationary, $\vec{v} = 0$, the force = 0

Q.3 Explain why magnetic force never does any work on moving charges.

Ans:

i. Magnetic force is given by $\vec{F} = q(\vec{v} \times \vec{B})$

ii. Then $\vec{v} \times \vec{B}$ will be a vector perpendicular to the plane containing the vectors $\vec{v}$ and $\vec{B}$

Thus the vectors $\vec{v}$ and $\vec{F}$ are always perpendicular to each other.

iii. Magnetic force is in turn perpendicular to displacement of charged particles.

work $= \vec{F}.\vec{s} = Fs\cos 90 = 0$

Hence, magnetic force never does any work on moving charges.

Q.4 Give units and dimension of magnetic field.

Ans:

i. SI unit of B is Tesla or $wb/m^{2.}$

ii. Convenient unit of B is Gauss

1tesla = 10^4 gauss

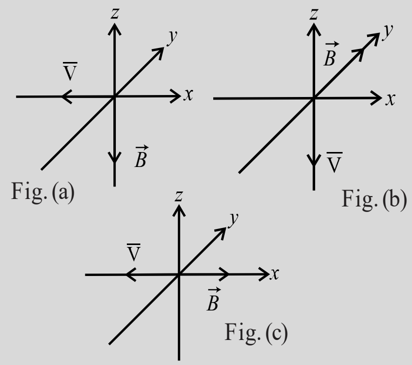
iii. Dimension of B is

$$[B] = \frac{[F]}{[q][v]} = \frac{[M^1L^1T^{-2}]}{[I^1T^1][M^0L^1T^{-2}]} = [M^1L^0T^{-1}I^{-1}]$$

INTEXT QUESTION

A charged particle travels with a velocity $\vec{v}$ through a uniform magnetic $\vec{B}$ field as shown the following figure, in three different situations.

What is the direction of the magnetic force $\vec{F}_m$ due to the magnetic field, on the particle?



Ans:

i. In figure (a), the direction of the vector $\vec{v} \times \vec{B}$ will be in the positive y direction. Hence, $\vec{F}_m$ will be in the **positive y direction.**

ii. In figure (b), $\vec{v} \times \vec{B}$ will be in the positive x direction. Hence the force $\vec{F}_m$ will be in the **positive x direction.**

iii. In figure (c), $\vec{v}$ and $\vec{B}$ are antiparallel, the angle between them is 180^o. As $\sin 180^o = 0$, $\vec{F}_m$ will be **equal to zero**.

Key Points

Consider a situation when force due to magnetic field is same as force due to electric field

$$qE = qvB$$

therefore, $v = \dfrac{E}{B}$

in this situation particle will pass through the field without any deviation in path

Type - I
Numerical based on Force acting on charged particle in magnetic field

Formulae used

1. $F = qvB\sin\theta$
2. $\vec{F} = q(\vec{v} \times \vec{B})$

1) A positive ion ($q = 3.2 \times 10^{-19}$ C) **from cosmic rays enters the earth's magnetic field of 1.6×10^{-6} wb/m^2 in a direction perpendicular to the field. If the force exerted on the ion by the magnetic field is 3.2×10^{-18} N. Find the speed of the ion.**

Data : $q = 3.2 \times 10^{-19}$ C, $B = 1.6 \times 10^{-6}$ wb/m^2

$F = 3.2 \times 10^{-18}$ N, $\theta = 0$

To Find: v

Formula: $F = q\,v\,B\sin\theta$

$$v = \frac{F}{q\,B\sin\theta}$$

Solution : $v = \dfrac{F}{q\,B\sin\theta}$

$$v = \frac{\cancel{3.2} \times 10^{-18}}{\cancel{3.2} \times 10^{-19} \times 1.6 \times 10^{-6} \times \sin 90^0}$$

$$= \frac{1}{1.6} \times 10^7 = 0.625 \times 10^7 \text{ m/s}$$

Ans : The speed of ion is 6.25×10^6 m/s

2) **A proton enters magnetic field of flux density 2.5 T with a velocity of 1.5×10^7 m/s at angle 30^0 with the field. Find the force on proton**

Data: $q = e = 1.6 \times 10^{-19}$ C, $B = 2.5$ T

$v = 1.5 \times 10^7$ m/s, $\theta = 30^0$

To find: F

Formula: $F = qvB\sin\theta$

Solution: $F = qvB\sin\theta$

$= 1.6 \times 10^{-19}\text{C} \times 1.5 \times 10^7 \times 2.5 \times \sin 30$

$= 3 \times 10^{-12}$ N

Ans : Force acting on proton is 3×10^{-12} N

Problem for Practice

1. A proton enters a magnetic field of flux density 1.5 T with a speed of 2×10^7 m/s at

angle of 30^0 with the field. Find the force on the proton

Ans : 2.4×10^{-12} N

2. An electron is moving northwards with a velocity of 10^7 m/s in a magnetic field of 3T, directed eastward. Calculate the instantaneous force on the electron.

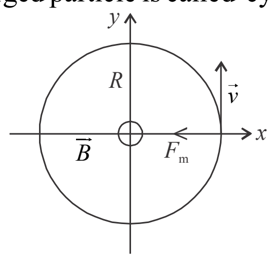
Ans : 4.8×10^{-12} N, vertically upward

10.3 Cyclotron motion

Q.5 What is Cyclotron motion. Obtain formula for Cyclotron motion.

Ans :

i. When a charged particle is projected into uniform magnetic field with its initial velocity perpendicular to field, the magnetic force acts on the charged particle perpendicular to both the magnetic field and the direction of motion. This force produces centripetal force to make the particle move in circular path. This motion of charged particle is called cyclotron motion.



ii. Consider a uniform magnetic field directed perpendicularly into the plane of the paper (parallel to the -ve z axis).
Consider a particle with charge q moving with a speed v in the plane of paper.

iii. When the initial velocity is perpendicular to the magnetic field. Here $\theta = 90^0$,
so $F = qvB \sin 90^0 = qvB$ = a maximum force.

iv. This force continuously deflects the particle sideways without changing its speed and the particle will move along a circle perpendicular to the field. Thus the magnetic force provides the centripetal force.
Let R be the radius of the circular path.
Now
Centripetal force = Magnetic force

$$\frac{mv^2}{R} = qvB$$

$$\boxed{p = mv = qBR}$$

This equation is known as cyclotron formula

Q.6 What is cyclotron? Discuss the principle, construction, working of a cyclotron. What is maximum kinetic energy acquired by the accelerated charged particles?

Ans :

i. **Cyclotron :** The cyclotron is a charged particle accelerator, accelerating charged particles to high energies. It was invented by Lawrence and Livingston in the year 1934

ii. **Principle :** Both electric as well as magnetic fields are used in a cyclotron, in combination. These are applied in directions perpendicular to each other and hence they are called **crossed fields. The magnetic field puts the particle (ion) into circular path and a high frequency electric field accelerates it.** Frequency of revolution of a charged particle is independent of its energy, in a magnetic field.

iii. **Construction :**

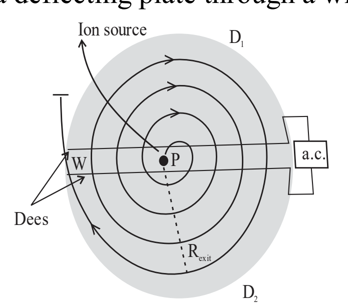
a. Cyclotron consists of two semi-circular disc-like metal chambers, D_1 and D_2, called the dees (Ds).

b. A uniform magnetic field B is applied perpendicular to plane of the Ds. This magnetic field is produced using an electromagnet producing a field upto 1.5 T

c. The Ds are connected to the source of high frequency alternating voltage.
(10000 V at high frequency, 10 MHz)

d. Positive ions are produced by a gas ionizing source kept at the point O in between the two Ds.

e. The charged particle is pulled out of Ds by a deflecting plate through a window.



Working.

a. Suppose a positive ion, say a proton, enters the gap between the two Ds and finds dee D_1 to be negative.

b. It gets accelerated towards dee D_1. As it enters the dee D_1, it does not experience any electric field due to shielding effect of the metallic dee.

c. The perpendicular magnetic field throws it into a circular path. At the instant the proton comes out of dee D_1, it finds dee D_1 positive and dee D_2 negative.

d. It now gets accelerated towards dee D_2. It moves faster through D_2 describing a larger semicircle than before.

e. Thus if the frequency of the applied voltage is kept exactly the same as the frequency of revolution of the proton, then every time the proton reaches the gap between the two Ds, the electric field is reversed and proton receives a push and finally it acquires very high energy. This is called the cyclotron's resonance condition.

f. The proton follows a spiral path. The accelerated proton is ejected through a window by a deflecting voltage and hits the target.

Mathematical Equation

a. Radius of circular path

Let r be the radius of the circular path.
Now
Centripetal force = Magnetic force

$$\frac{mv^2}{r} = qvB$$

$$mv = qBR$$

$$\boxed{r = \frac{mv^2}{qB}}$$

Thus the radius of the circular orbit is inversely proportional to the specific charge (charge to mass ratio q / m) and to the magnetic field.

b. Period of revolution

$$\text{Period of revolution} = \frac{\text{Circumference}}{\text{Speed}}$$

$$\boxed{T = \frac{2\pi r}{v} = \frac{2\pi}{v}\cdot\frac{mv}{qB} = \frac{2\pi m}{qB}}$$

Clearly, the time period is independent of v and r. If the particle moves faster, the radius is larger, it has to move along a larger circle so that the time taken is the same.

c. Frequency of revolution

The frequency of revolution is

$$\boxed{f_c = \frac{1}{T} = \frac{qB}{2\pi m}}$$

This frequency is called cyclotron frequency.

d. K.E of proton

The ions do not experience any electric field while they travel within the D. Their kinetic energy increases by eV every time they cross over from one D to the other. Here V is the voltage difference across the gap. The ions move in circular path with successively larger and larger radius to a maximum radius at which they are deflected by a magnetic field so that they can be extracted through an exit slit

$$v = \frac{qBR_{exit}}{m}$$

where R_{exit} is the radius of the path at the exit.

The kinetic energy of the ions/ protons will be

$$\boxed{K.E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{q^2B^2R_{exit}^2}{2m}}$$

Thus the final energy is proportional to the square of the radius of the outermost circular path (Rexit).

Note:

Limitations of cyclotron:

1. *According to the Einstein's special theory of relativity, the mass of a particle increases with the increases in its velocity as*

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the rest mass of the particle. At high velocities, the cyclotron frequency ($f_c = qB/2\pi m$) will decrease due to increase in mass. This will throw the particles out of resonance with the oscillating field. That is, as the ions reach the gap between the dees, the polarity of the dees is not reversed at that instant. Consequently the ions are not accelerated further.

The above drawback is overcome either by increasing magnetic field as in a ***synchrotron*** *or by decreasing the frequency of the alternating electric field as in a* ***synchro-cyclotron****.*

2. *Electrons cannot be accelerated in a cyclotron. A large increase in their energy increases their velocity to a very large extent. This throws the electrons out of step with the oscillating field.*
3. *Neutrons, being electrically neutral, cannot be accelerated in a cyclotron.*

Uses of cyclotron :

1. *The high energy particles produced in a cyclotron are used to bombard nuclei and study the resulting nuclear reactions and hence investigate nuclear structure.*
2. *The high energy particles are used to produce other high energy particles, such as neutrons by collisions. These fast neutrons are used in atomic reactors.*
3. *It is used to implant ions into solids and modify their properties or even synthesise new materials.*
4. *It is used to produce radioactive isotopes which are used in hospitals for diagnosis and treatment.*

Type - II

Numerical based on cyclotron

Formulae used

1. $v = \frac{qBR}{m}$

2. $r = \frac{mv^2}{qB}$

3. $T = \frac{2\pi m}{qB}$

4. $f_c = \frac{1}{T} = \frac{qB}{2\pi m}$

5. Time interval for reversing Ds (t)

$$t = \frac{T}{2} = \frac{\pi m}{qB}$$

6. $K.E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{q^2B^2R^2}{2m}$

★**1) For proton acceleration, a cyclotron is used in which a magnetic field of 1.4 wb/m² is applied. Find the time period for reversing the electric field between the two Ds.**

Data : $B = 1.4$ T, $m_p = 1.67 \times 10^{-27}$ kg

$q_p = e = 1.6 \times 10^{-19}$ C

To find: Time interval for reversing Ds (t)

Formula: $t = \frac{T}{2} = \frac{\pi m}{qB}$

Solution: $t = \frac{\pi m}{qB}$

$$t = \frac{3.142 \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 1.4}$$

= Anti log [table] $\times 10^{-8}$

log N		log D	
log 3.14	0.4969	log 1.6	0.2041
log 1.67	0.2227	log 1.4	0.1461
	0.7196		0.3502

$$= \text{Anti log}\begin{bmatrix} 0.7196 \\ -0.3502 \\ \hline 0.3694 \end{bmatrix} \times 10^{-8}$$

$= 2.341 \times 10^{-8}$ s

Ans : The time period is $= 2.341 \times 10^{-8}$ s

★2) **An alpha particle (the nucleus of helium atom) (with charge +2) is accelerated and moves in a vacuum tube with kinetic energy = 10 MeV. It passes through a uniform magnetic field of 1.88 T, and traces a circular path of radius 24.6 cm. Obtain the mass of the alpha particle. (1eV = 1.6×10^{-19}J, charge of electron = 1.6×10^{-19} C)**

Data : $q = +2\,e = +2 \times 1.6 \times 10^{-19}$ c

$KE = 10\text{ MeV} = 10 \times 10^{+6}$ eV

$= 10\times10^{+6} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-12}$ J

$B = 1.88$ T,

$R = 24.6\text{ cm} = 24.6 \times 10^{-2}$ m

To find: Mass of alpha-particle (m)

Formula: $K.E = \dfrac{q^2 B^2 R^2}{2m}$

$$m = \frac{q^2 B^2 R^2}{2(K.E.)}$$

Solution: $m = \dfrac{q^2 B^2 R^2}{2(K.E.)}$

$$m = \frac{(3.2\times10^{-19})^2 \times (1.88)^2 \times (24.6\times10^{-2})^2}{2\times1.6\times10^{-12}}$$

$$m = \frac{\cancel{3.2} \times 3.2 \times (1.88)^2 \times (24.6)^2 \times 10^{-38-4+12}}{\cancel{3.2}}$$

$= (3.2) \times (1.88)^2 \times (24.6)^2 \times 10^{-30}$

= At log

log 3.2	0.5051
log 1.88	0.2742
log 1.88	0.2742
log 24.6	+1.3909
log 24.6	1.3909
	3.8353

$\times 10^{-30}$

$= 6.844 \times 10^{-30+3}$

$= 6.844 \times 10^{-27}$ kg.

Ans : mass of α particle is 6.844×10^{-27}kg

★3) **In a cyclotron protons are to be accelerated. Radius of its D is 60 cm. and its oscillator frequency is 10 MHz. What will be the kinetic energy of the proton thus accelerated? (Proton mass = 1.67×10^{-27} kg, e = 1.60×10^{-19} C, eV = 1.6×10^{-19} J)**

Data: $R = 60\text{ cm} = 0.6$ m,

$f = 10\text{ MHz} = 10^7$ Hz,

$m_p = 1.67 \times 10^{-27}$ kh,

$q = 1.6 \times 10^{-19}$ C,

To find: Kinetic energy of proton

Formula: i. $\omega = 2\pi f$

ii. $v = R\omega$

iii. $KE = \dfrac{1}{2} mv^2$

Solution:

i. $\omega = 2\pi f = 2\pi \times 10^7$ rad/s

ii. $V = R\omega = 0.6 \times 2\pi \times 10^7$

$= 1.2\pi \times 10^7$ m/s

iii. $KE = \dfrac{1}{2} mv^2$

$$KE = \frac{1}{2}\times1.67\times10^{-27} \times (1.2\pi\times10^7)^2$$

$= 0.835 \times 1.44 \times (3.14)^2 \times 10^{-27+14}$

$= 11.85 \times 10^{-13}\text{ J} = 1.185 \times 10^{-12}$ J

$$= \frac{11.85\times10^{-13}}{1.6\times10^{-19}}\,eV = \frac{11.85\times5}{8}\times10^{-13+19}$$

$= 1.481\times5\times10^{-13+19} = 7.406 \times 10^6$ eV

= 7.406 MeV

Ans : The K.E. of the proton is 7.406 MeV

★4) **An electron is moving with a speed 3×10^7 m/s in a magnetic field of 6×10^{-4} T perpendicular to its path? What will be frequency and the energy in keV ? (Given: mass of electron = 9×10^{-31} kg, charge e = 1.6×10^{-19} C, 1 eV = 1.6×10^{-19} J)**

Data: $v = 3 \times 10^7$ m/s, $B = 6 \times 10^{-4}$ T

$m = 9 \times 10^{-31}$ kg, $q = 1.6 \times 10^{-19}$ C

To find: i. Radius of path (r)

ii. Frequency (f)

iii. Energy (in keV)

Formula: i. $r = \frac{mv}{qB}$

ii. $f = \frac{qB}{2\pi m}$

iii. $KE = \frac{1}{2}mv^2$

Solution:

i.. $r = \frac{mv}{qB}$

$r = \frac{9\times10^{-31}\times3\times10^{+7}}{1.6\times10^{-19}\times6\times10^{-4}}$

$= \frac{9}{3.2}\times10^{-31+7+19+4}$

$= 2.812 \times 10^{-1}$ m = 0.2812 m

ii. $f = \frac{qB}{2\pi m}$

$n = \frac{1.6\times10^{-19}\times6\times10^{-4}}{2\times3.142\times9\times10^{-31}}$

$= \frac{1.6}{3.142\times3}\times10^{8} = 16.97 \times 10^{6}$ Hz

= 16.97 MHz

iii. $KE = \frac{1}{2}mv^2$

$K = \frac{1}{2}\times9\times10^{-31}\times(3\times10^{7})^2$ J

$= \frac{81\times10^{-31+14}}{2\times1.6\times10^{-19}}$ eV $= \frac{81}{3.2}\times10^{2}$ eV

$= 25.31 \times 10^{2}$ eV

$= 2.531 \times 10^{3}$ eV = 2.531 keV

Ans : i. The radius of the path is 0.2812 m
ii. The frequency is 16.97 MHz
iii. Energy is 2.531 keV.

Problem for Practice

1. A chamber is maintained at a uniform magnetic field of 5×10^{-3} T. An electron with a speed of 5×10^{7} ms^{-1} enter the chamber in a direction normal to the field. Calculate (i) radius of the path and (ii) frequency of revolution of the electron.

Ans : (i) r = 5.7 cm,
(ii) *f* = 1.4 × 10^{8} Hz.

2. An electron travels in a circular path of radius 20 cm in a magnetic field 2×10^{-3} T. (i) Calculate the speed of the electron. (ii) What is the potential difference through which the electron must be accelerated to acquire this speed?

Ans : v = 7.0 × 10^{7} ms^{-1},
V = 14 kV

3. An electron after being accelerated through a potential difference of 10^{4} V enters a uniform magnetic field of 0.04 T perpendicular to its direction of motion. Calculate the radius of curvature of its trajectory.

Ans : r = 8.43 mm

4. A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons ? If the radius of the 'Ds' is 60 cm, what is the kinetic energy of the proton beam produced by the accelerator? (e = 1.60×10^{-19} C, $m_p = 1.67 \times 10^{-27}$ kg). Express your answer in units of MeV (1 MeV = 1.602×10^{-13} J).

Ans. i. B = 0.66 T,
ii. K.E = 7.4 MeV

5. In a cyclotron, magnetic field of 1.4 wb/m^{2} is used. To accelerate protons, how rapidly should the electric field between the Dees be reversed? (π = 3.142, $M_p = 1.67 \times 10^{-27}$ kg, e = 1.6×10^{-19} C) *(March-2018)*

Ans : 426 × 10^{5} times/s

6. A cyclotron is used to accelerate protons to a kinetic energy of 5 MeV. If the strength of magnetic field in the cyclotron is 2T, find the radius and the frequency needed for the applied alternating voltage of the cyclotron. (Given: Velocity of proton = 3×10^{7} m/s)

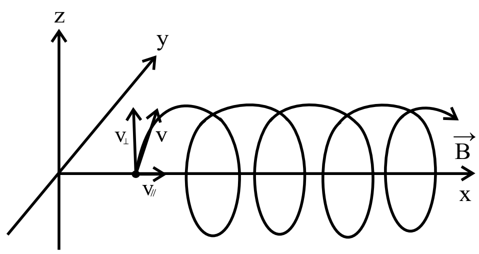
Ans: f = 0.3051 × 10^{8} Hz, r = 0.156 m

10.4 Helical Motion

Q.7 Explain helical motion of charged particle in magnetic field.

