

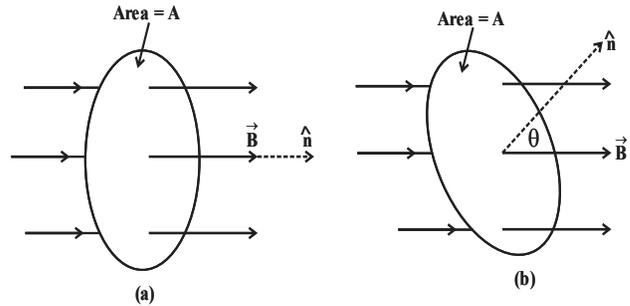
## Syllabus

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## 12.1 Introduction

**Magnetic Flux**

- i. The magnetic flux through any surface placed in a magnetic field is the total number of magnetic lines of force crossing this surface normally.
- ii. It is measured as the product of the component of the magnetic field normal to the surface and the surface area.
- iii. Magnetic flux is scalar quantity, denoted by  $\phi$  and  $\phi_B$ .
- iv. If a uniform magnetic field  $\vec{B}$  passes normally through a plane surface area  $A$ .



Then the component of the field normal to this area will be  $B \cos \theta$ , so that

$$\phi = B \cos \theta \times A$$

$$\phi = BA \cos \theta = \vec{B} \cdot \vec{A}$$

Here the direction of vector  $\vec{A}$  is the direction of the outward drawn normal to the surface.

v. **Dimension of magnetic flux.**

As we know that

$$\phi = BA$$

$$\therefore \phi = \frac{F}{qv \sin \theta} \cdot A \quad \dots \left( \because B = \frac{F}{qv \sin \theta} \right)$$

$$[\phi] = \frac{[M^1 L^1 T^{-2}]}{[A^1 T^1][L^1 T^{-1}]} \cdot [L]^2 = [M^1 L^2 T^{-2} A^{-1}]$$

vi. **SI unit of magnetic flux.**

The SI unit of magnetic flux is weber (Wb).

vii. **CGS unit of magnetic flux.**

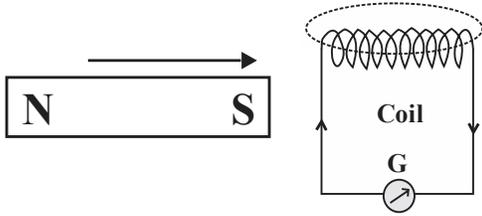
The CGS unit of magnetic flux is maxwell (Mx).

$$1 \text{ Wb} = 10^8 \text{ maxwell}$$

○ **The experimental observations of Faraday's experiment of coil and magnet**

- i. When a magnet approaches a closed circuit consisting of a coil, it produces a

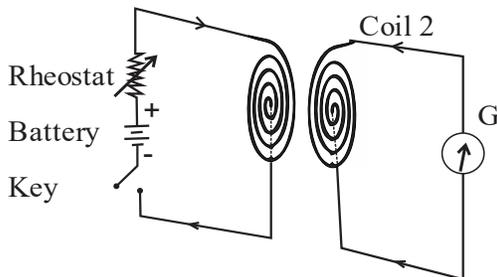
current in it. This current is known as induced current.



- ii. When the magnet is taken away from the closed circuit, a current is again produced but in the opposite direction with respect to the induced current produced that in experiment (i).
- iii. If instead of the magnet, the coil is moved towards the magnet or away from it, an induced current is produced in the coil (i.e., in the closed circuit).
- iv. If the polarity of approaching or receding magnet is changed the direction of induced current in the coil is also changed.
- v. The magnitude of induced current depends on the relative speed of the coil with respect to magnet. It also depends upon the number of turns in the coil.
- vi. The induced current exists so long as there is a relative motion between the coil and magnet.

○ **The experimental observations of Faraday's experiment of two coils with planes facing each other**

One coil is connected in series with a battery, rheostat and key while the ends of the other coil are connected to a galvanometer (G). The coil which consists of a source of emf (a battery) is termed as primary coil while the other as secondary coil. With these two coils, following observation are made.



- i. When the circuit in the primary coil is closed or broken, a momentary deflection is produced in the galvanometer at the time of make or break. When the circuit is closed or broken the directions of deflection in the galvanometer are opposite to each other.
- ii. When there is a relative motion between the two coils (with their circuits closed), an induced current is produced in the secondary coil but it exists so long as there is a relative motion between the coils.
- iii. Whenever the current in the primary coil is changed (either increased or decreased) by sliding the rheostat-jockey, a deflection is produced in the galvanometer.
- iv. This indicates the presence of induced current. The induced current exists so long as there is a change of current in the primary coil.