Ans:

i. Consider a charged particle q entering a uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ inclined at an angle θ with the direction of $\vec{B}$, as shown in fig.



ii. The velocity $\vec{v}$ can be resolved into two rectangular components:

a. The component $v_{||}$ along the direction of the field i.e., along x-axis. Clearly

$v_{||} = v\cos\theta$

The parallel component remains unaffected by the magnetic field and so the charged particle continues to move along the field with a speed of $v\cos\theta$.

b. The component $v_{\perp}$ perpendicular to the direction of the field i.e., along z-axis. Clearly

$v_{\perp} = v\sin\theta$

Due to this component of velocity, the charged particle experience a force $F = qv_{\perp}B$ which acts perpendicular to both $v_{\perp}$ and $\vec{B}$. This force makes particle to move in circular path

iii. Thus a charged particle moving in a uniform magnetic field has two concurrent motions: a linear motion in the direction of $\vec{B}$ and a circular motion in a plane perpendicular to $\vec{B}$. Hence the resultant path of a charged particle will be helix, with the axis along the direction of $\vec{B}$

Key Points

i. The radius of this helical path is

$$r = \frac{m(v\sin\theta)}{qB}$$

ii. Time period and frequency do not depend on velocity and so they are given by

$$T = \frac{2\pi m}{qB} \text{ and } \nu = \frac{qB}{2\pi m}$$

iii. The *pitch* of the *helix*, (*i.e.*, linear distance travelled in one rotation) will be given by

$$p = T(v\cos\theta) = 2\pi\frac{m}{qB}(v\cos\theta)$$

iv. If pitch value is *p*, then number of pitches obtained in length *l* given as

Number of pitches $= \frac{l}{p}$ and

time required $t = \frac{l}{v\cos\theta}$

MULTIPLE CHOICE QUESTIONS
Entrance Corner (Set 1)

1. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with certain velocity, then

a. its velocity will decrease
b. its velocity will increase
c. it will turn towards right of direction of motion
d. it will turn towards left of direction of motion

2. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in the region. The path of the particle will be

a. ellipse b. circle
c. helix d. straight line

3. A charged particle moves through a magnetic field perpendicular to its direction. Then

a. kinetic energy changes but the momentum is constant
b. the momentum changes but the kinetic energy is constant
c. both momentum and kinetic energy of the particle are not constant
d. both momentum and kinetic energy of the particle are constant

4. If an electron and a proton having same momentum enter perpendicularly to a magnetic field, then

a. curved path of electron and proton will be same (ignoring the sense of revolution)
b. they will move undeflected
c. curved path of electron is more curved than that of proton
d. path of proton is more curved.

5. A particle of charge -16×10^{-18} coulomb moving with velocity 10 ms^{-1} along the X-axis enters a region, where a magnetic field of induction B is along the Y-axis and an electric field of magnitude 10^4 Vm^{-1} is along the negative Z-axis, If the charged particle continues moving along the X-axis, the magnitude of B is

a. 10^3 wb m^{-2} b. 10^5 wb m^{-2}
c. 10^{16} wb m^{-2} d. 10^{-3} wb m^{-2}

6. Electron of mass m and charge q is travelling with a speed v along a circular path of radius r at right angles to a uniform magnetic field of intensity B. If the speed of the electron is doubled and the magnetic field is halved the resulting path would have a radius

a. 2 r b. 4 r
c. $\frac{r}{4}$ d. $\frac{r}{2}$

7. A proton and α-particle are projected normally into a magnetic field. What will be the ratio of radii of the trajectories of the proton and α-particle ?

a. 2 : 1 b. 1 : 2
c. 4 : 1 d. 1 : 4

8. An electric field of 1500 V/m and a magnetic field of 0.40 wb/m^2 act on a moving electron. The minimum uniform speed along a straight line the electron could have is

a. 1.6×10^{15} m/s b. 6×10^{-16} m/s
c. 3.75×10^3 m/s d. 3.75×10^2 m/s

9. A proton is about 1840 times heavier than an electron. Then it is accelerated by a potential difference of 1 kV, its kinetic energy will be

a. 1,840 keV b. $\frac{1}{1,840}$ keV
c. 1 keV d. 920 keV

10. A deuteron of kinetic energy 50 keV is describing a circular orbit of radius 0.5 m in a plane perpendicular to magnetic field $\vec{B}$. The kinetic energy of the proton that describes a circular orbit of radius 0.5 m in the same plane with the same $\vec{B}$ is

a. 25 keV b. 50 keV
c. 200 keV d. 100 keV

Try Yourself

11. A particle of mass M and charge Q moving with velocity *v* describes a circular path of radius R, when subjected to a uniform transverse magnetic field of induction B. The work done by the field, when the particle completes one full circle is

a. $\left(\frac{Mv^2}{R}\right) \times 2\pi R$ b. zero
c. BQ (2 π R) d. BQv (2 π R)

12. A charged particle of mass m and charge q moves along a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is

a. $\frac{2\pi mq}{B}$ b. $\frac{2\pi q^2 B}{m}$
c. $\frac{2\pi qB}{m}$ d. $\frac{2\pi m}{qB}$

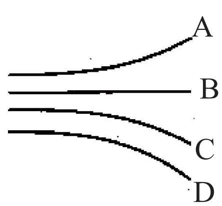
13. The time period of charged particle undergoing a circular motion in a uniform magnetic field is independent of its
a. speed
b. mass
c. charge
d. magnetic induction

14. A very high magnetic field is applied to a stationary charge. Then, the charge experiences.
a. a force in the direction of magnetic field
b. a force perpendicular to the magnetic field
c. a force in an arbitrary direction
d. no force

15. An electron enters into a magnetic field at an angle of 60°, its path will be
a. straight line b. circle
c. parabola d. helix

16. A proton and an α-particle with the same velocity enter into a uniform magnetic field, acting normal to the plane of their motion. The ratio of the radii of the circular paths described by the proton and α-particle is
a. 1 : 2 b. 1 : 4
c. 1 : 16 d. 4 : 1

17. The following diagram which particles has the highest e/m value ?



a. A b. B
c. C d. D

18. A uniform magnetic field acts at right angles to the direction of motion of electron. As a result, the electron moves in a circular path of radius 2 cm. If the speed of the electrons is doubled, then the radius of the circular path will be
a. 2.0 cm b. 0.5 cm
c. 4.0 cm d. 1.0 cm

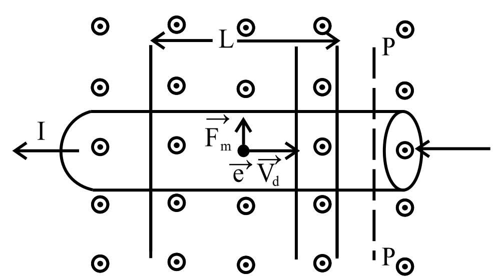
19. In a cyclotron, if a deuteron can gain an energy of 40 MeV, then a proton can gain an energy of
a. 40 MeV b. 80 MeV
c. 20 MeV d. 60 MeV

20. A proton and an α-particle follow the same circular path in a transverse magnetic field. Their kinetic energies are in the ratio
a. 1 : 4 b. $1:\sqrt{2}$
c. 1 : 2 d. 1 : 1

10.5 Magnetic force on a wire carrying a current

Q.8 Derive the relation for magnetic force acting on a straight wire carrying current.

i. Consider a straight wire of length L. An external magnetic field $\vec{B}$ is applied perpendicular to the wire, coming out of the plane of the paper.
Let a current I flow through the wire under an applied potential difference.



ii. If $\vec{v}_d$ is the drift velocity of conduction electrons in the part of length L of the wire, the charge q flowing across the plane of paper in time t will be, q = I t

∴ $q = \frac{IL}{V_d}$ $\left[\because t = \frac{L}{V_d}\right]$

iii. The magnetic force $\vec{F}_m$ on this charge, due to the applied magnetic field $\vec{B}$ is

$$\vec{F}_m = q(\vec{v}_d \times \vec{B}) = qv_d B \sin\theta \hat{n} \quad \text{... (1)}$$

Where $\hat{n}$ is a unit vector perpendicular to both $\vec{B}$ and $\vec{v}_d$,

iv. Substituting value of q in equation (1) we get,

$$\vec{F}_m = \left[\frac{IL}{\cancel{v_d}}\right] B \cancel{v_d} \sin\theta \hat{n}$$

$$\vec{F}_m = ILB \sin\theta \hat{n}$$

As $\vec{B}$ and $\vec{v}_d$ are perpendicular, $\theta = 90^0$

$$\vec{F}_m = ILB \sin 90 \hat{n} = ILB \hat{n}$$

This is, therefore, the magnetic force acting on the portion of the straight wire having length L.

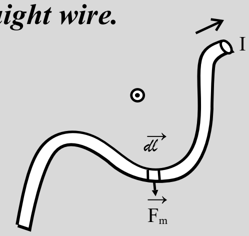
v. If $\vec{B}$ is not perpendicular to the wire, then magnetic force is,

$$\boxed{\vec{F}_m = I(\vec{L} \times \vec{B})}$$

Where, $\vec{L}$ is the length vector directed along the portion of the wire of length L.

Note:

Relation for magnetic force acting on an arbitrarily shaped wire assuming relation for a straight wire.



i. Consider a segment of infinitesimal length dl along the wire. If I in the current flowing, the magnetic force due to perpendicular magnetic field $\vec{B}$ (coming out of the plane of the paper) is given by

$$d\vec{F}_m = I(d\vec{l} \times \vec{B})$$

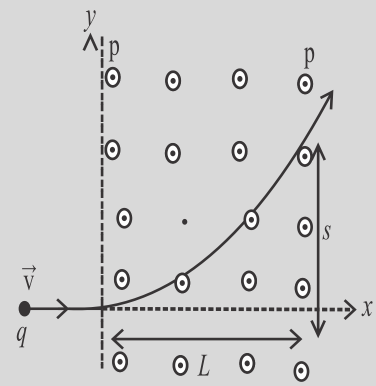
iii. The force on the total length of wire

$$\vec{F}_m = \int d\vec{F}_m = I(\int d\vec{l} \times \vec{B})$$

INTEXT QUESTION

A particle of charge q follows a trajectory as shown in figure. Obtain the type of the charge (positive or negatively charged). Obtain the momentum p of the particle in terms of B, L, s, q, where s being the distance travelled by the particle.

Particle trajectory : A uniform magnetic field $\vec{B}$ is applied in the region PP perpendicular to the plane of the paper, coming out of the plane of the paper.



i. From diagram, magnetic field $\vec{B}$ is coming out of the paper .

ii. Since particle is moving in upward direction force must be acting along positive direction of y-axis

iii. The velocity is in positive direction of x-axis

iv. Direction of $\vec{v} \times \vec{B}$ is along negative direction of y-axis . As force is acting on +y-axis which is opposite to that of $\vec{v} \times \vec{B}$, **therefore charge must be negative.**

v. Magnitude of force acting on charge is $F_{max} = qvB$ (along +y-axis)

vi. Acceleration of particle is given by

$$a = \frac{F_{max}}{m} = \frac{qvB}{m}$$

where m is mass of the charged particle

vii. To calculate distance (s) travelled by particle in y direction we apply 2nd Kinematic equation

$$s = ut + \frac{1}{2}at^2$$

$$\therefore \quad s = (0)t + \frac{1}{2}\left(\frac{qvB}{m}\right)t^2 = \frac{qvBt^2}{2m} \quad \text{... (1)}$$

viii. During time t, particle travels distance L along x-direction with uniform velocity

$L = vt$

$$t = \frac{L}{v} \quad \text{... (2)}$$

ix. Substituting equation (2) in equation (1),

$$\therefore \quad s = \frac{qvB}{2m}\left(\frac{L}{v}\right)^2 = \frac{qBL^2}{2mv} \qquad \therefore \quad v = \frac{qBL^2}{2ms}$$

x. Momentum,

$$p = mv = \cancel{m} \times \frac{qBL^2}{2\cancel{m}s} = \frac{qBL^2}{2s}$$

This is required relation.

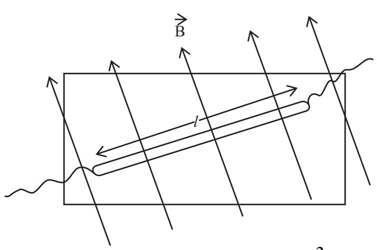
Type - III

Numerical based on force acting on a current carrying conductor in magnetic field

Formulae used

$F = BIl\sin\theta$

★1) A piece of straight wire has mass 20 g and length 1 m. It is to be levitated using a current of 1A flowing through it and a perpendicular magnetic field B in a horizontal direction. What must be the magnitude of B ?



Data: $m = 20 \quad g = 20 \times 10^{-3}$ kg, $\quad l = 1$ m,
$I = 1A, \quad \theta = 90^0$

To find: Magnitude of magnetic field (B)

Formula: i. $F_B = BIl$
ii. $F_G = mg$

Solution: At equilibrium,

$F_B = F_C$

$\therefore \quad BIl = mg$

$$B = \frac{mg}{Il}$$

$$= \frac{20 \times 10^{-3} \times 9.8}{1 \times 1}$$

$= 196 \times 10^{-3} = 0.196$ T

Ans :	Magnitude of magnetic field is 0.196 T

2) The horizontal component of the earth's magnetic field at a certain place is 3.0 × 10^5 T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1 A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is (a) east to west, (b) south to north?

Data: $B = 3.0 \times 10^{-5}$T, $I = 1$ A

To Find: Force per unit length (f)

Formula: $F = IlB \sin\theta$

Solution:

i. The force on a conductor of length l placed in a magnetic field B, and carrying current I, is

$F = IlB \sin\theta$

The force per unit length will be

$$f = \frac{F}{l} = IB\sin\theta$$

where θ is the angle that the conductor makes with the direction of $\vec{B}$.

ii. When the current flows east to west, $\theta = 90°$.

$f = IB \sin 90° = 1 \times 3.0 \times 10^{-5} \times 1$

$= 3.0 \times 10^{-5}\ Nm^{-1}$.

According to Fleming's left hand rule, this force acts vertically downwards.

iii. When the current flows from south to north, $\theta = 0°$

$f = IB \sin 0° = 0$

Thus the force per unit length of the conductor is zero.

Ans : a) When current flows from east to west force per unit length is 3×10^{-5}T
b) When current flows from east to west force per unit length is 0T

Problem for Practice

1. A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field $\vec{B}$. What is the magnitude of magnetic field

Ans: 0.65 T

2. A conductor of length 10 cm is placed perpendicular to uniform magnetic field of strength 100 oersted. If a charge of 5 C passes through it in 5 s. Find frequency of force experienced by the conductor

Ans: 10^{-3} N

3. A straight conductor of length 5 m and carrying a current of 0.1 A is kept in a uniform magnetic field of induction 0.5 Wb/m^2. If the length of the conductor is perpendicular to the direction of the field, find the force acting on the conductor.

Ans. 0.25 N

4. A straight conductor of length 1 m and mass 2 gm is kept horizontal in a uniform magnetic field of induction 2 x 10^{-3} T in the horizontal plane and at right angles to the length of the conductor. Find the current that should be passed through the conductor to balance it.

Ans : 9.8 A

MULTIPLE CHOICE QUESTIONS
Entrance Corner (Set 2)

1. A metallic rod of mass per unit length 0.5 kg m^{-1} is lying horizontally on a smooth inclined plane which makes an angle of 30° with the horizontal. The rod is not allowed to slide down by flowing a current through it when a magnetic field of induction 0.25 T is acting on it in the vertical direction. The current flowing in the rod to keep it stationary is

(a) 7.14 A (b) 5.98 A
(c) 11.32 A (d) 14.76 A

2. A force acting on a conductor of length 5 m carrying a current of 8 A kept perpendicular to the magnetic field of 1.5 T is

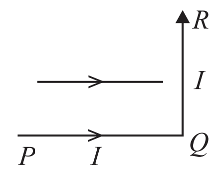
a. 100 N b. 60 N
c. 50 N d. 75 N

3. A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid air by a uniform horizontal magnetic field B.

a. 2 T b. 1.5 G
c. 0.55 G d. 0.65 T

4. A wire PQR is bent as shown in figure and is placed in a region of uniform magnetic field B. The length of PQ = QR = l. A current I ampere flows through the wire as shown. The magnitude of the force on PQ and QR will be

a. BIl, 0
b. 2 BIl, 0
c. 0, BIl
d. 0, 0



5. A wire of length l carries a current i along x-axis. A magnetic field exists given by $B = B_0(\hat{i}+\hat{j}+\hat{k})$ T. The magnitude of the magnetic force acting on the wire is

a. ilB_0 b. $\sqrt{3}\,ilB_0$
c. $2ilB_0$ d. $\sqrt{2}\,ilB_0$

Try yourself

6. A current is flowing in a linear conductor having a length of 40 *cm*. The conductor is placed in a magnetic field of strength 500 *gauss* and makes an angle of 30° with the direction of the field. It experiences a force of magnitude.

(a) 3×10^{4}N (b) 3×10^{2}N
(c) 3×10^{-2}N (d) 3×10^{-4}N

7. If a wire of length 1 meter placed in uniform magnetic field 1.5 Tesla at angle 30° with magnetic field. The current in a wire 10 amp. Then force on a wire will be

(a) 7.5 N (b) 1.5 N
(c) 0.5 N (d) 2.5 N

8. A one metre long wire is lying at right angles to the magnetic field. A force of 1 kg wt. is acting on it in a magnetic field of 0.98 Tesla. The current flowing in it will be

(a) 100 A (b) 10 A
(c) 1 A (d) Zero

9. A straight wire (conductor) of length 10 cm is kept in a uniform magnetic field of induction 0.02 T. The angle between the conductor and the field direction is 30°. A current of 5A is passed through the conductor. The force on the conductor is ___N.

(a) 4×10^{-3} (b) 5×10^{-3}
(c) 6×10^{-3} (d) 7×10^{-3}

10. A long straight wire carrying a current of 30A is placed in an external uniform magnetic field of induction 4×10^{-4}T. The magnetic field is acting parallel. to the direction of current. The magnitude of the resultant magnetic induction in tesla at a point 2.0 cm away from the wire is $[\mu_0 = 4\pi\times10^{-7}\,Hm^{-1}]$

(a) 10^{-4} (b) 3×10^{-4}
(c) 6×10^{-4} (d) 5×10^{-4}

10.6 Force on a closed circuit in magnetic field

Q.10 Shows that for uniform magnetic field, force on a closed circuit is zero

Ans:

i. Consider a closed circuit say of length C.

ii. We know that for an arbitrarily shaped wire,

$$\vec{F}_m = \int d\vec{F}_m = I\left(\int d\vec{l} \times \vec{B}\right)$$

iii. Therefore, for a closed wire circuit C,

$$\vec{F}_m = I\left(\oint d\vec{l} \times \vec{B}\right)$$

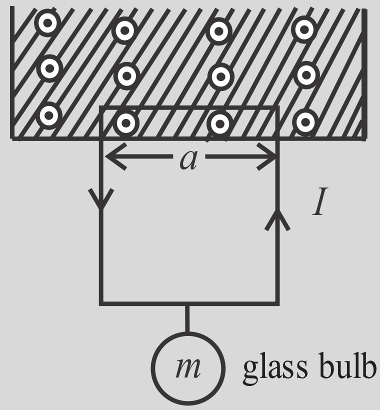
iv. $\oint d\vec{l} = 0 =$ sum of vectors along a closed circuit.

v. This makes, $\vec{F}_m = 0$

Hence, proved.

INTEXT QUESTION

Consider a square loop of wire loaded with a glass bulb of mass m hanging vertically, suspended in air with its one part in a uniform magnetic field $\vec{B}$ with its direction coming out of the plane of the paper ⊙. Due to the current I flowing through the loop, there is a magnetic force in upward direction. Calculate the current I in the loop for which the magnetic force would be exactly balanced by the force on mass m due to gravity.



Solution:

i. The current I in the loop with its part in the magnetic field B causes an upward force F_m in the horizontal part of the loop,

$F_m = IBa$,

Where a is the length of one arm of the loop.

ii. This force is balanced by the force due to gravity.

$\therefore F_m = IBa = mg$

$$\boxed{I = \frac{mg}{Ba}}$$

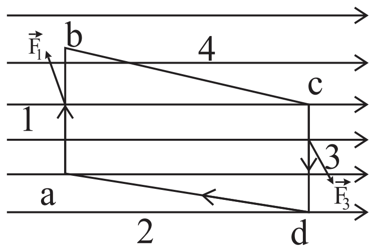
For this current, the wire loop will hang in air.

10.7 Torque on a current loop

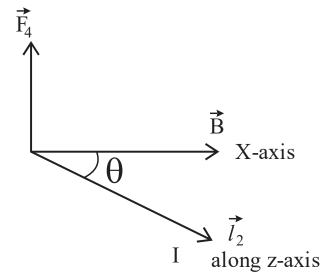
Q.11 Derive expression for torque acting on a plane coil in a uniform magnetic field.

Ans:

i. Consider rectangular loop abcd placed in a uniform magnetic field $\vec{B}$ such that the sides ab and cd are perpendicular to magnetic field but the sides bc and da are not.



ii Let force on side (bc) be F_4.



Magnetic field is along +x-axis and current is along +z-axis, so according to fleming left hand rule, force is acting along the +y-axis

$F_4 = BIl_2$

iii. Let force on side (ad) be F_2.

Magnetic field is along +x-axis and current is along - z-axis, so according to fleming left hand rule, force is acting along the -y-axis.