○ **Conclusion of above two experiments**

The above observations indicate that so long as there is a change of magnetic flux (produced either by means of a magnet or by a current carrying coil) inside a coil, an induced emf is produced. The direction of induced emf reverses if instead of increasing the flux, the flux is decreased or vice versa

**12.2 Faraday's Laws of Electromagnetic**

**Q.1 State and explain Faraday's law of electromagnetic induction**

**OR**

★ **State Faraday's law of electromagnetic induction**

**Ans:**

i. **First law:**

Whenever there is a change of magnetic flux in a closed circuit, an induced emf is produced in the circuit. Also, if a conductor cuts the lines of magnetic field, an e.m.f is induced between its ends.

This law is a qualitative law as it only indicates the characteristics of induced emf.

ii. **Second law:**

The magnitude of induced emf produced in the circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

This law is known as quantitative law as it gives the magnitude of induced emf.

iii. If  $\phi$  is the magnetic flux linked with the coil at any instant  $t$ , then the induced emf.

$$e \propto \frac{d\phi}{dt} \quad \dots(1)$$

$$\therefore e = K \frac{d\phi}{dt},$$

where  $K$  is constant of proportionality.

If  $e$ ,  $\phi$  and  $t$  are measured in the same system of units,  $K = 1$ .

$$e = \frac{d\phi}{dt} \quad \dots(2)$$

v. Combining this expression with the first law, we get,

$$e = -\frac{d\phi}{dt} \quad \dots(3)$$

vi. For a coil consisting of  $N$  turn, the total magnetic flux is given as,

$$e = -N \frac{d\phi}{dt} \quad \dots(4)$$

**Key Point**

i. **Induced emf**

$$i = -\frac{Nd\phi}{dt}$$

ii. **Induced current**

$$i = \frac{e}{R} = -\frac{N}{R} \frac{d\phi}{dt}$$

iii. **Induced charge**

$$\begin{aligned} dq &= idt \\ &= -\frac{N}{R} d\phi \end{aligned}$$

Induced charge is time independent

iv. **Induced power**

$$P = \frac{e^2}{R} = \frac{N^2}{R} \left( \frac{d\phi}{dt} \right)^2$$

It depends upon time and resistance

**Type - I**

**Numerical based on magnetic flux and Faraday's Law of electromagnetism**

**Formulae used :**

1.  $\phi = BA \cos \theta = \vec{B} \cdot \vec{A}$

2.  $e = -\frac{d\phi}{dt}$   
in terms of magnitude

$$|e| = \left| \frac{d\phi}{dt} \right|$$

3. For  $N$  turns,  $e = -N \frac{d\phi}{dt}$

4. Average emf induced

$$e = \frac{-N(\phi_2 - \phi_1)}{t}$$

5. Induced current

$$i = \frac{e}{R} = -\frac{1}{R} \frac{d\phi}{dt}$$

where  $R$  is resistance of coil

★ 1) A conducting loop of area  $1 \text{ m}^2$  is placed normal to uniform magnetic field  $3 \text{ Wb/m}^2$ . If the magnetic field is uniformly reduced to  $1 \text{ Wb/m}^2$  in a time of  $0.5 \text{ s}$ , calculate the induced emf produced in the loop.

**Data :**  $B_1 = 3 \text{ Wb/m}^2$ ,  $B_2 = 1 \text{ Wb/m}^2$   
 $A = 1 \text{ m}^2$ ,  $\Delta t = 0.5 \text{ s}$

**To find :**  $e$

$$\text{Formula : } |e| = \left| \frac{\Delta\phi}{\Delta t} \right| = \left| \frac{\phi_2 - \phi_1}{\Delta t} \right| = \left| \frac{B_2 - B_1}{\Delta t} \right| A$$

**Solutions:**

$$\begin{aligned} |e| &= \left| \frac{B_2 - B_1}{\Delta t} \right| \cdot A = \left| \frac{1 - 3}{0.5} \right| \times 1 \\ &= |-4| \times 1 = 4 \text{ V} \end{aligned}$$

**Ans :** Induced emf produced in the loop is  $4 \text{ V}$ .

- ★ 2) A coil consists of 400 turns of wire. Each turn is a square of side  $d = 20$  cm. A uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.8 s, what is the magnitude of induced emf in the coil while the field is changing?

**Data:**  $B_1 = 0$  T,  $B_2 = 0.50$  T,  $N = 400$   
 $d = 20$  cm = 0.2 m,  $\Delta t = 0.8$  s  
 $A = d^2 = (0.2)^2 = 0.04$  m<sup>2</sup>.

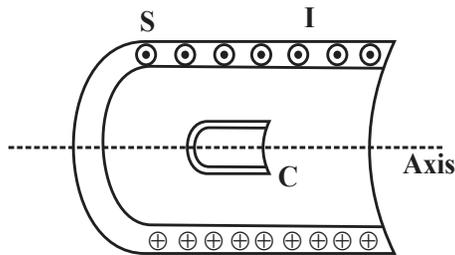
**To find :** Induced emf  $|e|$

**Formula :**  $|e| = N \left| \frac{\Delta\phi}{\Delta t} \right| = N \left| \frac{\phi_2 - \phi_1}{\Delta t} \right| = N \left| \frac{B_2 - B_1}{\Delta t} \right| A$

**Solution :**  $|e| = N \left| \frac{B_2 - B_1}{\Delta t} \right| A$   
 $= 400 \times \frac{(0.5 - 0)}{0.8} \times 0.04$   
 $= 16 \times \frac{5}{8} = 10$  V

**Ans :** The induced emf in the coil is 10V.

- ★ 3) A long solenoid S, as shown in the figure has 200 turns/cm and carries a current I of 1.4A. The diameter D of the solenoid is 3 cm. A coil C, having 100 turns and a diameter d of 2 cm is kept at the centre of the solenoid. The current in the solenoid is decreased steadily to zero in 20 ms. Calculate the magnitude of emf induced in the coil C when the current in the solenoid is changing.



⊙ Magnetic field  $\vec{B}$  out of paper  
⊗ Magnetic field  $\vec{B}$  into the paper

**Data :** For Solenoid

$$D = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$$

$$n = 200 \text{ turns/cm} = 2 \times 10^4 \text{ turns/m}$$

$$I_1 = 1.4 \text{ A}; I_2 = 0, \Delta t = 20 \text{ ms} = 20 \times 10^{-3} \text{ s}$$

**For Coil,**

$$N_c = 100; d_c = 2 \text{ cm} = 0.02 \text{ m},$$

$$r = 0.01 \text{ m} = 10^{-2} \text{ m}$$

**To find :**  $|e|$

**Formulae:** i.  $B = \mu_0 n I$       ii.  $\phi = BA$

iii.  $|e| = N \left| \frac{\Delta\phi}{\Delta t} \right|$

**Solution :**

- i. Initial magnetic field along the axis of solenoid

$$B_1 = \mu_0 n I_1$$

Initial flux associated with coil

$$\phi_1 = B_1 A \cos \theta = \mu_0 n I_1 A = \mu_0 n I_1 \pi r^2$$

$$\phi_1 = 4\pi \times 10^{-7} \times 2 \times 10^4 \times 1.4 \times (3.14) \times (1 \times 10^{-2})^2$$

$$\phi_1 = 4 \times 3.14 \times 2 \times 1.4 \times (3.14) \times 10^{-7}$$

$$= 1.104 \times 10^{-5} \text{ Wb}$$

- ii. Final magnetic field along the axis of solenoid

$$B_2 = \mu_0 n I_2 = 0$$

final flux associated with coil

$$\phi_2 = B_2 A \cos \theta = 0$$

iii.  $|e| = N \left| \frac{\Delta\phi}{\Delta t} \right| = N \left| \frac{\phi_2 - \phi_1}{\Delta t} \right|$

$$|e| = 100 \times \left| \frac{(0 - 1.104 \times 10^{-5})}{20 \times 10^{-3}} \right|$$

$$= \frac{10^5 \times 1.104 \times 10^{-5}}{20} = 55.2 \times 10^{-3} \text{ V}$$

$$= 55.2 \text{ mV}$$

**Ans :** The induced emf is 55.2 mV

- ★ 4) A metal disc is made to spin at 20 revolutions per second about an axis passing through its centre and normal to its plane. The disc has a radius of 30 cm. and spins in a uniform magnetic field of 0.20 T, which is parallel to the axis of

rotation. Calculate,

- i. The area swept out per second by the radius of the disc,
- ii. The flux cut per second by a radius of the disc,
- iii. The induced emf in the disc.

**Data :**  $f = 20$  rps,  $\omega = 2\pi \times 20 = 40\pi$  rad/s  
 $R = 30\text{cm} = 0.3$  m,  $B = 0.20\text{T}$ ,  $\Delta t = 1\text{s}$

**To find :** i.  $\Delta A$  (Area swept out in one second)  
ii.  $\Delta\phi$  (Change in flux in one second)  
iii.  $|e|$

**Formulae:** i. Area swept out (area of sector)

$$\Delta A = \pi R^2 \times \frac{\Delta\theta}{2\pi} = \frac{1}{2} R^2 \Delta\theta$$

$$\therefore \Delta A = \frac{1}{2} R^2 \frac{\Delta\theta}{\Delta t} \Delta t$$

$$\Delta A = \frac{1}{2} R^2 \omega \Delta t \quad \dots \left( \because \omega = \frac{\Delta\theta}{\Delta t} \right)$$

$$\text{ii. } \Delta\phi = B\Delta A$$

$$\text{iii. } |e| = \frac{\Delta\phi}{\Delta t}$$

**Solution:**

$$\text{i. } \Delta A = \frac{1}{2} R^2 \omega \Delta t \quad \dots \left( \because \omega = \frac{\Delta\theta}{\Delta t} \right)$$

$$\therefore \Delta A = \frac{1}{2} \times (0.3)^2 \times 40 \times 3.142 \times 1$$

$$= 5.6556 \text{ m}^2 = 5.65 \text{ m}^2$$

$$\text{ii. } \Delta\phi = B\Delta A = 0.20 \times 5.65 = 1.130 \text{ Wb}$$

$$\text{iii. } |e| = \frac{\Delta\phi}{\Delta t} = \frac{1.130}{1} = 1.130 \text{ V}$$