$F_2 = BIl_2$

iii. It is clear that both force F_4 and F_2 are equal and opposite and acting on same line of action. So they cancel each other.

iv. Let force acting on side (ab) be F_1.

Magnetic field is along +x-axis and current is along +y-axis, so according to fleming left hand rule, force is acting along the -z-axis.

$F_1 = BIl_1$

v. Let force acting on side (cd) be F_3.

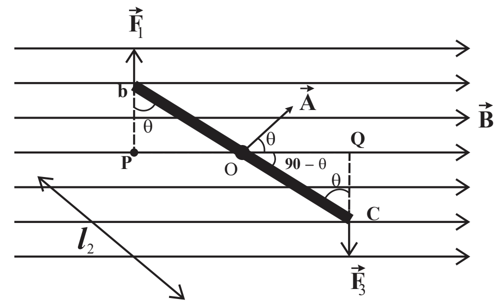
Magnetic field is along +x-axis and current is along -y-axis, so according to fleming left hand rule, force is acting along the +z-axis.

$F_3 = BIl_1$

These two forces do not act along the same line and hence they produce a net torque.

v. This torque results into rotation of the loop so that the loop is perpendicular to the direction of the magnetic field

vi. Let us consider side view of the loop abcd at angle θ



Torque = Force × moment arm

$$\tau = F_1 \times OP + F_2 \times OQ$$

$$\tau = BIl_1 \times \left(\frac{1}{2} l_2 \sin\theta\right) + BIl_1 \times \left(\frac{1}{2} l_2 \sin\theta\right)$$

$$\tau = BIl_1 l_2 \sin\theta$$

vii. If the current carrying loop is made up of multiple turns N, in the form of a flat coil, the total torque will be

$$\tau' = N\tau = NBIl_1 l_2 \sin\theta$$

here $A = l_1 l_2$ is the area enclosed by the coil

viii. The above equation holds good for all flat coils irrespective of their shape, in a uniform magnetic field.

Key Points

i. Torque : Consider a current carrying coil having N turns and area A, placed in a uniform field $\vec{B}$, in such a way that the normal $(\hat{n})$ to the coil makes an angle θ with the direction of. the coil experiences a torque given by

τ **= NBIA sin**θ .

Case 1 : τ is zero when $\theta = 0$, i.e.,when the plane of the coil is perpendicular to the field.

Case 2: τ is maximum when , i.e., the plane of the coil is parallel to the field.

τ_{max} **= BINA**

Type - III

Numerical based on torque acting on coil

Formulae used

1. τ = NIAB sinθ

2. τ_{max} = BINA

1) The maximum torque acting on a coil of effective are 0.02 m^2 is 2×10^{-8} Nm. When the current in it is 100 μA. Find the magnetic induction in which it is kept.

Data : $A = 0.02\ m^2$

$\tau_{max} = 2 \times 10^{-8}$ Nm

$I = 100\ \mu A$

$= 100 \times 10^{-6} = 10^{-4}$ A

$N = 1$

To find : B

Formula : $\tau_{max} = BINA$

Solution : $\tau_{max} = BINA$

$\therefore \quad B = \frac{\tau_{max}}{INA}$

$B = \frac{2\times10^{-8}}{10^{-4}\times1\times2\times10^{-2}}$

$B = 10^{-2}$ T

Ans : Magnetic induction is 10^{-2}T

2) Calculate the torque on a 100 turn rectangular coil of length 10 cm and breadth 20 cm carrying a current of 5 A, when placed making angle 60° with a magnetic field of 2 T

Data : N = 100, l = 10 cm, b = 20 cm

$A = 10 \times 20 = 200\ cm^2$

$= 200 \times 10^{-4}\ m^2 = 2 \times 10^{-2}\ m^2$

I = 5 A

$\theta = 90 - 60° = 30°$

(angle between $\vec{B}$ and normal to coil)

B = 2 T

To find : τ

Formula : $\tau = BINA\sin\theta$

Solution : $\tau = 2 \times 5 \times 100 \times 2 \times 10^{-2} \times \sin 30$

$= 20 \times \frac{1}{2} = 10\,Nm$

Ans : Torque acting on coil is 10 Nm.

Problem for Practice

1. Calculate the torque of 100 turns rectangular coil of length 40 cm and breadth 20 cm carrying a current of 10 A when placed making on angle of 60° with magnetic field of 5 T.

Ans : 346.41 Nm

2. Calculate the maximum torque experienced by a rectangular coil of length 10 cm and breadth 8 cm carrying current of 100 mA and has 1000 turns placed in a region of magnetic field of 0.2 T.

Ans : 0.16 Nm

3. A rectangular coil having length 15 cm breadth 10 cm, is kept in uniform magnetic field of 2×10^5 wb/m^2, The coil carries a current of 100 mA. Compute the torque acting on conductors of length 15 cm, when it is perpendicular to flux density.

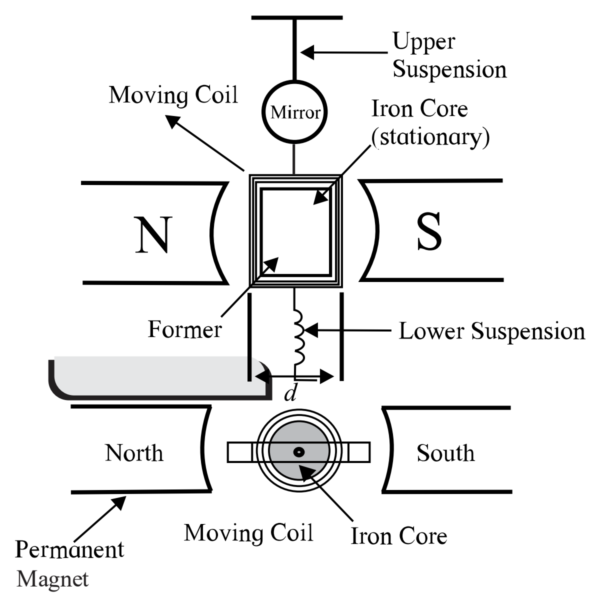
Ans:30×10^{-9} Nm

10.7.1 Moving coil Galvanometer

Q.11 State principle of moving coil galvanometer (M.C.G).Explain construction and working of Moving Coil Galvanometer (M.C.G.) with a neat labelled diagram.

Ans: Principal of moving coil galvanometer:

When a coil carrying an electric current is suspended in a uniform magnetic field, a torque acts on it. This torque tends to rotate the coil about the axis of suspension, so that the magnetic flux passing through the coil is maximum.



Moving coil galvanometer.

Construction

a) **Horse shoe magnet :** A strong horse shoe magnet with concave pole pieces is used to produce a radial magnetic field.

b) **Coil :**

i) A light rectangular coil of thin insulated copper wire with many turns wound on a non-magnetic frame.

ii) It is suspended between the poles of the horseshoe magnet and free to rotate about a vertical axis.

c) **Suspension fibre :**

i) The coil is suspended by means of a thin phosphor-bronze fibre.

ii) The lower end of the coil is connected to phosphor - bronze helical spring.

iii) The current enters the coil through the fibre and leaves through the spring

d) **Iron core :**

i) A soft iron cylinder is fixed, in the gap of the coil.

ii) The coil can freely rotate around the cylinder.

iii) As the permittivity of soft iron is high, the iron core increases the strength of the radial magnetic field.

e) **Lamp-scale arrangement :**

i) A small mirror is fixed to the suspension fiber.

ii) A lamp and scale is arranged in front of the mirror. The reflected bright spot gives the deflection.

f) **Helical spring :**

i) The lower end of coil is connected to phosphor bronze helical spring.

ii) Give to its elastic behaviour it helps the coil to bring back to its original position, when current cut -off

Working:

i. The coil rotates due to a torque acting on it as the current flows through it. Torque acting on current carrying coil is

$\tau = NIAB \sin\theta$

Here $\theta = 90^{o}$ as the field is radial.

$\therefore \quad \tau = NIAB$

Where A is the area of the coil, B the strength of the magnetic field, N the number of turns of the coil and I the current in the coil. This torque is deflecting torque

ii. This torque is counter balanced by a torque due to a spring fitted at the bottom at equillibrium

$$\begin{pmatrix}\text{Deflecting}\\\text{torque}\end{pmatrix} = \begin{pmatrix}\text{Restoring}\\\text{torque}\end{pmatrix}$$

$\therefore \quad \tau_R = nBIA$

iii. The fixed steady current I in the coil produces a steady angular deflection ϕ.Larger the current is, larger is the deflection and larger is the torque due to the spring.

Restoring torque $\alpha\,\phi$

$\therefore \quad \tau_R = K\phi$

$\therefore \quad nBIA = K\phi$

$$\therefore \quad \boxed{\phi = \frac{KI}{nBA}}$$

This means, the deflection ϕ is proportional to the current I

Key Point

Sensitivity of MCG

i. It is defined as the ratio of the change in the deflection of the galvanometer to the change in the current.

Sensitivity of MCG is given by

$$S_i = \frac{d\phi}{dI} \qquad \text{... (1)}$$

ii. The current flowing through MCG is

$$I = \left(\frac{K}{NBA}\right)\phi$$

Differentiating the equation w.r.t. ϕ

$$\therefore \quad dI = \left(\frac{K}{NBA}\right)d\phi \qquad . \qquad \text{... (2)}$$

From equation (i) and (ii)

Differentiating the equation w.r.t. I

$$\therefore \quad S_i = \frac{NBA}{c} \qquad \text{... (3)}$$

iii. Hence, the sensitivity can be increased by,

a. increasing the number of turns (n)

of the coil,

b. increasing the induction (B) of the magnetic field,

c. increasing area (A) of the coil,

d. decreasing torque per unit twist (c) of the fibre.

Voltage Sensitivity

i. It is defined as the deflection produced in the galvanometer when a unit potential difference is applied across its ends.

ii. $V_s = \frac{d\phi}{dV} = \frac{NBA}{KG}$

iii. $voltage\,Sensitivity = \frac{Current\ sensitivity}{G}$

Radial Magnetic Field

i. The radial field ensures that in any position the plane of the coil is parallel to the field, and the current flowing through the coil is directly proportional to the deflection of the coil. i.e., we can have a linear scale.

ii. In the absence of radial field the distance between two forces would be, $b\cos\theta$. The angle made by the plane of the coil with the magnetic induction will go on changing. This results in a varying torque acting on the coil, as the coil rotates.
Therefore, there shall not be a linear relation between the current and the deflection.

Phosphor bronze is used for suspension or hair spring because of several reason

i. It is good conductor of electricity.
ii. It dose not oxidise
iii. It is perfectly elastic
iv. It is non-magnetic
v. Of all material, it has the minimum value for restoring torque per unit twist i.e smallest torsional constant k

Type - IV
Numerical based on MCG

Formulae used

$$I = \left(\frac{K}{NBA}\right)\phi$$

★1) **A moving coil galvanometer has been fitted with a rectangular coil having 50 turns and dimensions 5cm×3 cm. The radial magnetic field in which the coil is suspended is of 0.05 Wb/m^2. The torsional constant of the spring is 1.5×10^{-9} Nm/degree. Obtain the current required to be passed through the galvanometer so as to produce a deflection of 30°.**

Data: $N = 50$ turns,
$A = 5\text{ cm} \times 3\text{ cm} = 15 \times 10^{-4}\text{ m}^2$.
$B = 0.05\text{ Wb/m}^2$,
$K = 1.5 \times 10^{-9}$ Nm/degree, $\phi = 30°$

To find: Electric current

Formula: $I = \frac{K}{NAB}\phi$

Solution: $I = \frac{K}{NAB}\phi$

$$I = \frac{1.5 \times 10^{-9}}{50 \times 15 \times 10^{-4} \times 0.05} \times 30$$

$$= \frac{3}{250} \times 10^{-3}$$

$= 12 \times 10^{-6}$ Hz $= 1.2 \times 10^{-5}$ A

Ans : Required electric current is 1.2×10^{-5}A

2) **A rectangular coil of area 5.0 × 10^{-4} m^2 and 60 turns is pivoted about one of its vertical sides. The coil is in a radial horizontal field of 90 G ('radial' here means the field lines are in the plane of the coil for any orientation). What is the torsional constant of the hair-springs connected to the coil if a current of 0.20 mA produces an angular deflection of 18° ?**

Data : $B = 90\text{ G} = 90 \times 10^{-4}$ T,
$A = 5.0 \times 10^{-4}\text{ m}^2$,
$I = 0.20$ m
$A = 0.20 \times 10^{-3}$ A, $N = 60$, $\phi = 18°$

To find: Torsional constant of the hair spring(K)

Formula: $I = \left(\frac{K}{NBA}\right)\phi$

Solution:

$$k = \frac{NIBA}{\phi}$$

$$= \frac{60\times0.2\times10^{-3}\times90\times10^{-4}\times5\times10^{-4}}{18}$$

$= 3.0 \times 10^{-9}$ Nm deg^{-1}

Ans : Torsional constant is 3.0×10^{-9} Nm deg^{-1}

Problem for Practice

1. A moving coil galvanometer has 10 turns each of length 12 cm and breadth 8 cm. The coil of M.C.G. carries a current of 125 mA. The coil is kept perpendicular to uniform magnetic field of induction 10^{-2} T. The twist constant of phosphor bronze wire is 12×10^{-9} N-m/degree. Calculate the deflection produced.

Ans: $\phi = 10^0$

2. A rectangular coil of MCG has 50 turns and an area of 12 cm^2. It is suspended in a radial magnetic field of induction 0.025 Wb/m^2. The torsional constant of suspension fibre is 1.5×10^{-9} Nm/degree. Calculate the current required to produce a deflection of 5^0.

Ans: 5 μA

3. A rectangular coil of effective area 0.1 m^2 is suspended freely in a radial magnetic field of induction 0.01 Wb/m^2. If the torque per unit twist of the suspension fibre is 5×10^{-9} Nm per degree, find the angle through which coil rotates when a current of 200 μA is passed through it.

Ans: $\theta = 40^0$

10.8 Magnetic dipole moment

Q.12 Write expression of torque acting on rotating current carrying coil in terms of its magnetic dipole moment.

Ans:

i. Magnetic dipole moment of current carrying coil is given by

$\mu = NIA$.... (1)

Where, N is the number of turns of the coil, I the current passing through the coil, A the area enclosed by each turns of the coil.

ii. When a current carrying coil is placed in magnetic field , torque acts on a coil which is given by

$\tau = BINA\sin\theta = (NIA)B\sin\theta = \mu B\sin\theta$

$\therefore \quad \vec{\tau} = \vec{\mu}\times\vec{B}$

Note:

Symbol of magnetic dipole moment is 'm' or 'μ'. Torque acting on a current carrying coil in magnetic field

$\vec{\tau} = \vec{\mu}\times\vec{B}$ or $\vec{\tau} = \vec{m}\times\vec{B}$

The above equation is analogous to equation of torque on an electric dipole exerted by an electric field

$\vec{\tau} = \vec{p}\times\vec{E}$

Where $\vec{p}$ *is electric dipole moment.*

Type - V
Numerical based on magnetic moment of coil

Formulae used

$\tau = \mu B\sin\theta$

$\vec{\tau} = \vec{\mu}\times\vec{B}$

★1) A circular coil of conducting wire has 500 turns and an area 1.26×10^{-4} m^2 is enclosed by the coil. A current 100 μA is passed through the coil. Calculate the magnetic moment of the coil.

Data: N = 500, A = 1.26×10^{-4} m^2, I = 100 μA

To find: Magnetic moment of the coil (μ)

Formula: $\mu = NIA$

Solution : $\mu = NIA$

$\mu = NIA = 500 \times 100 \times 10^{-6} \times 1.26 \times 10^{-4}$

$= 630 \times 10^{-8} = 6.3 \times 10^{-6}$ Am^2

Ans : The magnetic moment of the coil is 6.3×10^{-6} Am^2

Problem for Practice

1. A circular coil of 300 turns and average area 5×10^{-3} m^2 carries a current of 15 A. Calculate the magnitude of magnetic moment associated with coil.

Ans: 22.5 Am^2

2. A circular coil of 500 turns and area of 0.5 cm^2 carries a current of 10A. Find the magnetic moment associated with coil.

Ans: 0.25 Am^2

Q.13 Write a note on magnetic potential energy of a dipole.

Ans:

i. A magnetic dipole freely suspended in a magnetic field possesses magnetic potential energy because of its orientation in the field.

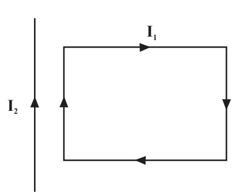
ii. The magnetic potential energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by $U = -\vec{\mu}.\vec{B} = -\mu B\cos\theta$ Where, θ is the angle between $\vec{\mu}$ and $\vec{B}$

Case 1 : If θ = 0, $U = -\mu B \cos 0^o = -\mu B$ In a magnetic field when $\vec{\mu}$ and $\vec{B}$ are parallel to each other, magnetic potential energy of a magnetic dipole is minimum.

Case 2 : If $\theta = 180^o$, $U = -\mu B \cos 180^o = \mu B$ When $\vec{\mu}$ and $\vec{B}$ are antiparallel to each other, magnetic potential energy of a magnetic dipole is maximum.

MULTIPLE CHOICE QUESTIONS
Entrance Corner (Set 3)

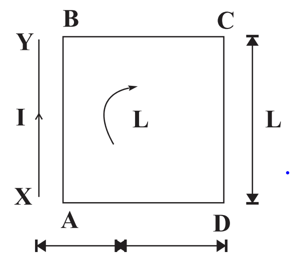
1. A rectangular loop carrying a current I_1 is situated near a long straight wire carrying a steady current I_2. The wire is parallel to one of the sides of the loop and is in the plane of the loop as shown in the figure.



Then the current loop will
a. rotate about an axis parallel to the wire
b. move towards the wire
c. move away from the wire
d. remain stationary.

2. A current loop of area 0.01m^2 and carrying a current of 10 ampere is held perpendicular to a magnetic field of intensity of 0.1 tesla. The torque (in N m) acting on the loop is
a. 0 b. 0.001
c. 0.01 d. 1.1

3. A square loop ABCD carrying a current i, is placed near and coplanar with a long straight conductor XY carrying a current I, the net force on loop will be:



a. $\frac{2\mu_0 Ii}{3\pi}$ b. $\frac{\mu_0 Ii}{2\pi}$
c. $\frac{2\mu_0 IiL}{3\pi}$ d. $\frac{\mu_0 IiL}{2\pi}$

4. A circular coil of radius 4 *cm* and of 20 turns carries a current of 3 amperes. It is placed in a magnetic field of intensity of 0.5 weber/m^2. The magnetic dipole moment of the coil is
a. 0.15 ampere –m^2 b. 0.3
c. 0.45 d. 0.6

5. A rectangular coil of wire of area 400 cm^2 contains 500 turns. It is placed in a magnetic field of induction 4×10^{-3} T and it makes an angle 60° with the field. A current of 0.2 A is passed through it. The torque on the coil is
a) $8\sqrt{3} \times 10^{-3}$ N m b) 8×10^{-3} N m
c) $8\sqrt{3} \times 10^{-4}$ N m d) 8×10^{-4} N m

6. A vertical rectangular coil of sides 5cm x 2cm has 10turns and carries a current of 2A.The torque(couple) on the coil when it is placed

in a uniform horizontal magnetic field of 0.1T with its plane perpendicular to the field is

a) 4×10^{-3}N-m b) Zero

c) 2×10^{-3}N-m d) 10^{-3}N-m

7. A rectangular coil of length 10cm and breadth 20cm is placed in uniform magnetic field of induction 20 wb/m^2. A current of 2A is passed through the coil. If it consists of 100 turns, the maximum torque experienced is

a) 40Nm b) 80Nm

c) 4000Nm d) 8000Nm

8. The area of the coil in a moving coil galvanometer is 15 cm^2 and has 20 turns. The magnetic induction is 0.2 T and the couple per unit twist of the suspended wire is $10^{-6}\,Nm$ per degree. If the deflection is 45^0, the current passing through it is

a) $75\times10^{-4}\,A$ b) $7.5\times10^{-4}\,A$

c) $0.75\times10^{-4}\,A$ d) $750\times10^{-4}\,A$

9. The area of the coil in a moving coil galvanometer is 80 cm^2 and it has 200 turns. The magnetic induction of the radial field is 0.2 T and the couple per unit twist of the suspension wire is 2×10^{-6} Nm per degree. If the deflection is 4°, the current passing through it is

a) 0.25 m A b) 2.5 m A

c) 0.025 mA d) 250 mA

10. The coil in a MCG has an area of 4 cm^2 and 500 turns. The intensity of magnetic induction is 2T. When a current of 10^{-4} A is passed through it, the deflection is 20^0. The couple per unit twist is (N-m)

a) 3×10^{-6} b) 2×10^{-6}

c) 4×10^{-6} d) 5×10^{-6}

Try yourself

11. In order to increase the sensitivity of a moving coil galvanometer, one should decrease

a. The strength of its magnet

b. The torsional constant of its suspension

c. The number of turns in its coil

d. The area of its coil

12. A circular loop of area 0.01 m2 carrying a current of 10 A, is held perpendicular to a magnetic field of intensity 0.1 T . The torque acting on the loop is

a. Zero b. 0.001Nm

c. 0.01 Nm d. 0.8 Nm

13. A current loop placed in a non-uniform magnetic field experiences

a. a force of repulsion

b. a force of attraction

c. a force but not force

d. a force and a torque

14. A current carrying coil is subjected to a uniform magnetic field. The coil will orient so that its plane becomes

a. inclined to 45^o to the magnetic field

b. inclined at any arbitrary angle to the magnetic field

c. parallel to the magnetic field

d. perpendicular to magnetic field

15. A conducting circular loop of radius *r* carries a constant current *i*. It is placed in a uniform magnetic field $\vec{B}$, such that is perpendicular to the plane of the loop. The magnetic force acting on the loop is

a. $ir\vec{B}$ b. $2\pi ri\vec{B}$

c. Zero d. $\pi ri\vec{B}$

16. The radius of a circular loop is *r* and a current *i* is flowing in it. The equivalent magnetic moment will be

a. ir b. $2\pi ir$

c. $i\pi r^2$ d. $\frac{1}{r^2}$

17. A circular loop has a radius of 5 *cm* and it is carrying a current of 0.1 amp. Its magnetic moment is

a. $1.32\times10^{-4}\,amp-m^2$

b. $2.62\times10^{-4}\,amp\text{-}m^2$

c. $5.25\times10^{-4}\,amp\text{-}m^2$

d. $7.85\times10^{-4}\,amp\text{-}m^2$

18. A current carrying loop is placed in a uniform magnetic field. The torque acting on it does not depend upon

a. Shape of the loop b. Area of the loop
c. Value of the current d. Magnetic field

19. To make the field radial in a moving coil galvanometer

a. The number of turns in the coil is increased
b. Magnet is taken in the form of horse-shoe
c. Poles are cylindrically cut
d. Coil is wounded on aluminium frame

20. In a moving coil galvanometer, the deflection of the coil θ is related to the electrical current i by the relation

a. $i \propto \tan\theta$ b. $i \propto \theta$
c. $i \propto \theta^2$ d. $i \propto \sqrt{\theta}$

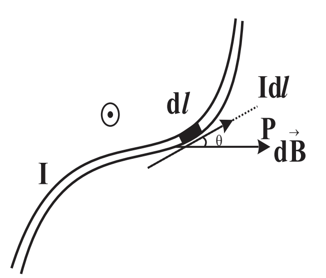
10.10 Magnetic field due to current Biot-Savart law

Q.14 Explain Biot-Savart law.