**Ans :** i. Area swept in one second is  $5.65\text{m}^2$   
ii. Flux cut per second is  $1.130\text{Wb}$ .  
iii. Induced emf in disc is  $1.130\text{V}$ .

- ★ 5) A long solenoid consisting of  $1.5 \times 10^3$  turns/m has an area of cross-section of  $25\text{cm}^2$ . A coil C, consisting of 150 turns ( $N_c$ ) is wound tightly around the centre of the solenoid. Calculate for a current of 3.0 A in the solenoid

- i. the magnetic flux density at the centre of the solenoid,
- ii. the flux linkage in the coil C,
- iii. the average emf induced in coil C if reversed in direction in a time of

0.5s. ( $\mu_0 = 4\pi \times 10^{-7} \text{Tm/A}$ )

**Data:**  $n = 1.5 \times 10^3$  turns/m  
 $A = 25\text{cm}^2 = 25 \times 10^{-4} \text{m}^2$ ,  
 $I = 3.0$  A,  $N_c = 150$ ,  $t = 0.5\text{s}$ ,

**To find:** i. Magnetic flux density (B)  
ii. Flux linkage ( $\phi$ ) with coil  
iii. Average emf ( $e_{\text{avg}}$ )

**Formulae:** i.  $B = \mu_0 nI$

$$\text{ii. } \phi = NBA$$

$$\text{iii. } e_{\text{avg}} = \frac{\text{Change in flux}}{\text{time}} = \frac{|\phi_2 - \phi_1|}{t}$$

**Solution :**

$$\text{i. } B = \mu_0 nI$$

$$B = 4 \times 3.142 \times 10^{-7} \times 1.5 \times 10^3 \times 3$$

$$= 56.56 \times 10^{-4} = 5.656 \times 10^{-3} \text{T}$$

$$= 5.66 \times 10^{-3} \text{T}$$

ii. Flux linked with coil

$$\phi = N(BA) = 150 \times 5.66 \times 10^{-3} \times 25 \times 10^{-4}$$

$$NBA = 21225 \times 10^{-7} = 2.12 \times 10^{-3} \text{ Wb}$$

iii. Initial flux ( $\phi_1$ ) =  $2.12 \times 10^{-3} \text{Wb}$

Now current is reversed in 0.5 seconds.

$$\therefore \phi_2 = -2.12 \times 10^{-3} \text{ Wb}$$

$$e_{\text{avg}} = \frac{|\phi_2 - \phi_1|}{t}$$

$$= \frac{|-2.12 \times 10^{-3} - 2.12 \times 10^{-3}|}{0.5}$$

$$= \frac{4.24 \times 10^{-3}}{0.5} = 8.48 \times 10^{-3} \text{V}$$

**Ans :** i. Magnetic field is  $5.66 \times 10^{-3} \text{T}$ .  
ii. Flux linkage is  $2.12 \times 10^{-3} \text{Wb}$ .  
iii. Induced emf is  $8.48 \times 10^{-3} \text{V}$ .

- ★ 6) A search coil having 2000 turns with area  $1.5 \text{ cm}^2$  is placed in a magnetic field of  $0.60 \text{ T}$ . The coil is moved rapidly out of the field in a time of  $0.2$  second. Calculate the induced emf across the search coil.

**Data :**  $N = 2000, A = 1.5 \text{ cm}^2 = 1.5 \times 10^{-4} \text{ m}^2$   
 $B = 0.60 \text{ T}, t = 0.2 \text{ s}$

**To find:**  $|e|$

**Formula:**  $|e| = \frac{|\phi_2 - \phi_1|}{t}$

**Solution:**

- i.  $\phi_1 = NBA$   
 $= 2000 \times 0.6 \times 1.5 \times 10^{-4} = 0.18 \text{ Wb}$
- ii. As coil is moved rapidly out of the field  
 $\phi_2 = 0$
- iii.  $|e| = \frac{|\phi_2 - \phi_1|}{t} = \frac{|0 - 0.18|}{0.2} = 0.9 \text{ V}$