Ans:

i. Consider an arbitrarily shaped wire carrying a current I.

ii. Let dl be a length element along the wire. The current in this element is in the direction of the length vector $\vec{dl}$ which produces differential magnetic field $\vec{dB}$ directed into the plane of paper as shown in figure below: Let the position vector of point P relative to element $\vec{dl}$ be $\vec{r}$. Let θ be angle between $\vec{dl}$ and $\vec{r}$.



iii. According to Biot-Savart's law which was formulated empirically in 1820,

a. $dB \propto I$ (The field is proportional to current).

b. $dB \propto d\ell$ (The field is proportional to the length of the current element)

c. $dB \propto \sin\theta$, (The field is directly proportional to sine of angle between $\vec{dl}$ and $\vec{r}$)

iv. $dB \propto \frac{1}{r^2}$, (The field falls off inversely with the square of the distance between the source of the field and the point at which the field is to be measured. It is also called **inverse square law**.)

Combining all the four factors, we get

$$dB \propto \frac{I\,d\ell \sin\theta}{r^2}$$

$$\therefore \quad dB = k\frac{I\,d\ell \sin\theta}{r^2}$$

where k is a constant of proportionality whose value depends on the medium
for free space and SI unit

$$K = \frac{\mu_0}{4\pi} \text{ wb/Am}$$

$$\therefore \quad dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell \sin\theta}{r^2}$$

In vector notation,

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I(\overrightarrow{d\ell} \times \vec{r})}{r^3}$$

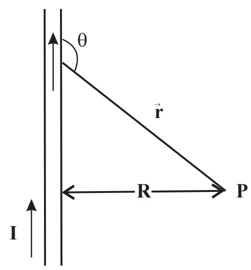
The direction of $\overrightarrow{dB}$ is the same as the direction $\overrightarrow{d\ell} \times \vec{r}$ and is given by right hand rule for the cross product of vectors. $\overrightarrow{dB}$ is perpendicular to both $\overrightarrow{d\ell}$ and $\vec{r}$

Q.15 Derive expression of magnetic field at a point near infinitely long straight wire. How will expression change for semi-infinite straight wire?

Ans:

i. Consider a straight wire of length l carrying current I.

ii. Let a point P situated at a perpendicular distance R form the wire as shown below.



iii. Consider infinitesimal length $\vec{d}l$ of wire carrying current I, then current element $= I\,\overrightarrow{dl}$.

iv. Current element is situated at distance r from point P making an angle θ, as shown in figure above.

v. Using Biot Savart law, magnetic field, produced $\overrightarrow{dB}$ at P due to current element $I\vec{d}l$ is,

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{d}l\sin\theta}{r^2} \qquad \ldots(1)$$

vi. According to properties of cross-product, $\vec{d}l \times \vec{r}$ indicates direction of $\overrightarrow{dB}$, in this case, is into the plane of paper.

vii. Summing up all current elements upper half of infinitely long wire,

$$B_{upper} = \int_0^\infty dB = \frac{\mu_0}{4\pi}\int_0^\infty \frac{I\vec{d}l\sin\theta}{r^2} \qquad \ldots(2)$$

viii. Taking into account symmetry of wire, current elements in lower half of infinitely long wire will also contribute same as upper half. i.e.,
$B_{lower} = B_{upper}$ ...(3)

ix. Adding contributions from lower and upper part, total magnetic field point P is

$$B = 2\int_0^\infty dB \qquad \ldots[\text{From (2)}]$$

$$= \frac{2\mu_0}{4\pi}\int_0^\infty \frac{Idl\sin\theta}{r^2} \qquad \ldots[\text{From (1)}]$$

But $r = \sqrt{l^2 + R^2}$ and

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{r} = \frac{R}{\sqrt{l^2 + R^2}}$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi}\int_0^\infty \frac{R\,dl}{(l^2 + R^2)\sqrt{l^2 + R^2}} = \frac{\mu_0 I}{2\pi}R\int_0^\infty \frac{dl}{(l^2 + R^2)^{\frac{3}{2}}}$$

Solving the integration,

$$B = \frac{\mu_0 I}{2\pi}R \times \frac{1}{R^2} = \frac{\mu_0 I}{2\pi R} \qquad \ldots(4)$$

This is the equation for magnetic field at a point situated at a perpendicular distance R from infinitely long wire carrying current I.

For semi-infinite straight wire:

For semi-infinite straight wire, instead of contributions from both upper and lower parts, only either part will contribute.

Hence, magnetic field for semi-infinite wire will be half of that of infinite wire. i.e.,

$$B_{semi} = \frac{B}{2} = \frac{\mu_0 I}{2\pi R} \times \frac{1}{2} = \frac{\mu_0 I}{4\pi R} \qquad \ldots[\text{From (4)}]$$

Type - VI

Numerical based on magnetic field due to straight conductor

Formula Used

$$B = \frac{\mu_0 I}{2\pi r}$$

★1) Calculate the value of magnetic field at a distance of 2 cm from a very long straight wire carrying a current of 5 A. (Given: $\mu_0 = 4\pi \times 10^{-7}$ Wb/Am).

Data: $I = 5$ A, $R = 2$ cm $= 2 \times 10^{-2}$ m
$\mu_0 = 4\pi \times 10^{-7}$ Wb/Am

To find: (B) Magnetic field

Formula: $B = \frac{\mu_0 I}{2\pi R}$

Solution:

$$B = \frac{\mu_0 I}{2\pi R}$$

$$B = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 2 \times 10^{-2}} = 5 \times 10^{-5}\ \text{T}$$

Ans: The value of magnetic field is 5×10^{-5} T

★2) A very long straight wire carries a current 5.2 A. What is the magnitude of the magnetic field at a distance 3.1 cm from

the wire? (Given: $\mu_0 = 4\pi \times 10^{-7}$ T m/A).

Date: $I = 5.2$ A,
$R = 3.1$ cm $= 3.1 \times 10^{-2}$ m,
$\mu_0 = 4\pi \times 10^{-7}$ T m/A

To find: B

Formula: $B = \dfrac{\mu_0 I}{2\pi R}$

Solution: $B = \dfrac{\mu_0 I}{2\pi R}$

$$B = \frac{4\pi \times 10^{-7} \times 5.2}{2 \times \pi \times 3.1 \times 10^{-2}} = \frac{10.4}{3.1} \times 10^{-7+2}$$

$= 3.3548 \times 10^{-5}$ T

Ans : The magnetic of magnetic field is 3.5×10^{-5}T

★3) **Magnetic field at a distance 2.4 cm from a long straight wire is 16 µT. What must be current through a wire?**

Data: $B = 16\mu T = 16 \times 10^{-6}$ T,
$R = 2.4$ cm $= 2.4 \times 10^{-2}$ m
$\mu_0 = 4\pi \times 10^{-7}$ Tm/A

To find: I

Formula: $B = \dfrac{\mu_0 I}{2\pi R}$

Solution: $B = \dfrac{\mu_0 I}{2\pi R}$

$$16 \times 10^{-6} = \frac{4\pi \times 10^{-7} \times 1}{2 \times \pi \times 2.4 \times 10^{-2}}$$

$$I = \frac{16 \times 10^{-6} \times 2.4 \times 10^{-2}}{2 \times 10^{-7}}$$

$= 8 \times 2.4 \times 10^{-6-2+7}$
$= 19.2 \times 10^{-1} = 1.92$ A

Ans : The magnetic of electric current is 1.92 A

Problem for Practice

1. A straight wire carries a current of 3 A. Calculate the magnitude of the magnetic field at a point 10 cm away from the wire

Ans: 6×10^{-6} T

2. A long straight wire carries a current of 35A. What is the magnitude of magnetic field B at a point 20 cm from the wire

Ans : 3.5×10^{-5} T

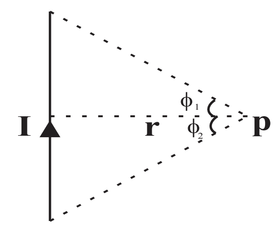
3. A long straight wire carries a current of 40A, then find the magnitude of the field at a point 15 cm away from the wire.

Ans : 5.34×10^{-5} T

Key Points

i. Magnetic field due to a straight conductor of infinite length

$$\boxed{B = \frac{\mu_0 I}{4\pi r}[\sin\phi_1 + \sin\phi_2]}$$



ii. For wire of infinite length

$\phi_1 = \phi_2 = 90^\circ$.

$$B = \frac{\mu_0 I}{4\pi r}[\sin 90 + \sin 90] = \frac{\mu_0 I}{4\pi r}(1+1)$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R}}$$

iii. For a wire of semi-infinite length when point lies near the end

$\phi_1 = 90^\circ$ and $\phi_2 = 0^\circ$

$$B = \frac{\mu_0 I}{4\pi r}[\sin 90^\circ + \sin 0^\circ] = \frac{\mu_0 I}{4\pi r}(1+0)$$

$$B = \frac{\mu_0 I}{4\pi r}$$

MULTIPLE CHOICE QUESTIONS
Entrance Corner (Set 4)

1. A current I ampere flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is

a. infinite b. Zero

c. $\frac{\mu_o}{4\pi}\times\frac{2I}{r}$ tesla d. $\frac{2I}{r}$ tesla

2. A current I flows along the length of an infinitely long, straight thin walled pipe Then

a. The magnetic field at all points inside the pipe is the same, but not zero

b. The magnetic field is zero only on the axis of the pipe

c. The magnetic field is different at different points inside the pipe

d. The magnetic field at any point inside the pipe is zero

3. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1' and COD carries a current I_2. The magnetic field on a point lying at a distance 'd' from O, in a direction perpendicular to the plane of the wires AOB and COD,will be given by

a. $\frac{\mu_o}{2\pi}\left(I_1^2+I_2^2\right)$ b. $\frac{\mu_o}{2\pi}\left(\frac{I_1+I_2}{d}\right)2$

c. $\frac{\mu_o}{2\pi d}\left(I_1^2+I_2^2\right)^{1/2}$ d. $\frac{\mu_o}{2\pi d}\left(I_1+I_2\right)$

4. A long straight wire of radius 'a' carries a steady current I . The current is uniformly distributed across its cross-section The ratio of the magnetic field at $\frac{a}{2}$ and 2a is

a. $\frac{1}{2}$ b. 1

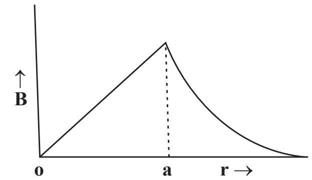
c. 2 d. $\frac{1}{4}$

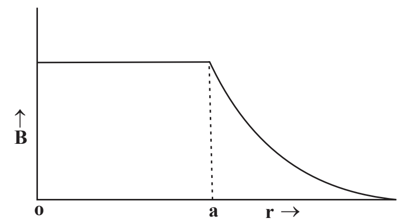
5. Two long straight wires are set parallel to each other. Each carries a current i in the same direction and separation between them is 2r. Intensity of magnetic field midway between them is

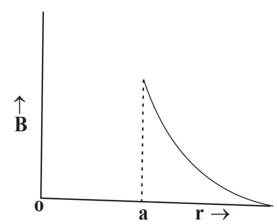
a. $\frac{\mu_o i}{r}$ b. Zero

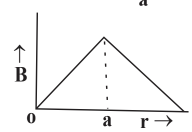
c. $\frac{4\mu_o i}{r}$ d. $\frac{\mu_o i}{4r}$

6. The magnetic field due to a straight conductor of uniform cross-section of radius a and carrying a steady current is represented by

a. 

b. 

c. 

d. 

7. The magnetic field dB due to a small current elements dl at a distance

a. $\vec{dB}=\frac{\mu_o}{4\pi}\times\frac{Id\vec{I}\times\vec{r}}{r}$

b. $\vec{dB}=\frac{\mu_o}{4\pi}\times\frac{Id\vec{I}\times\vec{r}}{r^2}$

c. $\vec{dB}=\frac{\mu_o}{4\pi}\times\frac{Id\vec{I}\times\vec{r}}{2r^2}$

d. $\vec{dB}=\frac{\mu_o}{4\pi}\times\frac{Id\vec{I}\times\vec{r}}{r^3}$

8. The magnetic field at a distance r from a long wire carrying current I is 0.4 tesla. The magnetic field at a distance 2r is

a. 0.1 tesla b. 0.2 tesla

c. 0.8 tesla d. 1.6 tesla

9. A straight wire of distance 0.5mm carrying a current of 1 A is replaced by another wire of 1 mm diameter carrying the same current. The strength of the magnetic field for away is

a. twice the earlier value
b. one half of the earlier value
c. one quarter of the earlier value
d. same as the earlier value

10. Two wires are held perpendicular to the plane of paper at 5m part. They carry currents of 2.5 A and 5A in same direction. Then the magnetic field strength (B) at a point midway between the wires will be

a. $\frac{\mu_o}{4\pi}T$ b. $\frac{\mu_o}{2\pi}T$

c. $\frac{3\mu_o}{2\pi}T$ d. $\frac{3\mu_o}{4\pi}T$

11. Two long parallel wires P and Q are both perpendicular to the plane of the paper with distance of 5m between them .If P and Q carry currents of 2.5 A and 5 A respectively in the same direction, then the magnetic field at a point half-way between the wires is

a. $\frac{\sqrt{3\mu_o}}{2\pi}$ b. $\frac{\mu_o}{17}$

c. $\frac{3\mu_o}{2\pi}$ d. $\frac{\mu_o}{2\pi}$

12. 20 amp current is flowing in a long straight wire The intensity of magnetic field at a distance of 10cm from the wire will be

a. 4×10^{-5} Wb/m^2 b. 2×10^{-5} Wb/m^2
c. 3×10^{-5} Wb/m^2 d. 8×10^{-5} Wb/m^2

13. The magnetic field at a distance r from a long wire carrying current I is 0.4 tesla. The magnetic field at a distance 2r is

a. 0.2 tesla b. 0.8 tesla
c. 0.1 tesla d. 1.6 tesla

14. A vertical straight conductor carries a current vertically upwards. A point p lies to the east of it at a small distance and another point Q lies to the west at the same distance. The magnetic field at p is

a. greater than at Q
b. same as at Q
c. less than at Q
d. greater or less that at Q depending upon the strength of current

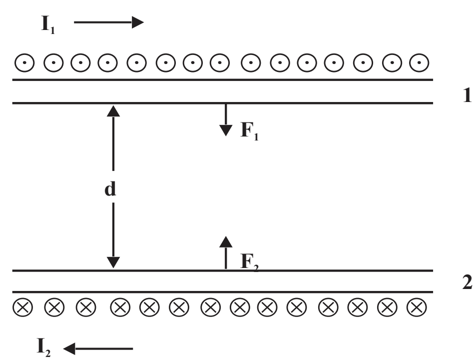
10.11 Force of attraction between two long parallel wire

Q.16 Derive expression for force between two long current carrying parallel wires separated by small distance.

Ans:

Case I : Both wires carry current in same direction.

i. Consider two long parallel wires separated by distance d and carrying I_1 and I_2 respectively same direction as same as shown in figure below:



Two long parallel wires, distance d apart

ii. The magnetic field at the second wire due to the current I_1 in the first one,
According to Biot - Savart law is

$$B_2 = \frac{\mu_0 I_1}{2\pi d} \quad ...(1)$$

By the right hand rule, the direction of this field is into the plane of the paper.

iii. Force on the wire 2, because of the current I_2 and the magnetic field B_1 due to current in wire 1 is

$$F_2 = B_1 I_2 \int dl \sin 90$$

The direction of this force is towards wire 1

$$F_2 = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2 \int dl \quad \text{(from 1)}$$

Force per unit length of the wire will be

$$\frac{F_2}{L} = \frac{\mu_0 I_1 I_2}{2\pi d} \qquad ....(2)$$

iv. Similarly the magnetic field at the first wire due to the current I_2 in the second one, According to Biot - Savart law is

$$B_1 = \frac{\mu_0 I_2}{2\pi d} \qquad ...(3)$$

By the right hand rule, the direction of this field is out of plane of the paper.

v. Force on the wire 1, because of the current I_1 and the magnetic field B_2 due to current in wire 2 is

$$F_1 = B_2 I_1 \int dl \sin 90$$

The direction of this force is towards wire 2

$$F_1 = \left(\frac{\mu_0 I_2}{2\pi d}\right) I_1 \int dl \qquad \text{(from 3)}$$

Force per unit length of the wire will be

$$\frac{F_1}{L} = \frac{\mu_0 I_1 I_2}{2\pi d} \qquad ...(4)$$

vi. From (3) and (4) we can conclude that, Force acting per unit length of wire is

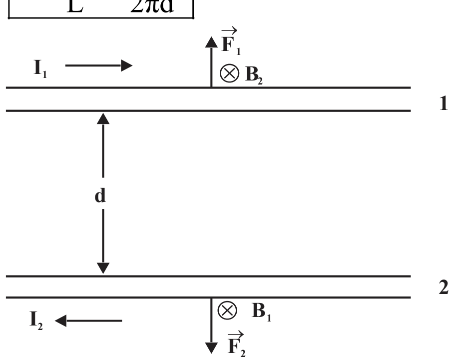
$$\boxed{f = \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}}$$

from direction of force F_1 and F_2 we can conclude that when current in wires are in same direction both wires will attract each

Case II : Two wires carry current in opposite direction.

If the currents are in the opposite direction to each other, there will be force of repulsion between the two conductors.

$$\boxed{f = \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}}$$



Q.17 Define one ampere

Ans:

i. Consider two parallel wires separated by 1 m and carrying a current of 1 A each.
Then $I_1 = I_2 = 1$ A and d = 1 m

$$\therefore \quad \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

$$\therefore \quad \frac{F}{L} = \frac{4\pi \times 10^{-7} \times 1 \times 1}{2\pi \times 1}$$

$$\frac{F}{L} = 2 \times 10^{-7}\ \text{N/m}$$

This is used to formally define the unit 'ampere' of electric current.

ii. If two parallel, long wires, kept 1m apart in vacuum, carry equal currents in the same direction and there is a force of attraction of 2×10^{-7} newton per metre of each wire, the current in each wire is said to be 1 ampere.

Type - VII

Numerical based on force acting between two long parallel wire

Formula Used

$$f = \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

★1) Current of equal magnitude flows through two long parallel wires having separation of 1.35 cm. If the force per unit length on each of the wires in 4.76 × 10^{-2} N, what must be I?

Data: $I_1 = I_2 = I$, $\frac{F}{L} = 4.76 \times 10^{-2}$ N

d = 1.35 cm = 1.35 × 10^{-2} m

To find: I

Formula: $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

Solution: $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

$$4.76 \times 10^{-2} = \frac{4\pi \times 10^{-7} \times I \times I}{2 \times \pi \times 1.35 \times 10^{-2}}$$

$$I^2 = \frac{4.76 \times 10^{-2} \times 1.35 \times 10^{-2}}{2 \times 10^{-7}}$$

$= 1.35 \times 2.38 \times 10^{-2-2+7}$

$I = \sqrt{1.35 \times 2.38 \times 10^{3}}$

$= \sqrt{13.5 \times 2.38} \times 10$

$= \sqrt{32.13} \times 10$

$= anti\log\left[\frac{1}{2}\log 32.13\right] \times 10$

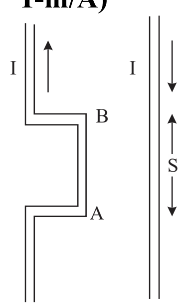
$= anti\log\left[\frac{1}{2}1.5069\right] \times 10$

$= anti\log[0.7534] \times 10$

$= 5.668 \times 10 = 56.68$ A

Ans : The electric current is 56.8 A

★2) Two wires shown in the figure are connected in a series circuit and the same amount of current of 10 A passes through both, but in opposite directions. Separation between the two wires is 8 mm. The length AB is S = 22 cm. Obtain the direction and magnitude of the magnetic field due to current in wire 2 on the section AB of wire 1. Also obtain the magnitude and direction of the force on wire 1. ($\mu_0 = 4\pi \times 10^{-7}$ T-m/A)



Data : $I_1 = I_2 = I = 10$ A
$d = 8\text{ mm} = 8 \times 10^{-3}$ m,
$L = 22\text{ cm} = 22 \times 10^{-2}$ m,

To find: i. Magnitude and direction of magnetic field
ii. Magnitude and direction of force

Formula: i. $B = \frac{\mu_0 I}{2\pi d}$

ii. $F = \frac{\mu_0 I_1 I_2}{2\pi d} \times L$

Solution: i. $B = \frac{\mu_0 I}{2\pi d}$

$B = \frac{4\pi \times 10^{-7} \times 10}{2 \times \pi \times 8 \times 10^{-3}} = 0.25 \times 10^{-3}$

By using right hand thumb rule, direction of magnetic field due to current in wire 2 on section AB of wire 1 is into the plane of paper.

ii. $F = \frac{\mu_0 I_1 I_2}{2\pi d} \times L$

$F = \frac{4\pi \times 10^{-7} \times 10 \times 10}{2 \times \pi \times 8 \times 10^{-3}} \times 22 \times 10^{-2}$

$= 0.25 \times 22 \times 10^{-7+2-2+3}$

$= 5.5 \times 10^{-4}$ N

From Flemings' left hand rule, force is repulsive i.e., in leftward of wire 1.

Ans : i. The magnitude of magnetic field is 2.5×10^{-11}T and its direction is into the plane of paper.
ii. The force is repulsive and acting leftward to wire 1 with magnitude 5.5×10^{-4} N.

Problem for Practice

1. A current of 5.0 A flows through each of two parallel long wires the wire 2.5 cm apart. Calculate the force acting per unit length of each wire. Use the standard value of constant required. What will be the nature of the force, if both currents flow in the same direction?

Ans: 2×10^{-4} Nm^{-1}

2. A long horizontal wire P carries a current of 50 A. It is rigidly fixed. Another fine wire Q is placed directly above and parallel to P. The weight of the wire Q is 0.075 Nm^{-1} and it carries a current of 25 A. Find the position of the wire Q from the wire P so that Q remains suspended due to the magnetic repulsion. Also indicate the direction of current in Q with respect to p

Ans : 3.33 mm

3. Calculate the force per unit length on a long straight wire carrying current of 4 A due to a parallel wire carrying 6 A . Distance between the wires =3cm

Ans : 1.6×10^{-4} Nm^{-1}

4. Two straight wires A and B of lengths 10m and 12m carrying currents of 4.0 A and 6.0 A respectively in opposite direction, lie parallel to each other at a distance of 3.0 cm. Estimate the force on a 15 cm section of the wire B near its centre.

Ans : 2.4×10^{-5} N, Repulsive

MULTIPLE CHOICE QUESTIONS
Entrance Corner (Set 5)

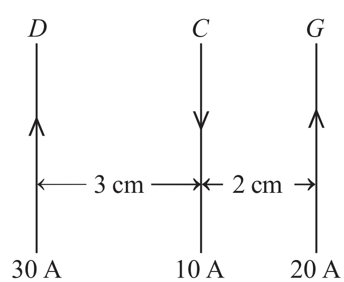
1. Two long conductors, separated by a distance d carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to 3d. The new value of force between them is

a. $-2F$ b. $\frac{F}{3}$
c. $\frac{-2F}{3}$ d. $\frac{-F}{3}$

2. If two electron beams travel in the same direction, they will

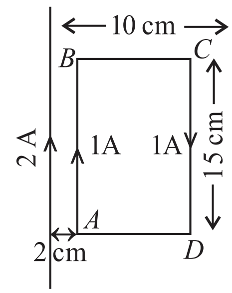
a. attract each other
b. repel each other
c. nothing will happen
d. none of the above

3. Three long, straight parallel wires, carrying current, are arranged as shown in figure. The force experienced by a 25 cm length of wire C is



a. 10^{-3} N b. 2.5×10^{-3} N
c. zero d. 1.5×10^{3} N

4. What is the net force on the rectangular coil?

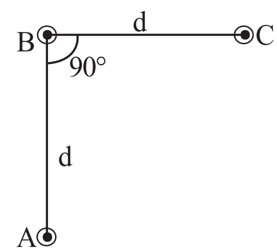


a. 25×10^{-7} N towards wire
b. 25×10^{-7} N away from wire
c. 35×10^{-7} N towards wire
d. 35×10^{7} N away from wire

5. Two parallel wires of length 9 m each are separated by a distance 0.15 m. If they carry equal currents in the same direction and exert a total force of 30×10^{-7} N on each other, then the value of current must be

a. 2.5 amp b. 3.5 amp
c. 1.5 amp d. 0.5 amp

6. An arrangement pf three parallel straight wires placed perpendicular to plane of paper carrying same current 'I along the same direction is shown in fig. Magnitude of force per unit length on the middle wire 'B' is given by



a. $\frac{2\mu_0 i^2}{\pi d}$ b. $\frac{\sqrt{2}\mu_0 i^2}{\pi d}$
c. $\frac{\mu_0 i^2}{\sqrt{2}\pi d}$ d. $\frac{\mu_0 i^2}{2\pi d}$

7. Currents of 10 A , 2 A are passed through two parallel wires A and B respectively in opposite directions. If the wire A is infinitely long and the length of the wire B is 2 metre, the force on the conductor B, which is situated at 10cm distance from A will be

a) 8×10^{-5} newton b) 5×10^{-5} newton
c) $8\pi \times 10^{-7}$ newton d) $4\pi \times 10^{-7}$ newton

Try yourself

8. Two thin long parallel wires, separated by a distance d carry a current of IA in the same direction. They will

a. attract each other with a force of $\frac{\mu_0 I^2}{2\pi d}$

b. repel each other with a force of $\frac{\mu_0 I^2}{2\pi d}$

c. attract each other with a force of $\frac{\mu_0 I^2}{2\pi d^2}$

d. repel each other with a force of $\frac{\mu_0 I^2}{2\pi d^2}$

9. When two parallel wires carry currents in the same direction,

a. they attract each other
b. they repel each other
c. magnetic forces on two wires are perpendicular to each other
d. they do not experience any magnetic force.

10. Two parallel beams of positrons moving in the same direction will

a. repel each other
b. will not interact with each other
c. attract each other
d. be deflected normal to the plane containing the two beams.

11. Two long parallel wires are at a distance of 1 m. If both of the them carry 1 A of current in some direction, then the force of attraction per unit length between the two wires is

a. 2×10^{-7} Nm^{-1} b. 2×10^{-8} Nm^{-1}
c. 5×10^{-8} Nm^{-1} d. 5×10^{-7} Nm^{-1}

12. Two parallel wires in free space are 10 cm apart and each carries a current of 10 A in the same direction. The force exerted by one wire on the other (per metre length) is

a. 2×10^{-4} N (attractive)
b. 2×10^{-7} N (attractive)
c. 2×10^{-4} N (attractive)
d. 2×10^{-7} N (repulsive)

13. Two thin long parallel wires separated by a distance b are carrying a current i A each. The magnitude of the force per unit length will be

a. $\frac{\mu_0 i^2}{b^2}$ b. $\frac{\mu_0 i^2}{2\pi b}$
c. $\frac{\mu_0 i}{2\pi b^2}$ d. $\frac{\mu_0 i}{2\pi b}$

14. When two infinitely long parallel wires separated by a distance of 1m, each carry a current of 3 ampere, the force in newton/metre length experienced by each will be,

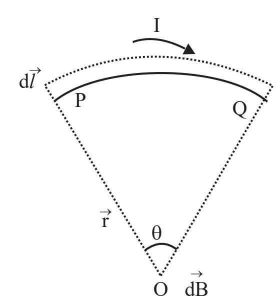
a) $2\text{x}10^{-7}$ b) $3 \text{ x } 10^{-7}$
c) $6\text{x } 10^{-7}$ d) $18 \text{ x } 10^{-7}$

15. A horizontal wire carries 200 amp current below which another wire of linear density $20\text{x}10^{-3}$ kgm^{-1} carrying a current is kept at 2 cm distance. If the wire kept below hangs in air. The current in this wire is

a) 4.8A b) 9.8 A
c) 98A d) 48A

10.12 Magnetic field produced by a current carrying are of a wire

Q.18 Derive an expression for a magnetic field produced at the centre of current carrying arc of wire. Also obtain expression for magnetic field at the centre of circular coil ?



Ans:

i. Consider circular arc of wire (PQ), carrying a current I.

ii. The circular arc PQ subtends an angle θ at the centre O of the circle with radius r of which the arc is a part.

iii. Consider length element $\vec{dl}$ lying always perpendicular to $\vec{r}$

Using Biot-Savart law, magnetic field produced at O is:

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I(\vec{d\ell}\times\vec{r})}{r^3}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,r\,dl\,sin90}{r^3}$$

$$dB = \frac{\mu_0 I\,dl}{4\pi r^2} \quad ...(1)$$

iv. Equation (1) gives the magnitude of the field. The direction of the field is given by the right hand rule. Thus, the direction of each of the dB is into the plane of the paper.

The total field at O is

$$B = \int dB = \frac{\mu_0 I}{4\pi}\int_P^Q \frac{dl}{r^2}$$

$$= \int dB = \frac{\mu_0 I}{4\pi}\int_0^\theta \frac{r\,d\theta}{r^2}$$

$$\boxed{B = \frac{\mu_0 I}{4\pi r}\theta} \quad ...(2)$$

Where, the angle θ is in radians.

Magnetic field at the centre of the circle

v. For a full circular wire (instead of arc) $\theta = 2\pi$

Substitute $\theta = 2\pi$ in equation (2)

$$B = \frac{\mu_0 I}{4\pi r}(2\pi)$$

$$B = \frac{\mu_0 I}{2r}$$

For N turns

$$\boxed{B = \frac{\mu_0 NI}{2r}}$$

This is magnetic field at the centre of of circular coil carrying current.

Type - VIII

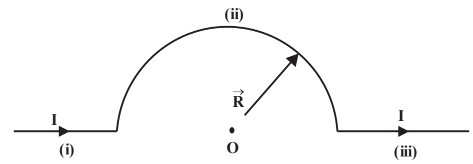
Numerical based on current carrying arc

Formula Used

1. $B = \frac{\mu_0}{4\pi}\frac{I}{r}\theta$
2. At the centre of circular coil

$$B = \frac{\mu_0 NI}{2r} \text{ (for N turns)}$$

1) A wire has 2 straight sections and one are as shown in the figure



Determine the direction and magnitude of the magnetic field produced at the centre O of the semicircle by the three section individually and the total.

Solution :

Applying Bio-Savart law to the 3 section of the wire For the section (i) and (ii) the angle between the current - length elements $Id\vec{l}$ and $\vec{R}$ is 180° and 0° respectively

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin(180°)}{R^2} = \frac{\mu_0}{4\pi}\frac{Idl\sin(0°)}{R^2} = 0$$

$$\Rightarrow B_I = B_{III} = 0$$

For section (ii), dl is always perpendicular to $\vec{R}$

$$\therefore \quad dB = \frac{\mu_0}{4\pi}\frac{Idl\sin(90°)}{R^2} = \frac{\mu_0}{4\pi}\frac{Idl}{R^2}$$

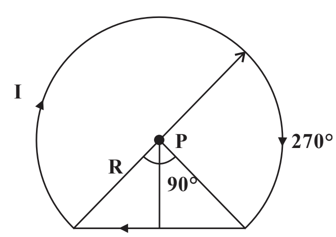
Integrating, $(B)_{II} = \frac{\mu_0}{4\pi}\frac{I}{R^2}\int_0^{\pi R} dl = \frac{\mu_0}{4\pi}\frac{I}{R^2}\pi R$

$$\therefore \quad (B)_{II} = \frac{\mu_0}{4}\frac{I}{R}$$

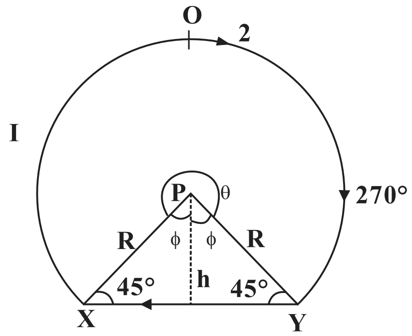
Total $B = B_I + B_{II} + B_{III}$

$$= 0 + \frac{\mu_0 I}{4R} + 0 = \frac{\mu_0 I}{4R}$$

★2) A wire loop of the form shown in the figure carries a current I. Obtain the magnitude and direction of the magnetic field at P.



Solution:



Magnetic induction field at P is sum of 2 sections as shown in the figure

$B = B_1 + B_2$ ... (i)

where,

B_1 will be magnetic field due to straight current carrying wire segment XY situated at perpendicular distance of h.

$$B_1 = \frac{\mu_0 I}{4\pi h}(2\sin\phi) = \frac{\mu_0 I}{4\pi h}(2\times\sin 45°)$$

$$= \frac{\sqrt{2}\,\mu_0 I}{4\pi h}$$

But $h = R\sin 45° = \frac{R}{\sqrt{2}}$

$$B_1 = \frac{2\mu_0 I}{4\pi R} \quad \text{...(ii)}$$

B_2 will be magnetic field due to are XOY subtending angle θ at P.

$$\theta = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{R}\theta$$

$$= \frac{\mu_0 I}{4\pi R}\left(\frac{3\pi}{2}\right) \quad \text{... (iii)}$$

Using equations (i), (ii) and (iii),

$$B = \frac{\mu_0 I}{4\pi R}\left[2 + \frac{3\pi}{2}\right]$$

Direction of B is inside the plane of paper.

3) Consider a closely wound 1000 turn coil, having radius of 1 m. If a current of 10 A passes through the coil, what will be the magnitude of the magnetic field at the centre?

Data : N = 1000, R = 100 cm = 1 m, I = 10 A.

To find: Magnetic field (B)

Formula:

$$B = \frac{\mu_0 NI}{2R}$$

$$B = \frac{4\pi\times10^{-7}\times10^3\times10}{2\times1}$$

$= 2\pi\times10^{-3}$

$= 6.282\times10^{-3}$ T.

Ans: Magnetic field is 6.282×10^{-3} T

4) A circular coil of wire is made up of 100 turns, each of radius 8.0 cm. If a current of 0.40 A passes through it, what be the magnetic field at the centre of the coil?

Data: N = 100, R = 8.0 cm = 8 × 10–2 m, I = 0.40 A,

We know that, $\mu_0 = 4\pi\times10^{-7}$ T m/A

To find: Magnetic field at the centre of coil.

Formula: $B_c = \frac{\mu_0 NI}{2R}$

Solution : $B_c = \frac{\mu_0 NI}{2R}$

$$B_c = \frac{4\pi\times10^{-7}\times100\times0.40}{2\times8\times10^{-2}}$$

$$= \frac{160}{16}\pi\times10^{-7+2}$$

$= \pi\times10^{-5+1}$

$= 3.142\times10^{-4}$ T

Ans: Magnetic field at the centre is 3.142×10^{-4} T

Problem for Practice

1. In a Bohr model of hydrogen atom an electron revolve around a nucleus in a circular orbit of 0.53 A^0 at a frequency of 6.8×10^{15} Hz. What is the magnetic field set up at the centre of the orbit ?

Ans: 13.4 T

2. Consider a tightly wound 100 turns coil of radius 10 cm, carrying a current of 1 A. What is the magnitude of magnetic field at the centre of a circular coil.

Ans: 6.28×10^{-4}T

3. The wire carries a current of 10 A. Determine the magnitude of the magnetic field at the centre. Given that radius of bent coil is 3 cm.

Ans: 1.57×10^{-4} T

4. A semicircular arc of radius 20 cm carries a current of 10 A. Calculate the magnitude of magnetic field at the centre of the arc.

Ans: 1.57×10^{-5} T

10.13 Axial magnetic field produced by a current in a circular loop

Q.19 Derive an expression for axial magnetic field produced by current in a circular loop.

Ans:

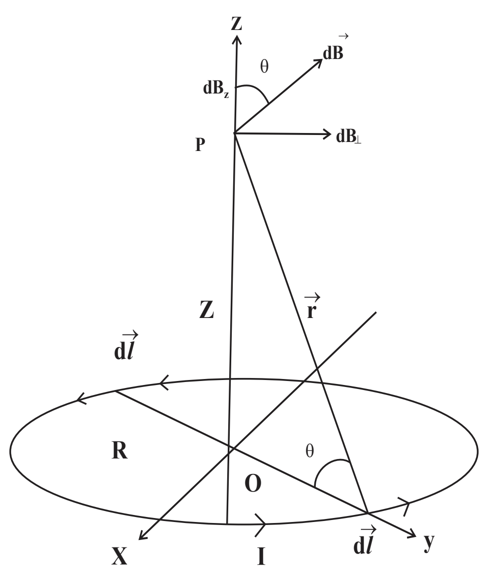
i. Consider loop of radius R carrying current I in anticlockwise direction and placed in x-y plane with its centre at origin O.

ii. Consider current element $\vec{dl}$ at point A.

Let P be any point on z-axis at a distance $\vec{r}$ from current element $\vec{dl}$ of the loop. We suppose to find $\vec{B}$ at axial point P.

iii. Using Biot-Savart law, the magnitude of the magnetic field dB is given by

$$dB = \frac{\mu_0}{4\pi}\frac{I|\vec{dl}\times\vec{r}|}{r^3} \quad ...(1)$$



iv. Current element ($\vec{dl}$) is always perpendicular to $\vec{r}$. Now the element is in the x-y plane, while the vector is in the y-z plane. Hence

$|\vec{dl}\times\vec{r}|$= dl r sin90=dl r

substituting in equation (1)

$$\therefore \quad dB = \frac{\mu_0}{4\pi}\frac{Idl\,r}{r^3} = \frac{\mu_0}{4\pi}\frac{Idl}{r^2} \quad ...(2)$$

Now by applying Pythagoras theorem we get, $r^2=R^2+z^2$

Substituting in eq (2)

$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{(R^2+z^2)} \quad ...(3)$$

v. Now, direction of $\vec{dB}$ is perpendicular to the plane formed by $\vec{dl}$ and $\vec{r}$ (shown in figure).

Resolving $\vec{dB}$ into two perpendicular component

a. dB_z is along z-axis and (dBcosθ)

b. $\vec{dB_\perp}$ perpendicular to z-axis.(dBsinθ)

vi For two diametrically opposite element of the loop, the component perpendicular to the axis of the loop (dBsinθ) will be equal and opposite and will cancel out. Their axial component (dBcosθ) will be in same direction and get added up.

vii. Therefore, the net contribution along the z-axis is obtained by integrating dB_z= dB cos θ over the entire loop

$$B = \int dB_z = \int dB\cos\theta$$

$$B = \frac{\mu_0 I}{4\pi}\int\frac{dl}{(z^2+R^2)}\cos\theta \quad \text{(From 3)}$$

since, $\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{(z^2+R^2)}}$

$$\therefore \quad B = \frac{\mu_0 I}{4\pi}\int\frac{dl}{(z^2+R^2)}\frac{R}{\sqrt{(z^2+R^2)}}$$

$$\therefore \quad B = \frac{\mu_0 I}{4\pi}\int\frac{dl\,R}{(z^2+R^2)^{3/2}}$$

$\therefore \quad B = \frac{\mu_0 I}{4\pi}\frac{R}{(z^2+R^2)^{3/2}}\int dl$

$\therefore \quad B = \frac{\mu_0 I}{4\pi}\frac{R}{(z^2+R^2)^{3/2}} \times 2\pi R \quad \left(\because \int dl = 2\pi R\right)$

$\therefore \quad B = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}$

For N turns

$$\boxed{B = \frac{\mu_0 N I R^2}{2(z^2+R^2)^{3/2}}}$$

This is the magnitude of the magnetic field due to current I in the loop of radius R, on a point at P on the z axis of the loop.