**Ans:** emf induced in search coil is  $0.9 \text{ V}$ .

### Problem for Practice

1. The magnetic flux through a coil perpendicular to the plane is varying according to the relation:  
 $f = (5t^3 + 4t^2 + 2t - 5) \text{ Wb}$   
Calculate the induced current through the coil at  $t = 2 \text{ s}$ , if the resistance of the coil is  $5 \text{ W}$ .  
**Ans:  $15.6 \text{ A}$**
2. A square loop of side  $10 \text{ cm}$  and resistance of  $0.70 \text{ W}$  is placed vertically in the east-west plane. A uniform magnetic field of  $0.10 \text{ T}$  is set up across the plane in the north-east direction. The magnetic field is decreased to zero in  $0.70 \text{ s}$  at a steady rate. Determine the magnitudes of induced emf and current during this time-interval.  
**Ans:  $1.4 \text{ mA}$**
3. A wire  $88 \text{ cm}$  long bent into a circular loop is placed perpendicular to the magnetic field of flux density  $2.5 \text{ Wbm}^{-2}$ . Within  $0.5 \text{ s}$ , the loop is changed into  $22 \text{ cm}$  square and flux density is increased to  $3.0 \text{ Wbm}^{-2}$ . Calculate the value of the emf induced.  
**Ans:  $0.0176 \text{ V}$**
4. The magnetic flux the reading a coil change

from  $12 \times 10^{-3} \text{ Wb}$  in  $0.01 \text{ s}$ . Calculate the induced emf.

**Ans:  $0.6 \text{ V}$**

5. A  $70$  turn coil with average diameter of  $0.02 \text{ m}$  is placed perpendicular to magnetic field of  $9000 \text{ T}$ . If the magnetic field is changed to  $6000 \text{ T}$  in  $3 \text{ s}$ , what is the magnitude of the induced emf?  
**Ans:  $2.2 \text{ V}$**
6. A rectangular coil of length  $0.5 \text{ m}$  and breadth  $0.4 \text{ m}$  has resistance of  $5 \text{ W}$ . The coil is placed in a magnetic induction of  $0.05 \text{ T}$  and the direction is perpendicular to the plane of the coil. If the magnetic induction is uniformly reduced to zero in  $5$  mili seconds, find the emf and current induced in the coil.  
**Ans:  $1 \text{ mV}, 0.2 \text{ mA}$**

7. The magnetic flux through a loop varies according to the relation  $f = 8t^2 + 6t + C$ , where 'C' is constant, 'f' is in miliweber and 't' is in second. What is the magnitude of induced e.m.f. in the loop at  $t = 2$  second?  
**Ans:  $38 \text{ Wbms}^{-1}$**
8. The magnetic flux through a loop is varying according to a relation  $f = 6t^2 + 7t + 1$  where f is in miliweber and t is in second. What is the e.m.f. induced in the loop at  $t = 2$  second?  
**Ans:  $31 \text{ mV}$**

### 12.3 Lenz's law

**Q.2 State and explain Lenz's law**

**OR**

★ **State and explain lenz law in accordance to law of conservation of energy?**

**Ans:** **Lenz's law :** The direction of the induced emf is such as to oppose the change in magnetic flux which produces it.

Combination of Faraday's and Lenz's laws can be written as,

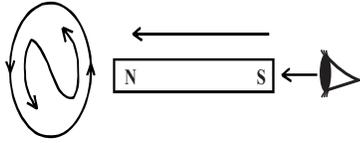
$$e = -\frac{d\phi}{dt}$$

**Explanation :** Magnet and Coil experiment

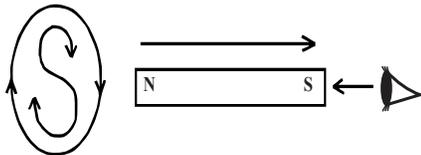
- i. When N-pole of the magnet is moved towards the coil, the flux linked with coil

increases and current is induced in the coil.

- ii. The current induced in the coil is in the anticlockwise direction such that face of the coil behaves as N-pole and tries to repelled the magnet. Therefore work has to be done against the force of repulsion in bringing the magnet closer to the coil.



- iii. When N-pole of the magnet is moved away from the coil, the flux linked with coil decreases and current is induced in the coil.
- iv. The current induced in the coil is in the clockwise direction such that face of the coil behaves as S-pole and tries to attract the magnet. Therefore work had to be done against the force of attraction, in taking magnet away from coil.



- v. Thus, the direction of the induced current is such that it tries to opposes the change which is responsible for its own production.
- vi. From this it is clear that mechanical work done in moving the magnet with respect to the coil that changes into electric energy producing induced current. Thus energy is being transformed only.
- vii. When we do not move the magnet, the work done is zero. Therefore induced current is not produced
- viii. Hence Lenz's law is in accordance with the law of conservation of energy.