Key Points

i. **At centre of a circular coil** $z = 0$

$B_{centre} = \frac{\mu_0}{4\pi}.\frac{2\pi NI}{R} = \frac{\mu_0 NI}{2R} = B_{max}$

ii. **The ratio of magnetic field at the centre of circular coil and on it's axis is given by**

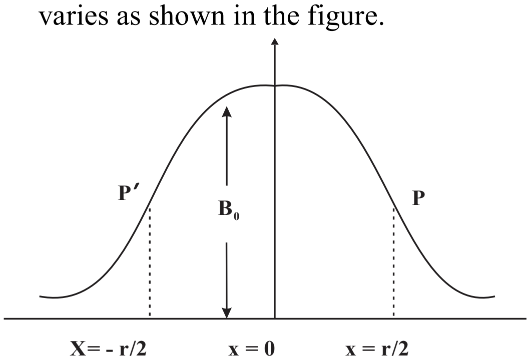
$\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{Z^2}{R^2}\right)^{3/2}$

iii. **At the axial point lying far away from the coil**, If $z >> R$

$B_{axis} = \frac{\mu_0}{4\pi}.\frac{2\pi NiR^2}{z^3} = \frac{\mu_0}{4\pi}.\frac{2NIA}{z^3}$

where $A = \pi r^2$ = Area of each turn of the coil.

iv. **B-x curve** : The variation of magnetic field due to a circular coil as the distance z varies as shown in the figure.



B varies non-linearly with distance *z*

a. B is maximum when $z^2 = \min = 0$, i.e., the point is at the centre of the coil

b. B is zero at $x = \pm\infty$.

v. Point of inflection (P and P′)**:** Also known as points of curvature change or points of zero curvature.

a. At these points *B* varies linearly with

$x \frac{dB}{dx} =$ constant

$\frac{d^2B}{dx^2} = 0$.

These are located at $x = \pm\frac{R}{2}$ from the centre of the coil and the magnetic field is

$B = \frac{4\mu_0 NI}{5\sqrt{5}R}$

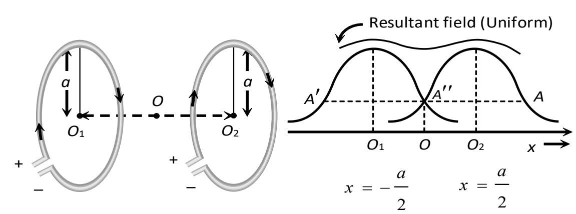
Helmholtz coils

i. This is the set-up of two coaxial coils of same radius such that distance between their centres is equal to their radius.

ii. At axial mid point *O*, magnetic field is given by

$B = \frac{8\mu_0 NI}{5\sqrt{5}R} = 0.716\frac{\mu_0 NI}{R} = 1.432B$

where $B = \frac{\mu_0 Ni}{2R}$



iii. **Current direction is same in both coils otherwise this arrangement is not called Helmholtz's coil arrangement.**

Q.20 Discuss magnetic field due to a circular current loop in terms of magnetic moment.

Ans: Magnetic induction at an axial point is

$B = \frac{\mu_0 NIR^2}{2(z^2+R^2)^{3/2}}$

Case 1: At the centre of the loop, z = 0

$\therefore \quad B = \dfrac{\mu_0 NIR^2}{2R^3}$

$\therefore \quad B = \dfrac{\mu_0 NI}{2R}$

In terms of area ($A = \pi R^2$) of the circular current loop, the above result may be written as under :

$$B = \frac{\mu_0 NIA}{2R(\pi R^2)} = \frac{\mu_0}{2\pi}\frac{NIA}{R^3}$$

$\therefore \quad B = \dfrac{\mu_0}{4\pi}\dfrac{2NIA}{R^3}$

The quantity NIA is known as the magnetic dipole moment m of the current loop.

$\therefore \quad B = \dfrac{\mu_0}{4\pi}\dfrac{2m}{r^3}$

Case - II : If the observation point is far away f rom the coil, then R << z. So, R^2 can be neglected in comparison to z^2.

$\therefore \quad B_z = \dfrac{\mu_0 NIR^2}{2z^3}$

Multiply numerator and denominator by π

$$B_z = \frac{\mu_0 NIR^2}{2z^3}\frac{\pi}{\pi}$$

$$B_z = \frac{NIA}{z^3} = \frac{\mu_0}{4\pi}\frac{2NIA}{z^3}$$

In terms of magnetic dipole moment,

$$B_z = \frac{\mu_0}{4\pi}\frac{2m}{z^3}$$

In terms of vector

$$\vec{B}_z = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{z^3}$$

From this it is clear that $\vec{B}_z$ and $\vec{m}$ are in same direction i.e perpendicular to the plane of the loop

Note :

Using electrostatic analogue

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$

which is electric field on axial point.

The direction of $\vec{E}$ is along the dipole axis.

Type - IX

Numerical based on magnetic field along the axis of circular coil carrying current

Formulae used

1. Magnetic induction at an axial point is

$$B = \frac{\mu_0 NIR^2}{2(z^2 + R^2)^{3/2}}$$

2. Magnetic induction at the centre of circular coil

$$B = \frac{\mu_0 NI}{2R}$$

in terms of magnetic moment

$$B = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$$

3. For far away point on the axis

$$B_z = \frac{\mu_0}{4\pi}\frac{2m}{z^3}$$

★1) The magnetic field at the centre of a circular current carrying loop of radius 12.3 cm is 6.4 × 10^{-6} T. What will be the magnetic moment at the loop?

Data: $B = 6.4 \times 10^{-6}$ T,

$R = 12.3$ cm $= 12.3 \times 10^{-3}$ m

To find: m

Formula: Magnetic induction at the centre of circular coil

$$B = \frac{\mu_0 NI}{2R} = \frac{\mu_0}{2\pi}\frac{NIA}{R^3}$$

$\therefore \quad B = \dfrac{\mu_0 m}{2\pi R^3}$

$\therefore \quad m = \dfrac{B \times 2\pi R^3}{\mu_0}$

Solution: $m = \dfrac{B \times 2\pi R^3}{\mu_0}$

$= \dfrac{\overset{1.6}{\cancel{6.4}} \times 10^{-6} \times 2 \times \cancel{\pi} \times (12.3 \times 10^{-2})^3}{\cancel{4}\,\cancel{\pi} \times 10^{-7}}$

$= 3.2 \times (12.3)^3 \times 10^{-6+7-6}$

$= 5.954 \times 10^{-2}\ Am^2$

Ans :	Magnetic momet of the loop is $5.954 \times 10 Am^2$

★2) A circular loop of radius 9.7 cm carries a current 2.3 A. Obtain the magnitude of the magnetic field
i. at the centre of the loop and
ii. at a distance of 9.7 cm from the centre of the loop but on the axis.

Data: $R = 9.7\ cm = 9.7 \times 10^{-2}\ m, I = 2.3\ A$,
$z = 9.7\ cm = 9.7 \times 10^{-2}\ m$

To find: Magnetic field
i. at the centre of the loop
ii. on the axis

Formula: i. $B_c = \dfrac{\mu_0 I}{2R}$

ii. $B_a = \dfrac{\mu_0 I R^2}{2\left(z^2 + R^2\right)^{3/2}}$

Solution: i. $B_c = \dfrac{\mu_0 I}{2R}$

$B_c = \dfrac{\overset{2}{\cancel{4}}\pi \times 10^{-7} \times 2.3}{\cancel{2} \times 9.7 \times 10^{-2}}$

$= \dfrac{4.6 \times 3.142}{9.7} \times 10^{-7+2}$

$= 1.49 \times 10^{-5}\ T$

ii. $B_a = \dfrac{\mu_0 I R^2}{2\left(z^2 + R^2\right)^{3/2}}$

$B_a = \dfrac{4\pi \times 10^{-7} \times 2.3 \times (9.7 \times 10^{-2})^2}{2\left[(9.7 \times 10^{-2})^2 + (9.7 \times 10^{-2})^2\right]^{3/2}}$

$= \dfrac{\overset{2}{\cancel{4}}\pi \times 10^{-7} \times 2.3 \times (9.7 \times 10^{-2})^2}{\cancel{2} \times 2^{3/2} \times (9.7 \times 10^{-2})^3}$

$= \dfrac{2\pi \times 2.3 \times (9.7)^2 \times 10^{-7-4}}{2\sqrt{2} \times (9.7)^3 \times 10^{-6}}$

$= \dfrac{3.14 \times 2.3}{1.414 \times 9.7} \times 10^{-5} = \dfrac{7.222}{13.71} \times 10^{-5}$

$= 5.268 \times 10^{-6}\ T$

Ans :	i. Magnetic field at the centre of 1.49×10^{5} T
	i. Magnetic field at the centre of 5.268×10^{6} T

Problem for Practice

1. A circular coil of radius 10 cm and having 100 turns carries a current of 2 A. Find the magnetic induction at a point on the axis of the coil at a distance of $20\sqrt{2}$ cm from the centre of the coil.
   Ans: 4.652×10^{-5} wb/m²

2. The magnetic field due to current carrying circular loop of radius 12 cm is 0.5×10^{-4} T.Find the magnetic field due to this loop at apoint on the axis at a distance of 5 cm form the centre.
   Ans: 0.4×10^{-4} T

3. The magnetic field due to current carrying circular loop of radius 10cm is 0.6×10^{-4} T. Find the magnetic field due to this loop at a point on the axis at a distance of 4 cm fom the centre.
   Ans: 4×10^{-5} T

4. Two flat circular coils having 50 turns and each of 6 cm radius are kept parallel. Distance between the two coils is 16cm. Find resultant magnetic induction at a point midway on their axis. Given currents of 3A and 4.5 A flow through the two coils in the same direction.
   Ans: 16.96×10^{-7} T

MULTIPLE CHOICE QUESTIONS
Entrance Corner (Set 6)

1. If in a circular coil A of radius R, current I is flowing and in another coil B of radius 2 R, a current 2 I is flowing; then the ratio of the

magnetic fields B_A and B_B produced by them will be

a. 1 b. 2

3. 1/2 d. 4

2. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μ T. What will be its value at the centre of the loop ?

a. 250 μ T b. 150 μ T

c. 125 μ T d. 75 μ T

3. Two concentric coils each of radius equal to 2π cm are placed at right angle to each other. 3 A and 4 A are the currents flowing in each coil respectively. The magnetic induction (in Wb m^{-2}) at the centre of the coils ($\mu_0 = 4\pi \times 10^{-7}$ Wb A^{-1} m^{-1}) will be

a. 12×10^{-5} b. 10^{-5}

c. 5×10^{-5} d. 7×10^{-5}

4. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns.
The magnetic field at the centre of the coil will be

a. n B b. n^2B

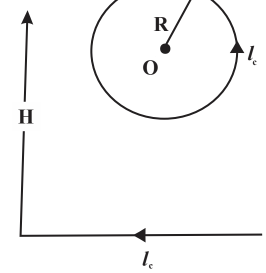
c. 2 nB d. $2n^2B$

5. A circular current carrying coil has a radius. *R*.The distance from the centre of the coil on the axis of the coil, where the magnetic induction is 1/8th of its value at the centre of the coil is

a. $\sqrt{3}R$ b. $\frac{R}{\sqrt{3}}$

c. $\left(\frac{2}{\sqrt{3}}\right)R$ d. $\frac{R}{2\sqrt{3}}$

6. Circular loop of a wire and a long straight wire carry currents I_c and I_e respectively as shown in the figure. Assuming that these are placed in the same plane, the magnetic fields will be zero at the centre O of the loop, when the separation H is



a. $\frac{I_eR}{I_c\pi}$ b. $\frac{I_cR}{I_e\pi}$

c. $\frac{I_c\pi}{I_eR}$ d. $\frac{I_e\pi}{I_cR}$

7. Two circular coils 1 and 2 are made from the same wire but the radius of the first coil is twice that of the second coil. what ratio of the potential difference (in volt) should be applied across them, so that the magnetic field at their centres is the same

a. 2 b. 3

c. 4 d. 6

8. A circular loop of radius 5cm is placed in a plane where magnetic field intensity is 5.6×10^{-5} T which is perpendicular to the plane of circular loop. what current is required to neutralize the effect of the magnetic field ?

a. 20.5 A b. 0.4A

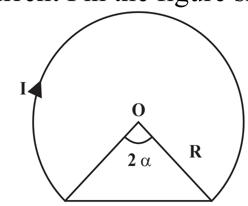
c. 4A d. 40A

9. A particle having charge 100 times that of an electron is revolving in a circular path of radius 0.8m with one rotation per sec. The Magnetic field produced at a centre of particle is

a. $10^{-3}\mu_o$ b. $10^{-11}\mu_o$

c. $10^{-7}\mu_o$ d. $10^{-17}\mu_o$

10. The magnetic field intensity due to a thin wire carrying current I in the figure shown is



a. $\frac{\mu_o I}{2\pi R}(\pi - \alpha + \tan\alpha)$

b. $\frac{\mu_o I}{2\pi R}(\pi - \alpha)$ c. $\frac{\mu_o I}{2\pi R}(\pi + \alpha)$

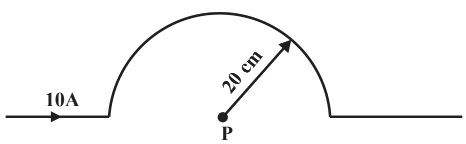
d. $\frac{\mu_o}{2\pi R}(\pi + \alpha - \tan\alpha)$

Try yourself

11. In a circular coil of radius r, the magnetic field at the centre is proportional to

a. r^2 b. r
c. $1/r$ d. $1/r^2$

12. A current of 10 A is passing through a long wire which has semicircular loop of the radius 20cm as



Shown in the figure. Magnetic field produced at the centre of the loop is

a. $10\pi\mu T$ b. $5\pi\mu T$
c. $4\pi\mu T$ d. $2\pi\mu T$

13. A wire of certain length is bent a circular coil of a single turn. If the same wire is bent into a coil of smaller radius so as to have turns, then magnetic field produced at the centre by the same value of current is

a. one quarter of its value in first case
b. one half of its value in first case
c. two times its value in first case
d. four times its value in first case.

14. A circular coil A has a radius R and the current flowing through it is I. Another circular coil B has a radius $2R$ and if 2I is the current flowing through it , then the magnetic fields at the centre of the circular coils are in the ratio of

a. 1:1   b. 2:1
c. 3:1   d. 4:1

15. The magnetic field of a given length of wire carrying a current for a single turn circular coil at centre is B. Then its value for two turns coil of the same wire, when the same current passes through it, is

a. $\frac{B}{4}$ b. $\frac{B}{2}$
c. 2B   d. 4B

16. Magnetic field at the centre of a current carrying circular loop having 1A current and number of turns one will be (radius of the loop is 1m)

a. $\frac{\mu_o}{2}$ b. $2\mu_o$
c. $\frac{\mu_o}{4}$ d. $4\mu_o$

17. To produce a magnetic field of π tesla at the centre of circular loop of diameter 1m, the current flowing through loop is

a. 5×10^6A   b. 10^7A
c. 2.5×10^6A   d. 2×10^6A

18. Two circular coils of diameter 10 and 20 cm have same number of turns. The ratio of the magnetic field inductions produced at the centre of coils when connected in series is

a. 1:2   b. 3:2
c. 2:1   d. 2:3

19. A circular loop of radius 5cm is placed in a plane where magnetic field intensity is 5.6×10^{-5} T which is perpendicular to the plane of circular loop. what current is required to neutralize the effect of the magnetic field ?

a. 20.5 A   b. 0.4A
c. 4A   d. 40A

20. A particle a having charge 100 times that of an electron is revolving in a circular path of radius 0.8m with one rotation per sec Magnetic field produced at a centre of particle is

a. $10^{-3}\mu_o$ b. $10^{-11}\mu_o$
c. $10^{-7}\mu_o$ d. $10^{-17}\mu_o$

Q.20 State and explain Ampere's law.

Ans. Statement:

The line integral of magnetic field of induction $\vec{B}$ around any closed path in free of space is equal to absolute permeability of free space (μ_0) times the total current flowing through area bounded by the path.

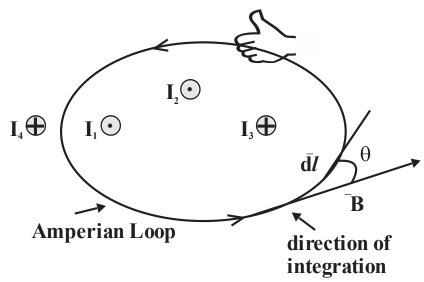
Mathematically,

$$\oint \vec{B}.d\vec{\ell} = \mu_0 I$$

Where, the sign $\oint$ indicates that the integral is to be evaluated over a closed loop called Amperian loop.

The current I on the right hand side is the net current encircled by the Amperian loop.

Explanation:



i. Consider cross-sections of four long straight wires carrying currents I_1, I_2, I_3, I_4 into or out of the plane of the paper.

ii. An Amperian loop is drawn to encircle 3 of the current wires and not the fourth one. As the current goes perpendicular to the plane of the paper, $\vec{B}$ is in the plane of the paper even if its direction is unknown.

iii. The length element on the Amperian loop is $d\vec{l}$ (in the plane of the paper).

$$\oint \vec{B}.d\vec{\ell} = \mu_0 I$$

$$\oint B\, dl \cos\theta = \mu_0 (I_1 + I_2 - I_3)$$

Where, I_1 and I_2 are coming out of paper so positive and I_3 is going into the paper so it is negative.

Q.21 Derive an expression for magnetic induction at a point near infinitely long straight conductor carrying an electric current on the basis of Ampere's law.

Ans:

i. Consider, an infinitely long straight conductor XY carrying an electric current I as shown in figure (a).

fig.(a)

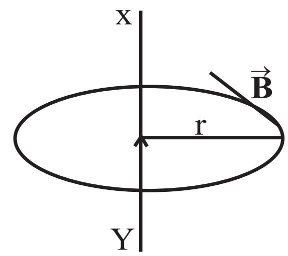


fig.(b)

ii. Let, P be a point at a distance r from the conductor.

iii. To determine the magnetic induction at point P, consider an Amperian loop as an imaginary circle with r as radius perpendicular to straight conductor as show in fig (b).

iv. Let, $\vec{B}$ - magnetic induction at point P

$\vec{dl}$ - length of small element of circle sss

v. According to Ampere's law

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I \quad \text{... (i)}$$

$$\therefore \oint B\cdot dl = B\oint dl$$

$$= \oint B\, dl \cos\theta$$

As angle θ between $\vec{B}$ and $\vec{dl}$ is 0^0

vi. Due to symmetry, B is same at all point along close path.

$$\therefore \oint \vec{B}\cdot\vec{dl} = B\oint dl$$

$$= B \times 2\pi r \quad \text{... (ii)}$$

Where,

$\oint dl = 2\pi r$ = circumference of circular loop.

vi. From equation (i) and (ii),

$\therefore$ $B.2\pi r = \mu_0 I$

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{... (iii)}$$

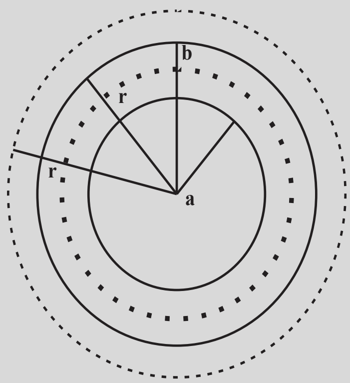
or $$B = \frac{\mu_0}{4\pi}\left(\frac{2I}{r}\right) \quad \text{...(iv)}$$

This is the required equation of Magnetic field at any point due to straight conductor

INTEXT QUESTION

A coaxial cable consist of a central conducting core wire of radius a and a coaxial cylindrical outer conductors of radius b (see figure). The two conductors carry an equal current I in

opposite directions in and out of the plane of the paper. What will be the magnitude of the magnetic field B for
i. a < r < b and
ii. b < r?
What will be its direction ?



Ans:

i By symmetry, B will be tangent to any circle centred on the central conductor.

ii For a < r < b
Amperian loop is a circle of radius r such that a < r < b,

By applying Ampere circuit law

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

$$\therefore \quad \oint B\,dl\cos 0 = \mu_0 (I)$$

$$\therefore \quad B\oint dl = \mu_0 I$$

$$\therefore \quad B\times 2\pi r = \mu_0 I$$

$$\therefore \quad B\times 2\pi r = \mu_0 I$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi r}$$

iii. For r >b

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 (I - I) = 0$$

The two current ar equal and opposite

$$B\times 2\pi r = 0$$

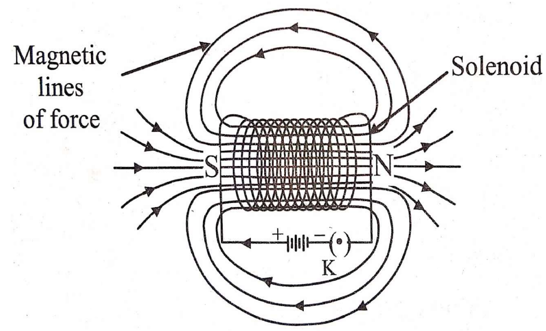
$$\therefore \quad B = 0$$

10.16 Magnetic field of a solenoid and a toroid

Solenoid :

i. When a copper wire with a resistive coating is wound in a chain of loops (like a spring), it is called as solenoid.

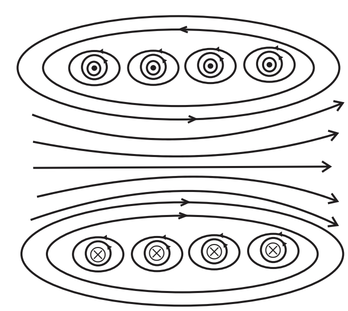
ii. Whenever electric current passed through a solenoid, magnetic of force are produced in a pattern as shown in the figure.



Magnetic field produced by a current carrying solenoid

iii. Consider a long, closely wound helical coil of a conducting wire such that the diameter of the coil is much smaller than its length. This forms a solenoid.

iv. The density of the magnetic field lines along the axis of the solenoid within the solenoid and at a certain distance away from the wire, is uniform as show in figure below.



Hence the magnetic field B parallel to the axis of the solenoid.

v. The lines are widely spaced outside the solenoid and hence the magnetic field is weak there.

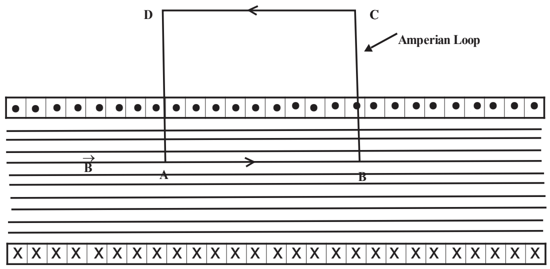
vi. For a real solenoid of finite length, magnetic field is uniform and has a good strength at the centre and comparatively weak at the outside of the coil.

Q.22 Obtain an expression for magnetic field due to the axis of an ideal long straight solenoid.

Ans:

i. Consider a long closely wound helical coil of conducting wir. Let us consider diameter of coil is much smaller than its length.

Let, $n = \frac{N}{L}$ - be the number of turns per unit length.



ii. The dots ⊙ show that the current is coming out of the plane paper the crosses ⊗ show that the current is going into the plane of the paper, both in the coil of square cross section wire.

iii. **Step 1: Select Amperian loop**
Consider an imaginary rectangular loop of length L as an amperian loop such that side ab coincide with axis of solenoid.

iv. Step 2 To Find: $\oint \vec{B}\cdot\vec{dl}$
Now, for closed loop ABCDA ,

$$\oint_{ABCD} \vec{B}\cdot\vec{dl} = \int_A^B \vec{B}\cdot\vec{dl} + \int_B^C \vec{B}\cdot\vec{dl} + \int_C^D \vec{B}\cdot\vec{dl} + \int_D^A \vec{B}\cdot\vec{dl} \quad \text{... (i)}$$

a. As $\vec{B} \perp$ BC and DA. Their dot product is zero.

$$\therefore \quad \int_B^C \vec{B}\cdot\vec{dl} = \int_D^A \vec{B}\cdot\vec{dl} = 0 \quad \text{... (ii)}$$

b. Also, As length of solenoid is much larger than its diameter, the magnetic field at the points outside is zero.

$$\therefore \quad \int_C^D \vec{B}\cdot\vec{dl} = 0 \quad \text{... (iii)}$$

From (i) , (ii) and (iii), we can say

$$\oint_{ABCD} \vec{B}\cdot\vec{dl} = \int_A^B \vec{B}\cdot\vec{dl} = \int_A^B \vec{B}\,\vec{dl}\cos\theta$$

As angle θ between $\vec{B}$ and $\vec{dl}$ is 0^0
As, magnetic induction at any axial point is same.

$$\therefore \quad \oint \vec{B}\cdot\vec{dl} = B\int_A^B dl = BL \quad \text{... (iv)}$$

Since, $\int_A^B dl = L$, Length of solenoid

v. **Step 3 Apply Ampere circuit law**

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 NI = \mu_0 (nLI)$$

$$\therefore \quad BL = \mu_0 (nLI)$$

$$\therefore \quad B = \mu_0 nI$$

Key Point

i. The magnetic induction at a point near either of the end is,

$$B = \frac{1}{2}\mu_0 nI$$

ii. Relation between magnetic induction at the centre and at near the ends is,

$$B_{centre} = 2 \times B_{at\ end}$$

iii. $n = \frac{N}{L}$ - be the number of turns per unit length .

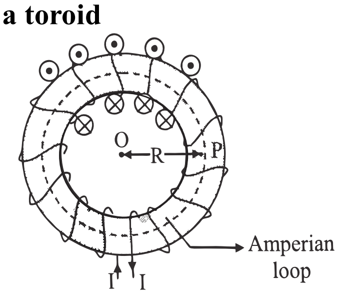
Toroid

i. A hollow circular ring in the shape of a doughnut that has many turns of enamelled wire which are wound so close to each other that there is negligible space between the two turns.

ii. A toroid can be considered as a circular solenoid that is used in an electric circuit, as an inductor at low frequencies when large inductances are required.

iii. The first toroid was invented in 1830 by the physicist Michael Faraday.

Q.23 Using Ampere's law, derive an expression for magnetic induction at a point along the axis of a toroid



Ans: Expression for magnetic induction at a point along the axis of a toroid:

i. A toroid is a solenoid of finite length bent into a hollow circular tube

ii. The magnetic field around the toroid consist of concentric circular lines of force around it. Magnetic field is produced when a steady current 'I' flows through toroid.

iii. The direction of magnetic field at a point is along the tangent to the circular path at that point.

iv. **Step 1: Select Amperian loop**

Consider Amperian loop as a circle of radius R. This loop is concentric with the axis of toroid. P is a point on the loop.

v. **Step 2 To Find:** $\oint \vec{B} \cdot \vec{dl}$

$\oint B \cdot dl = \oint B\, dl \cos\theta = \oint B\, dl$

As angle θ between $\vec{B}$ and $\vec{dl}$ is 0^0

As, magnetic induction along the axial point is same.

$\oint \vec{B} \cdot \vec{dl} = B \oint dl = B \times 2\pi R$ ......(i)

Where,

$\oint dl = 2\pi R$ - circumference of circular loop

Step 3 Apply Ampere circuit law

$\oint \vec{B} \cdot \vec{dl} = \mu_0(NI)$ ....(ii)

Where,

N = total number of turns in the toroid.

I = total current flowing through toroid.

$B \times 2\pi R = \mu_0(NI)$

∴ $B = \frac{\mu_0 NI}{2\pi r}$

∴ $B = \mu_0 nI$ ....(iii)

This is expression for magnetic induction along the axis of toroid.

Where, $n = \frac{N}{2\pi r}$ - number of turns per unit length of toroid.

Type - X
Numerical based on solenoid and toroid

Formulae used

1. For solenoid

$B = \mu_0 nI = \frac{\mu_0 NI}{L}$

2. For Toroid

$B = \mu_0 nI = \frac{\mu_0 NI}{2\pi r}$

★1 A solenoid of length π m and 5 cm in diameter has winding of 1000 turns and carries a current of 5 A. Calculate the magnetic field at its centre along the axis.

Data : L = πm, D = 5 cm, N = 1000 turns, I = 5 A

To find: B along the axis

Formula: $B = \mu_0 nI = \frac{\mu_0 NI}{L}$

Solution: $B = \mu_0 nI = \frac{\mu_0 NI}{L}$

$B = 4\pi \times 10^{-7} \times \frac{1000}{\pi} \times 5$

$= 20 \times 10^{-7+3} = 2 \times 10^{-3}$ T

Ans : The magnetic field is 2×10^{-3} T

★2) A solenoid of length 25 cm has inner radius of 1 cm and is made up of 250 turns of copper wire. For a current of 3 A in it, what will be the magnitude of the magnetic field inside the solenoid?

Data: L= 25 cm = 0.25 m, r = 1 cm, N = 250 turns, I = 3 A

To find: B inside the solenoid

Formula: $B = \mu_0 nI = \frac{\mu_0 NI}{L}$

Solution : $B = \frac{\mu_0 NI}{L}$

$B = 4\pi \times 10^{-7} \times \frac{250}{0.25} \times 3$

$B = 4\pi \times 10^{-7} \times 10^3 \times 3$

$B = 3.77 \times 10^{-3}$ T

Ans : Magnetic field inside the solenoid is 3.77×10^{-3}T

★3) A toroid of narrow radius of 10 cm has 1000 turns of wire. For a magnetic field of 5×10^{-2} T along its axis, how much current is required to be passed through the wire?

Data: R = 10 cm = 10×10^{-2} m = 10^{-1} m, N = 1000, B = 5×10^{-2} T

To find: I

Formula: $B = \mu_0 nI = \frac{\mu_0 NI}{2\pi r}$

Solution: $B = \frac{\mu_0 NI}{2\pi r}$

$5\times10^{-2} = 4\pi\times10^{-7} \times \frac{10^4}{2\pi} \times I$

$$I = \frac{5\times10^{-2}}{2\times10^{-3}} = 2.5 \times 10 = 25\text{ A}$$

Ans : The electric current is 25 A

AnswerKey

Set - 1 (MCQ)

1	**a**	2	**d**	3	**b**	4	**a**	5	**a**
6	**b**	7	**b**	8	**c**	9	**c**	10	**d**

Try Yourself

11	**b**	12	**d**	13	**a**	14	**d**	15	**d**
16	**a**	17	**d**	18	**c**	19	**b**	20	**d**

Set - 2 (MCQ)

1	**c**	2	**b**	3	**d**	4	**c**	5	**d**

Try Yourself

6	**c**	7	**a**	8	**b**	9	**b**	10	**d**

Set - 3 (MCQ)

1	**b**	2	**a**	3	**a**	4	**b**	5	**b**
6	**b**	7	**b**	8	**a**	9	**c**	10	**b**

Try Yourself

11	**b**	12	**c**	13	**d**	14	**d**	15	**c**
16	**c**	17	**d**	18	**a**	19	**c**	20	**b**

Set - 4 (MCQ)

1	**b**	2	**d**	3	**c**	4	**b**	5	**b**
6	**a**	7	**d**	8	**b**	9	**d**	10	**b**
11	**d**	12	**a**	13	**a**	14	**b**		

Set - 5 (MCQ)

1	**c**	2	**a**	3	**c**	4	**a**	5	**d**
6	**c**	7	**a**						

Try Yourself

8	**a**	9	**a**	10	**c**	11	**a**	12	**a**
13	**b**	14	**d**	15	**c**				

Set - 6 (MCQ)

1	**a**	2	**a**	3	**c**	4	**b**	5	**a**
6	**b**	7	**c**	8	**c**	9	**d**	10	**a**

Set 1 :

1. **Solution (a)**

When projected along the direction of the magnetic field, the electron experiences opposite force.

As the electron is negatively charged, its motion gets retarded when projected along the direction of the electric field. Hence its velocity will decrease

2. **Solution (d)**

As the directions of electric and magnetic fields are parallel to the direction in which the charged particle is released the particle will move along a straight line.

3. **Solution (b)**

The magnetic field exerts a force perpendicular to the direction of motion of the charged particle. It continuously deflects the particle from its path but does no work on it. Hence momentum of the particle changes but kinetic energy remains same.

4. **Solution (a)**

Radius of the circular path of a charged particle in a perpendicular magnetic field.

$$r = \frac{mv}{qB}$$

For both electron and proton quantities mv, q and B are all same. Hence radius r will be same.

5. **Solution (a)**

Given $q = -16 \times 10^{-18}$ C, $v = 10\hat{i}$ ms^{-1}.

$$\vec{B} = B\hat{j}\text{ Wb}, \vec{E} = -10^4\hat{k}\text{ Vm}^{-1}$$

$$\vec{F_e} = q\vec{E} = -16\times10^{-18}\times(-10^4\hat{k})$$

$$= 16\times10^{-14}\hat{k}\text{ N}$$

$$\vec{F_m} = q(\vec{v}\times\vec{B}) = -16\times10^{18}(10\hat{i}\times B\hat{j})$$

$$= -16\times10^{-17}\ B\hat{k}\,N$$

As the particle continues to move along the same direction,

$F_m = F_e$

$16 \times 10^{-17} B = 16 \times 10^{-14}$

$B = 10^3$ Wb m^{-2}.

6. Solution(b)

$$r = \frac{mv}{qB}$$

When the speed is doubled and magnetic field is halved,

$$r' = \frac{m \times 2v}{q(B/2)} = 4r$$

7 Solution (b)

$$r_p = \frac{m_p v}{qB} = \frac{m_p v}{eB}$$

$$r_\alpha = \frac{m_\alpha v}{q_\alpha B} = \frac{4m_p v}{2eB} = 2r_p$$

$$\frac{r_p}{r_\alpha} = \frac{1}{2} = 1:2$$

8. Solution (c)

For the electron moving along the straight line,

$F_m = F_e$ or $evB = eE$

$$v = \frac{E}{B} = \frac{1500}{0.40} = 3.75 \times 10^3 \text{ m/s}$$

9. Solution (c)

K.E. gained = qV = e × 1 kV = 1 keV.

10. Solution (d)

$$\text{Here } \frac{mv^2}{r} = qvB \quad \text{or} \quad v = \frac{qrB}{m}$$

$$K = \frac{1}{2}mv^2 = \frac{q^2r^2B^2}{2\,m}$$

$$\frac{K_p}{K_d} = \frac{e^2r^2B^2}{2m_p} \cdot \frac{2 \times 2m_p}{e^2r^2B^2} = 2$$

$K_p = 2\,K_d = 2 \times 50 = 100$ keV

11. Solution (b)

As the magnetic force acts perpendicular to the direction of motion of the charged particle, the work done by the field is zero.

12. Solution (d)

$$\text{As } r = \frac{mv}{qB}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}.$$

13. Solution (a)

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}.$$

T is independent of speed v.

14. Solution (d)

As $\vec{v} = 0$,

So $\vec{F} = q(\vec{v} \times \vec{B}) = 0$

15 Solution (d)

The component vsin60° of velocity v makes the particle move along the field while the component vsin60° throws it into circular motion. The resultant motion is helix

16. Solution (a)

$$\text{Radius, } r = \frac{mv}{qB}$$

For same v and B, $r \propto \frac{m}{q}$

$$\therefore \frac{r_p}{r_\alpha} = \frac{m_p}{m_\alpha} - \frac{q_\alpha}{q_p} = \frac{m_p}{4m_p} \cdot \frac{2e}{e} = \frac{1}{2} = 1:2$$

17 Solution (d)

$$\text{Here, } evB = \frac{mv^2}{r} \quad \text{or} \quad \frac{e}{m} = \frac{v}{rB}$$

For same v and B, $\frac{e}{m} \propto \frac{1}{r}$

The radius of curvature is minimum for D. Hence its e/m is highest.

18. Solution (c)

As $r = \frac{mv}{qB}$ i.e., $r \propto v$

When v is doubled, r also gets doubled.

19 Solution (b)

$$R = \frac{\sqrt{2mK}}{qB} \quad \text{or} \quad \sqrt{2mK} = qBR$$

Now, $q_p = q_d$ and B and R are fixed,

$$\sqrt{2mK} = \text{constant} \quad \text{or} \quad K \propto \frac{1}{m}$$

$$\frac{K_p}{K_d} = \frac{m_d}{m_p} = \frac{2}{1}$$

or $K_p = 2\,K_d = 2 \times 40 = 80$ MeV.

20. Solution (d)

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \frac{1}{2}\frac{q^2B^2r^2}{m}$$

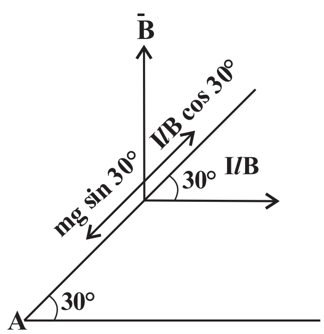
Here r and B are same for both particles.

$$\therefore \frac{K_p}{K_\alpha} = \frac{q_p^2}{q_\alpha^2}\cdot\frac{m_\alpha}{m_p} = \frac{e^2}{(2e)^2}\cdot\frac{4m_p}{m_p} = 1:1$$

➤ Set 2 :

1 . Solution (c) :

From fig. for Equilibrium,



mg sin 30° = I/B cos 30°

$$I = \frac{mg}{\ell B}\tan 30°$$

$$= \frac{0.5 \times 9.8}{0.25 \times \sqrt{3}} = 11.32\text{ A}$$

2. Solution (b)

$F = IlB\sin 90^0 = 8\times5\times1.5\times1 = 60$ N.

3. Solution (d)

For equilibrium of wire in mid-air,

Weight of wire = Force exerted by magnetic field

$mg = IlB\sin 90^0$

$$B = \frac{mg}{Il} = \frac{200\times10^{-3}\times9.8}{2\times1.5} = 0.65\text{ T}$$

4. Solution (c)

$F_{PQ} = IlB\sin 0^0 = 0$

$F_{QP} = IlB\sin 90^0 = IlB.$

5. Solution (d)

$$\vec{F} = i(\vec{l}\times\vec{B}) = i\left[l\hat{i}\times B_0(i+\hat{j}+\hat{k})\right]$$

$$= ilB_0\left[\hat{i}\times\hat{i} + i\times\hat{j} + \hat{i}\times\hat{k}\right] = ilB_0\left[\hat{k}-\hat{j}\right]$$

$$\left|\vec{F}\right| = il\,B_0\sqrt{(1)^2+(-1)^2} = \sqrt{2}il\,B_0.$$

⇨ Try Yourself :

6. Solution (c)

$F = Bil\sin\theta$

$$= 500\times10^{-4}\times3\times(40\times10^{-2})\times\frac{1}{2} = 3\times10^{-2}\,N$$

7. Solution (a)

F = BiL sin θ

7.5 = 2 × 5 × 1.5 × sin θ

sin θ = 0.5

θ = $\sin^{-1}$ 0.5

θ = 30

$$= 1.5\times10\times1\times\frac{1}{2} = 7.5$$

8. Solution (a)

$F = Bil \Rightarrow 1\times9.8 = 0.98\times i\times1 \Rightarrow i = 10A$

9. Solution (b)

$\vec{F} = 9\,(\,l\times\vec{B})$

F = Bill 69 n θ

= 0.02 × 5 × 0.1 × 59 n 30°

= 5×10^{-3} N

10. Solution : (d)

$$B_1 = \frac{\mu_0 i}{2\pi r} = \frac{2\times10^{-7}\times30}{2\times10^{-2}}$$
$$= 3\times10^{-4}\ T$$

$$\therefore \quad B = \sqrt{B_1^2 + B_2^2}$$
$$\sqrt{(3)^2+(4)^2}\times10^{-4}$$
$$= 5\times10^{-4}\ T$$

➢ Set 3 :

1. Solution : (b)
The current loop will move towards the wire

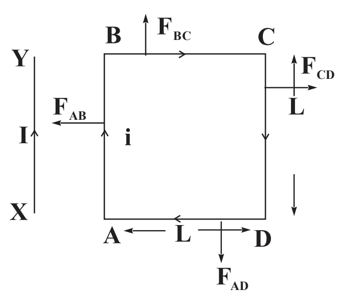
2. Solution : (a)

Normal to the loop area will be parallel to the magntic field , i.e., $\theta = 0^o$

$\therefore \quad \tau = IBA\sin 0^o = 10\times0.1\times0.01\times0 = \mathbf{0}$

3. Solution. (a)

The direction of current in conductor



XY and AB is same

$\therefore \quad F_{AB} = i\ell B$ (attractive)

$$F_{AB} = i(L).\ \frac{\mu_o I}{2\pi\left(\frac{L}{2}\right)}(\leftarrow) = \frac{\mu_0 il}{\pi}(\leftarrow)$$

$F_{BC}\ (\uparrow)$ and $F_{AD}\ (\downarrow)$

$\Rightarrow$ cancels each other

$F_{CD} = i\ell B$ (replusive)

$$F_{CD} = i(L)\ \frac{\mu_o il}{\pi} - \frac{\mu_0 il}{3\pi} = \frac{2\mu_0 il}{3\pi}$$

Therefore the net force on the loop $F_{net} = F_{AB} + F_{BC} + F_{CD} + F_{AD}$

$$F_{net} = \frac{\mu_o il}{\pi} - \frac{\mu_0 il}{3\pi} = \frac{2\mu_0 il}{3\pi}$$

4. Solution. (b)

$$M = NiA = 20\times\frac{22}{7}(4\times10^{-2})^2 3 = 0.3\,A-m^2$$

5. Solution (b)

Given : A = 400×10^{-4} m²

n = 500

B = 4×10^{-3}

Area makes complementary angle

$\therefore$ Angle between magnetic field and normal of coil is $\theta = 90° - 60° = 30°$

i = 0.2

$\tau = M \times B$

$= n\, i\, AB \sin\theta$

$= 500\times0.2\times400\times10^{-4}\times4\times10^{-3}\times\frac{1}{2}$

$= 8\times10^{-3}$ Nm

6. Solution : (b)

7. Solution : (b)

$\tau = niAB\sin\theta$

$\sin\theta = I$

$\therefore \quad \tau = niAB = 100\times2\times20\times10\times20\times10^{-4}$
$= 80$ Nm

8. Solution : (a)

Given : Area of coil an moving coil Galvanometer = 15cm² = 15×10^{-4} m²

n = 20

Magnetic induction = 0.2T

C = 10^{-6} N/m / kg

Deflection = 45 = $\frac{\pi}{4}$

c = nBIA

$\Rightarrow \quad 10^{-6} = 20\times0.2\times I\times15\times10^{-4}$

$$I = \frac{1}{6 \times 10^3}$$

Deflection is 45°

Total cirrent = I × total deficetion

$$I = \frac{1}{6 \times 10^3} \times 45$$

$I_{Total} = 75\times10^{-4}$ A.