**Q.3 Explain how lenz's law is incorporated in faraday's law**

**Ans:**

- i. According to Faraday's laws of electromagnetic induction,

$$e = -\frac{d\phi}{dt} \quad \dots(1)$$

- ii. Consider that area vector  $\vec{A}$  of the loop perpendicular to the plane of the loop is fixed and oriented parallel ( $\theta = 0$ ) to magnetic field  $\vec{B}$ . The magnetic field  $\vec{B}$  increases with time,

- iii. By definition of flux  $\phi = \vec{B} \cdot \vec{A}$

$$\therefore e = -\frac{d}{dt}(\vec{B} \cdot \vec{A}) = -|\vec{A}| \frac{d|\vec{B}|}{dt} \quad \dots(1)$$

Here,  $|\vec{A}|$  is positive and  $\frac{dB}{dt}$  is positive as B increases with time.

- iv. The screw driver rule fixes the positive sense of circulation around the loop as the clockwise direction.

- v. As the sense of the induced current in the loop is counter clockwise (negative), the sense of induced emf also is negative(-ve). That is, the LHS of equation (1) is indeed a negative (-ve) quantity in order to be equal to the RHS.

- vi. Thus the negative (-ve) sign in the equation

$$e = -\frac{d\phi}{dt} \text{ incorporates Lenz's law into Faraday's law.}$$

**MULTIPLE CHOICE QUESTIONS**  
**Entrance Corner (Set 1)**

1. A coil having n turns and resistance  $R\Omega$  is connected with a galvanometer of resistance  $4R\Omega$ . This combination is moved in time t seconds from a magnetic flux  $\phi_1$  weber to  $\phi_2$  weber. The induced current in the circuit is

a.  $\frac{\phi_2 - \phi_1}{5Rnt}$       b.  $-\frac{n(\phi_2 - \phi_1)}{5Rt}$   
c.  $-\frac{n(\phi_2 - \phi_1)}{Rnt}$       d.  $-\frac{n(\phi_2 - \phi_1)}{Rt}$

2. A coil having 500sq loops of side 10 cm is placed normal to magnetic flux which increases at a rate of 1 T/s. The induced emf is

- a. 0.1 V                      b. 0.5 V  
c. 1 V                         d. 5 V
3. A physicist works in a laboratory where the magnetic field is 2 T. She wears a necklace enclosing area  $0.01 \text{ m}^2$  in such a way that the plane of the necklace is normal to the field and is having a resistance  $R = 0.01 \Omega$ . Because of power failure, the field decays to 1 T in time  $10^{-3}$  sec. Then what is the total heat produced in her necklace?  
a. 10J                         b. 20J  
c. 30 J                        d. 40J
4. A conducting circular loop is placed in a uniform magnetic field of induction B tesla with its plane normal to the field. Now, the radius of the loop starts shrinking at the rate  $\left(\frac{dr}{dt}\right)$ . Then the induced emf at that instant when the radius is r, is  
a.  $\pi rB\left(\frac{dr}{dt}\right)$                       b.  $2\pi rB\left(\frac{dr}{dt}\right)$   
c.  $\pi r^2\left(\frac{dB}{dt}\right)$                       d.  $\left(\frac{\pi r^2}{2}\right)B\left(\frac{dr}{dt}\right)$
5. The magnetic flux linked with a coil (in Wb) is given by the equation  $\phi = 5t^2 + 3t + 16$  The induced emf in the coil in the fourth second will be  
a. 10 V                        b. 108 V  
c. 145 V                       d. 210 V
6. The current flows from A to B as shown in the figure. The direction of the induced current in the loop is  
a. clockwise  
b. anticlockwise  
c. straight line  
d. None of these
- 
7. Metal ring is held horizontally and bar magnet is dropped through the ring with its length along the axis of the ring. The acceleration of the falling magnet is  
a. equal to g                      b. less than g

- c. more than g                d. either (a) or (c)
8. A closed coil with a resistance  $R$  is placed in a magnetic field. The flux linked with the coil is  $\phi$ . If the magnetic field is suddenly reversed in direction, the charge that flows through the coil will be  
a.  $\phi/2R$     b.  $\phi/R$     c.  $2\phi/R$     d. zero
9. A coil of 1200 turns and mean area of  $500 \text{ cm}^2$  is held perpendicular to a uniform magnetic field of induction  $4 \times 10^{-4} \text{ T}$ . The resistance of the coil is 20 ohms. When the coil is rotated through  $180^\circ$  in the magnetic field in 0.1 seconds the average electric current (in mA) induced is :  
a. 12            b. 24            c. 36            d. 48
10. A magnetic field in a certain region is given by  $B = (40\hat{i} - 15\hat{k}) \times 10^{-4} \text{ T}$ . The magnetic flux passes through a loop of area  $5.0 \text{ cm}^2$  is placed flat on xy plane is  
a. 750nWb                      b. -750nWb  
c. 360nWb                      d. -360nWb

**Try yourself**

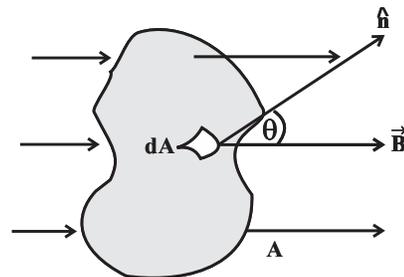
11. In lenz's law, there is conervation of  
a. chage                        b. momentum  
c. energy                       d. current
12. A magnetic field of  $2 \times 10^{-2} \text{ T}$  acts at right angles to a coil of area  $100 \text{ cm}^2$  with 50 turns. The average emf induced in the coil is 0.1 V, when it is removed from the field in the time t. The value of t is  
a. 0.1 s                        b. 0.01 s  
c. 1 s                            d. 10 s
- 13.. A rectangular, a square, a circular and an elliptical loop, all in the (x-y) plane, are moving out of a uniform magnetic field with a constant velocity  $\vec{V} = v\hat{i}$ . The magnetic field is directed along the negative z-axis direction. The induced emf, during the passage of these loops, out of the field region, will not remain constant for :  
a. the circular and the elliptical loops  
b. only the elliptical loop

- c. any of the four loop  
d. the rectangular, circular and elliptical loops
14. A conducting circular loop is placed in a uniform magnetic field 0.04 T with its plane perpendicular to the magnetic field. The radius of the loop starts shrinking at 2 mm/s. The induced emf in the loop when the radius is 2 cm is  
a.  $4.8 \pi \mu\text{V}$                       b.  $0.8 \pi \mu\text{V}$   
c.  $1.6 \pi \mu\text{V}$                          d.  $3.2 \pi \mu\text{V}$
15. In a coil of area  $10\text{cm}^2$  and 10 turns with magnetic field directed perpendicular to the plane and is changing at the rate of  $10^8$  gauss/second. The resistance of the coil is  $20 \Omega$ . The current in the coil will be  
a. 0.5A    b. 5A    c. 50A    d.  $5 \times 10^8$  A
16. A magnetic flux of 500 micro-webers passing through a 200 turns coil is reversed in  $20 \times 10^{-3}$  seconds. The average emf induced in the coil in volts, is :  
a. 2.5    b. 5.0    c. 7.5    d. 10.0
17. To measure the field 'B' between the poles of an electromagnet, a small test loop of area  $1 \text{ cm}^2$ , resistance  $10 \Omega$  and 20 turns is pulled out of it. A galvanometer shows that a total charge of  $2 \mu\text{C}$  passed through the loop. The value of 'B' is  
a. 0.001 T                              b. 0.01 T  
c. 0.1 T                                    d. 1.0 T
18. A circular coil of 'n' turns is kept in a uniform magnetic field such that the plane of the coil is perpendicular to the field. The magnetic flux associated with the coil is now  $\phi$ . Now the coil is opened and made into another circular coil of twice the radius of the previous coil and kept in the same field such that the plane of the coil is perpendicular to the field. The magnetic flux associated with this coil now is  
a.  $\phi$     b.  $2\phi$     c.  $\frac{\phi}{4}$     d.  $\frac{\phi}{2}$

19. A 800 turn coil of effective area  $0.05 \text{ m}^2$  is kept perpendicular to a magnetic field  $5 \times 10^{-5}$  T. When the plane of the coil is rotated by  $90^\circ$  around any of its coplanar axis in 0.1 s, the emf induced in the coil will be:  
a. 2 V    b. 0.2 V    c.  $2 \times 10^{-3}$  V    d. 0.02 V
20. A long solenoid of diameter 0.1 m has  $2 \times 10^4$  turns per meter. At the centre of the solenoid, a coil of 100 turns and radius 0.01 m is placed with its axis coinciding with the solenoid axis. The current in the solenoid reduces at a constant rate to 0A from 4A in 0.05 s. If the resistance of the coil is  $\pi^2 \Omega$ . the total charge flowing through the coil during this time is:-  
a.  $16 \mu\text{C}$                                  b.  $32 \mu\text{C}$   
c.  $16 \pi \mu\text{C}$                                 d.  $32 \pi \mu\text{C}$

**12.4 Flux of the field**

i. In general, the field  $\vec{B}$  over an area  $\vec{S}$  may not be uniform. However, over a small area element  $d\vec{A}$ , the field  $\vec{B}$  may be assumed to be uniform.