9. Solution : (c)
Couple per unit twist = 2×10^{-6} N/m

Deflection = 4

= Total couple = 8×10^{-6}

τ = ni AB

$8\times10^{-6} = 200 \times 80 \times 10^{-4} \times 0.2 \times i$

$i = 2.5 \times 10^{-4}$ A.

10. Solution : (b)
Given : A = 4 cm^2 = 4×10^{-4} m^2
N = 500
B = 2T
I = 10–4 A
θ = 20°
NBIA = Cθ

$$C = \frac{NBIA}{\theta}$$

$$C = \frac{(500)\,(2)\,(4\times10^{4})\,(10^{4})}{20}$$

$$C = \frac{4000 \times 10^{-8}}{20}$$

$= 2\times 10^{-6}$ Nm

⇨ Try Yourself :

11. Solution. (b)

$$\text{Sensitivity} = \frac{NAB}{C}$$

$$\text{Sensitivity } \alpha \frac{1}{c}$$

So, as C decreases sensitivity will ancrease.

12. Solution : (c)

$$\tau = NIBA\sin\theta$$

$$= 1\times10\times0.1\times0.01\sin 90^\circ = \mathbf{0.01Nm}.$$

13. Solution : (d)

The forces on opposite sides of the loop are unequal but opposite in a non-uniform magnetic field. so that current experiences both a force and a torque

14. Solution : (d)

The plane of coil will orient itself so that its area vector becomes parallel to the magnetic field Then $\tau = mB\sin 0^\circ = 0$

15. Solution. (c)

Net force on a current carrying closed loop is always zero, if it is placed in an uniform magnetic field.

16. Solution. (c)

$$M = i\pi r^2$$

17. Solution. (d)

$$M = iA = 0.1\times\pi\times(0.05)^2$$

$$= (0.1)\times3.14\times25\times10^{-4} = 7.85\times10^{-4}\,amp - m^2$$

18. Solution. (a)

Because τ = NiABcosθ

19. Solution. (c)

20. Solution. (b)

$$i = \frac{C\theta}{NAB} \Rightarrow i \propto \theta$$

➤ Set 4 :

1. Solution : (b) Zero

There is current flowing only on the surface id the tube. If we sraw any circuital loop inside the wire, at does not enclose any current. Hence, according to Ampere's circuital law, there is no induced magnetec field inside.

2. Solution : (d) The magnetic field at any point inside the pipe is zero.

Using Ampere's law $\oint Bdl = \mu_0 I$ the magnetic field insif=de the pipe is zero because at enclose no current i.e I = 0.

3. **Solution : (c)**

$$\frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{\frac{1}{2}}$$

As OP is perpendicular to the plane of wires means plane of papers so the magnetic field B1 due to AOB is towards left and field B2 due to COD is downwards

The required field $B = \sqrt{B_1^2 + B_2^2}$

$$B = \sqrt{\left(\frac{\mu_0 I_1}{2\pi d}\right)^2 + \left(\frac{\mu_0 I_2}{2\pi d}\right)^2}$$

$$= \frac{\mu_0}{2\pi d}\sqrt{I_1^2 + I_2^2} = \frac{\mu_0}{2\pi d}\left(I_1^2 + I_2^2\right)^{\frac{1}{2}}$$

4. **Solution : (b)**

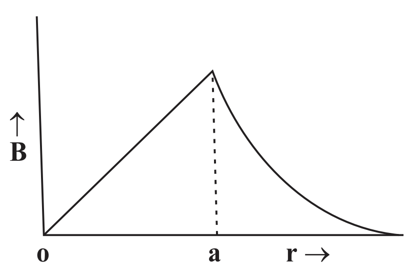
$$B_1 = \frac{\mu_o I}{2\pi a^2}.\frac{a}{2} \text{ and } B_2 = \frac{\mu_o I}{2\pi(2a)}.$$

$$\therefore \frac{B_1}{B_2} = \mathbf{1.}$$

5. **Solution: (b)**
The fields of the two wires will be in the opposite directions at the midway point.

$$\therefore B = B_1 - B_2 = \frac{\mu_o i}{2\pi r}\frac{\mu_o i}{2\pi r} = 0.$$

6. **Solution : (a)**



From ampere's low, field inside the wire is

$\mu_0(q\pi r^2) = B\,(2\pi r)$

$$\Rightarrow B = \frac{\mu_0 ir}{2}$$

So at varies linearly upto a.
From amperes low field outside the wire is

$\mu_0(q\pi a^2) = B\,(2\pi r)$

$$\Rightarrow B = \frac{\mu_0 ia^2}{2r}$$

For r < a, $B \propto r$ and for r > a,

7. **Solution : (d)**

$$d\vec{B} = \frac{\mu_0}{4\pi} \times \frac{Id\vec{l} \times \hat{r}}{r^3}$$

$$dB = \frac{\mu_0 i}{4\pi}\int\frac{\left(d\vec{l} \times \hat{r}\right)}{r^2}$$

We know that = $\hat{r} \Rightarrow \frac{\vec{r}}{r}$

$$dB = \frac{\mu_0 i}{4\pi}\int\frac{\left(d\vec{l} \times \hat{r}\right)}{r^3}$$

8. **Solution: (b)**
For a straight current carrying wire,

$$B \propto \frac{1}{4} \quad \therefore \frac{B'}{B} = \frac{r}{2r} = \frac{1}{2}$$

$$B' = \frac{1}{2}B = \frac{1}{2} \times \mathbf{0.4 = 0.2T}$$

9. **Solution: (d)**

The magnetic field of a current carrying wire does not depend on its diameter.

10. **Solution: (b)**

According to right hand thumb rule, the magnetic fields of the wires will be in opposite directions at the midway point.

$$\therefore B = B_2 - B_1 = \frac{\mu_o I_2}{2\pi r_2} - \frac{\mu_o I1}{2\pi r_1}$$

$$= \frac{\mu_o}{2\pi}\left[\frac{5}{2.5} - \frac{2.5}{2.5}\right] = \frac{\mu_o}{2\pi}T.$$

11. **Solution : (d)** $\frac{\mu_0}{2\pi}$

The fields created by q_1 and q_2 will be in opposite direction.

$$B = \frac{\mu_0}{4\pi}\frac{2q_2}{r/2} - \frac{\mu_0}{4\mu}\frac{2q_1}{r/2}$$

$$= \frac{\mu_0 4}{4\pi r}(q_2 - q_1)$$

$$= \frac{\mu_0 4}{4\pi 5}(5 - 2.5) = \frac{\mu_0}{2\pi}$$

Same reasoning as in the above problem.

12. Solution : (a)

$$B = \frac{\mu_o I}{2\pi r} = \frac{2\times10^{-7}\times20}{0.10} = 4\times10^{-5}\ \text{Wb/m}^2$$

13. Solution: (a)

$$\frac{B_2}{B_1} = \frac{r_1}{r_2} = \frac{r}{2r} = \frac{1}{2}$$

$$B_2 = \frac{1}{2}B_1 = \frac{1}{2}\times0.4 = \mathbf{0.2T}$$

14. Solution: (b)

$$B = \frac{\mu_o I}{2\pi r}$$

As I and r are same , so $B_p = B_Q$

➢ Set 5 :

1. Solution : (c)

In first case,

$$F = \frac{\mu_0}{2}\frac{I_1 I_2}{d} \quad \text{(attractive)}$$

In second case,

(repulsive)

$$F = \frac{\mu_0}{2\pi}\frac{(-2I_1)I_2}{3d} = -\frac{2}{3}F$$

2. Solution: (a)

Two electron beams travelling in the same direction produce currents in the same direction, so they attract each other.

3. Solution : (c)

The wires D and G exert repulsive forces on wire C. The net force is

$$F = \frac{\mu_0}{4\pi}\left[\frac{I_1 I_2}{r_1} - \frac{I_2 I_3}{r_2}\right] l$$

$$= 10^{-7}\left[\frac{30\times10}{0.03} - \frac{10\times20}{0.02}\right]\times0.25\ \text{N} = 0.$$

4. Solution: (a)

$$F_{AB} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r} l$$

$$= 2\times10^{-7}\times\frac{2\times1}{2\times10^{-2}}\times15\times10^{-2}$$

$= 30 \times 10^{-7}$ N (attractive)

$$F_{CD} = 2\times10^{-7}\times\frac{2\times1}{12\times10^{-2}}\times15\times10^{-2}\ \text{N}$$

$= 5 \times 10^{-7}$ N (repulsive)

$$F_{net} = F_{AB} - F_{CD} = 25\times10^{-7}\ \text{N}$$

(towards wire)

5. Solution: (d)

$$F = \frac{\mu_0}{4\pi}\cdot\frac{2I^2 l}{r}$$

$$30\times10^{-7} = 10^{-7}\times\frac{I^2\times9}{0.15}$$

$$I^2 = \frac{30\times10^{-7}}{120\times10^{-7}} = \frac{1}{4}$$

or $I = \frac{1}{2}\text{A} = 0.5\ \text{A}$

6. Solution (c)

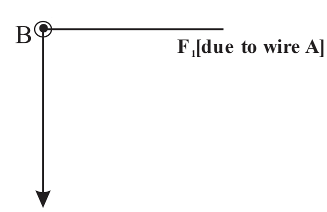
Force per unit length between two parallel current carrying conductors,

$$F = \frac{\mu_0 i_1 i_2}{2\pi d}$$

Since same current flowing through both the wires

$i_1 = i_2 = i$

so $F_1 = \frac{\mu_0 i^2}{2\pi d} = F_2$



Magnitude of force per unit length on the miffle wire 'B'

$$F_{net} = \sqrt{F_1^2 + F_2^2} = \frac{\mu_0 i^2}{\sqrt{2}\pi d}$$

7. Solution (a)

$$F = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r} \times 1$$

$$= \frac{10^{-7} \times 2 \times 10 \times 2}{0.1} \times 2 = 8 \times 10^{-5} N$$

⇨ Try Yourself :

8. Solution: (a)

The force between two parallel wires carrying currents I_1 and I_2 in the same direction is

$$F = \frac{\mu_0}{2}\frac{I_1 I_2}{d} \quad \text{(attractive)}$$

But $I_1 = I_2 = I$

$$F = \frac{\mu_0 I^2}{2\pi d} \quad \text{(attractive)}$$

9. Solution: (a)

Two wires carrying currents in the same direction attract each

10. Solution : (c)

Two parallel beams of positions moving in the same direction set up two parallel currents flowing in the same direction. Hence they attract each other.

11. Solution: (a)

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}$$

$$= 10^{-7} \times \frac{2 \times 1 \times 1}{1} = 2 \times 10^{-7}\ Nm^{-1}.$$

12. Solution: (a)

$$F = 10^{-7} \times \frac{2 \times 10 \times 10}{0.10} = 2 \times 10^{-4}\ Nm^{-1}.$$

13. Solution: (b)

Force per unit length,

$$f = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{\mu_0 i \cdot i}{2\pi b} = \frac{\mu_0 i^2}{2\pi b}$$

14. Solution (d)

18×10^{-7}

F = ILB

$$B = \frac{\mu_0 I_2}{2\pi d}$$

Given = L = 1

I = 3A

$I_2 = 3A$

d = 1m

$$F = \frac{3 \times 1 \times 4\pi \times 10^{-7} \times 31}{2\pi} = 18 \times 10^{-7} N$$

$$F = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r} \times l$$

15. Solution : (c)

98 A

Since ware hangs an the air

mg = bq$l$

$$\frac{m}{l} = \frac{Bq}{g}$$

$$20 \times 10^{-3} = \quad B = \left(\frac{\mu_0 q}{92\pi r}\right)^q$$

q = 98 A

$$\frac{\mu_0 i_1 i_2}{2\pi r} = \frac{mg}{l}$$

➤ Set 6 :

1. Solution : (a)

$$B_A = \frac{\mu_0 I}{2R}; \quad B_B = \frac{\mu_0 I}{2 \times 2R}$$

$$\frac{B_A}{B_B} = 1$$

2. Solution : (a)

250 μT

Given = B = 54 × 10^{-6}

a = 3 cm = 3×10^{-2}

x = 4cm = 4×10^{-2}m

$\mu_0 = 10^{-7}$

Formula : $$\frac{\mu 0 I_2 \pi}{4\pi}\frac{(a^2)}{(a^2 + x^2)^{\frac{3}{2}}}$$

$\therefore \quad 54\times10^{-6} = \frac{10^{-7}\times I\times 2\pi\times(3\times10^{-2})^2}{[(3\times10^{-2})^2+(4\times10^{-2})^2]^{\frac{3}{2}}}$

$\therefore \quad I = \frac{54\times10^{-7}(25\times10^{-4})^{\frac{3}{2}}}{10^{-7}\times3.14\times2\times9\times10^{-4}}$

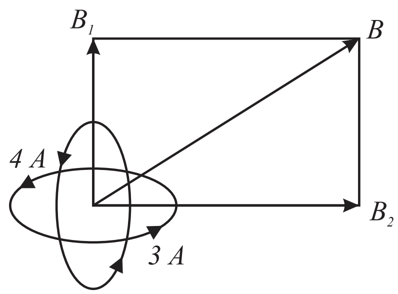
I = 11.94 A

Magnetic field at the

centre $= \frac{\mu 0 I}{4\pi a} = 2\pi$

$= \frac{10^{-7}\times11.94\times2\pi}{3\times10^{-2}} = 250\ \mu T$

3. Solution : (c)



$B_1 = \frac{\mu_0 I_1}{2r} = \frac{4\pi\times10^{-7}\times3}{2\times2\pi\times10^{-2}} = 3\times10^{-5}\ Wbm^{-2}$

$B_2 = \frac{\mu_0 I_2}{2r} = \frac{4\pi\times10^{-7}\times4}{2\times2\pi\times10^{-2}} = 4\times10^{-5}\ Wbm^{-2}$

$B = \sqrt{B_1^2+B_2^2} = \sqrt{3^2+4^2}\times10^{-5} = 5\times10^{-5}\ Wbm^{-2}$

4. Solution : (b)

The carcum forence of the first loop as the length of the wire = 2 π r

Magnetic field at the centre of his wqre

$B = \frac{\mu 0 I}{2R}$

The same wqre as bent to n xqeular coils

n × 2 π r = 2R

$r = \frac{R}{n}$

Two magnetic field at the the centre of the loops

$= \frac{n\mu_0 I}{2r}$

$= \frac{n_2\mu_0 I}{2R} = n^2 B$

5 Solution: (a)

$B_{axial} = \frac{1}{8}B_{centre}$

or $\frac{\mu_o IR^2}{2(R^2+r^2)^{3/2}} = \frac{1}{8}\frac{\mu_o I}{2R}$

or $\frac{R^2}{(R^2+r^2)^{3/2}} = \frac{1}{8R}$ or $(R^2+r^2)^{3/2} = 8R^3$

or $R^2+r^2 = 4R^2$ or $r = \sqrt{3}$**R**.

6. Solution: (b)

Magnetic field due to the circular loop at center O,

$B_1 = \frac{\mu_o I_c}{2R}$

Magnetic field due to the straight wire at point O,

$B_2 = \frac{\mu_o I_e}{2\pi H}$

As these two fields act in opposite directions, so

$B = B_1 - B_2 = 0$

or $B_1 = B_2$

or $\frac{\mu_o I_c}{2R} = \frac{\mu_o I_e}{2\pi H}$

or $H = \frac{I_e R}{I_c \pi}$

7. Solution: (c)

Let the radii of the two coils be 2*a* and *a*, then their resistances will be 2R and R respectively.

Given $B_1 = B_2$

or $\frac{\mu_o I_1}{2\times2a} = \frac{\mu_o I_2}{2a}$

or $\frac{\mu_o}{4a}.\frac{V_1}{2R} = \frac{\mu_o}{2a}.\frac{V_2}{R}$

or $\frac{V_1}{V_2} = 4$

8. Solution: (c)

$$B = \frac{\mu_o NI}{2a}$$

or $5.6\times10^{-5} = \frac{4\pi\times10^{-7}\times1\times I}{2\times0.05}$

$$I = \frac{56}{4\pi} = 4.45 \approx \mathbf{4A}$$

9. Solution: (d)

A revolving charge is equivalent to current loop.Here

I = frequency ×charge =100 e ×1 =100 e

$$B = \frac{\mu_o I}{2a} = \frac{\mu_o\times100\times1.6\times10^{-19}}{2\times0.8} = \mathbf{10^{-17}}\,\mu_o$$

10. Solution: (a)

$$\frac{\mu_0 q}{2\pi R}(\pi - \alpha + \tan\alpha)$$

Magnetic field qntensqty sue to are part aof wire

$$B_1 = \frac{\mu_0 I}{4\pi R}(2\pi - 2\alpha)$$

Magnetic field intensity due to line part of wire

$$B_2 = \frac{\mu_0 I(\sin\alpha+\sin\alpha)}{4\pi R\cos\alpha}$$

$$B_{net} = B_1 + B_2$$

$$B_{net} = \frac{\mu_0 I}{4\pi R}(2\pi - 2\alpha) + \frac{\mu_0 I(\sin\alpha+\sin\alpha)}{4\pi R\cos\alpha}$$

$$B_{net} = \frac{\mu_0 I}{2\pi R}(\pi - \alpha + \tan\alpha)$$

⇨ Try Yourself :

11. Solution : (c)

Here, $B = \frac{\mu_0 NI}{2r}$ i.e., $B \propto \frac{1}{r}$

12. Solution: (b)

$$B = \frac{\mu_o I}{4R} = \frac{4\pi\times10^{-7}\times10}{4\times0.20}$$

$$= 5\pi\times10^{-6}\,T = 5\pi\mu\mathbf{T}.$$

13. Solution : (b)

Case 1 : Number of terms = 1

Current = I

length of wire = I

radius $= \frac{1}{2\pi} = r_1$

Case 2 : Number of terms = n

Current = I

length of wire = I

radius $= \frac{1}{2\pi n} = r_2$

using relation $B = \frac{\mu_0 nI}{2r}$

$$\frac{B_1}{B_2} = \frac{\mu_0 I}{2n}\times\frac{2r_2}{\mu_0 nI}$$

$$= \frac{r_2}{nr_1} = \frac{1}{2\pi n}\times\frac{1}{n}\times\frac{2\pi}{1} = \frac{1}{n^2}$$

14. Solution: (a)

Magnetic field at the centre of aoil as given by

$$B = \frac{\mu_o 2\pi n}{4\pi R} = I$$

$\therefore$ $B\ \alpha\ \frac{1}{R}$

$$\frac{B_A}{B_B} = \frac{I_A R_B}{I_B R_A} = \frac{I/R}{2I/2R} = I$$

15. Solution : (d)

The magnetic field at the centre of the coil is $B = \frac{\mu nI}{2a}$

Total length of a coil is in I turn the radius

$2\pi a = I$

$a = \frac{1}{2\pi}$

If number of turns becom twice

$= 4\pi r = L$

$= r = \frac{L}{4\pi} = \frac{a}{2}$

n = 2 radius become $\frac{a}{2}$

$$B_1 = \frac{\mu 2I}{\frac{a}{2}} = \frac{4\mu n I}{a} = 4B$$

16. Solution: (a)

$$B = \frac{\mu_o NI}{2a} = \frac{\mu_o \times 1 \times 1}{2 \times 1} = \frac{\mu_o}{2}$$

17. Solution: (c)

As $B = \frac{\mu_o NI}{2a}$

$\therefore \quad \pi = \frac{4\pi \times 10^{-7} \times 1 \times I}{2 \times 0.5}$

or $\quad I = 2.5 \times \mathbf{10^6\ A}$.

18. Solution: (c)

For the coils connected in series, current I is same. Also N is given to be same

$\therefore \quad B \propto \frac{1}{a}$ or $\frac{B_1}{B_2} = \frac{a_2}{a_1} = \frac{20}{10} = \mathbf{2:1}$

19. Solution: (c)

$$B = \frac{\mu_o NI}{2a}$$

or $\quad 5.6 \times 10^{-5} = \frac{4\pi \times 10^{-7} \times 1 \times I}{2 \times 0.05}$

$$I = \frac{56}{4\pi} = 4.45 \approx \mathbf{4A}$$

20. Solution: (d)

A revolving charge is equivalent to current loop. Here

I = frequency ×charge =100 $e \times 1 = 100\ e$

$$B = \frac{\mu_o I}{2a} = \frac{\mu_o \times 100 \times 1.6 \times 10^{-19}}{2 \times 0.8} = \mathbf{10^{-17}}\mu_o$$

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