- ii. As shown in Figure, if  $\theta$  is the angle between  $\vec{B}$  and the normal drawn to area element  $d\vec{A}$ , then the component of  $\vec{B}$  normal to  $d\vec{A}$  will be  $B \cos \theta$ .
- iii. Flux through area element  $d\vec{A}$  is  
$$d\phi = B_{\perp} dA = B \cos \theta dA$$
  
$$= B dA \cos \theta = \vec{B} \cdot d\vec{A}$$
- iv. Then the flux of  $\vec{B}$  through the whole area  $\vec{S}$  is  
$$\phi = \int_s \vec{B} \cdot d\vec{A}$$
- v. According to Faraday's law,

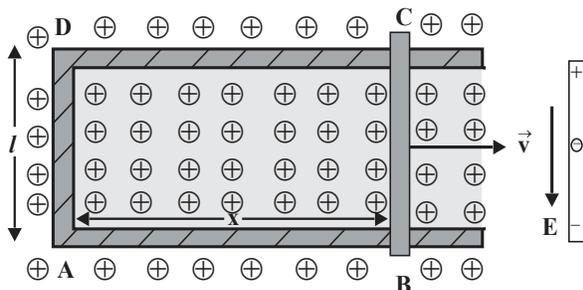
$$e = -\frac{d\phi}{dt}$$

$$e = -\frac{d}{dt} \int_s \vec{B} \cdot d\vec{A}$$

**12.5 Motional EMF**

**Q.4** Determine the motional emf induced in a straight conductor moving in uniform magnetic in uniform magnetic field with constant velocity on the basis of Lorentz Force

**Ans:**



i. Consider a rectangular frame of wires ABCD. Let 'l' be the length of wire and x be the breadth of wire frame on which wire BC is sliding.

Area of rectangular frame = lx

ii. As wire BC of length 'l' is moved out with velocity  $\vec{v}$  to increase x, the area of loop ABCD is also increased.

Thus the flux also increases with time.

iii. According to Faraday's law, emf is induced in wire causes current in the loop.

Magnitude of induced emf is

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(Blx)$$

$$|e| = Bl \frac{dx}{dt} = Blv \quad \dots(1)$$

iv. Due to current in loop let us assume charge q is moving through wire BC which experiences Lorentz force.

$$\vec{F} = q(\vec{v} \times \vec{B})$$

This force  $\vec{F}$  is perpendicular to both  $\vec{v}$  and  $\vec{B}$  and hence is parallel to BC.

This force  $\vec{F}$  is constant along the length 'l' of wire BC.

v. When the charge q moves a distance l along the wire, the work done by Lorentz force is  $W = f \cdot l = qvB \sin \theta \cdot l$

Where  $\theta$  is angle between  $\vec{v}$  and  $\vec{B}$ .

vi. By definition of potential

$$e = \frac{W}{q} = \frac{qvB \sin \theta \cdot l}{q}$$

$$\therefore e = vB \sin \theta \cdot l$$

vii. For maximum induced emf,

$$\sin \theta = 1$$

$$e_{\max} = Blv$$

This is motional emf induced in a straight conductor moving in uniform magnetic field with constant velocity.

**Type - II**

**Numerical based on**

**Motional EMF due to straight conductor**

**Formula used**

$$e = Blv$$

★ 1) An aircraft of wing span of 50 m flies horizontally in earth's magnetic field of  $6 \times 10^{-5}$  T at a speed of 400 m/s. Calculate the emf generated between the tips of the wings of the aircraft.

**Data :**  $l = 50$  m,  $B = 6 \times 10^{-5}$  T,  $v = 400$  m/s

**To find:** e

**Formula:**  $e = Blv$

**Solution:**  $e = Blv$

$$e = 6 \times 10^{-5} \times 50 \times 400 = 120000 \times 10^{-5} = 1.2 \text{ V}$$

**Ans :** Emf Induced between tips of wings 1.2V.

★ 2) Calculate the value of induced emf between the ends of an axle of a railway carriage 1.75 m long travelling on level ground with a uniform velocity of 50 km

per hour. The vertical component of Earth's magnetic field ( $B_v$ ) is given to be  $5 \times 10^{-5} \text{T}$ .

**Data:**  $l = 1.75 \text{m}$ ,  $B_v = 5 \times 10^{-5} \text{T}$ .

$$v = 50 \text{km/h} = \frac{25}{18} \times \frac{5}{9} = \frac{125}{9} \text{m/s}$$

**To find:**  $e$

**Formula:**  $e = Bv$

**Solution:**  $e = Bv$

$$e = 5 \times 10^{-5} \times 1.75 \times \frac{125}{9}$$

$$= 121.53 \times 10^{-5} \text{V} = 1.22 \text{mV}$$

**Ans:** Emf induced in railway axel is 1.22 mV

★ 3) A horizontal wire 20m long extending from east to west is falling with a velocity of 10 m/s normal to the Earth's magnetic field of  $0.5 \times 10^{-4} \text{T}$ . What is the value of induced emf in the wire?

**Data:**  $l = 20 \text{m}$ ,  $B = 0.5 \times 10^{-4} \text{T}$ .  
 $v = 10 \text{m/s}$ ,

**To find:**  $e$

**Formula:**  $e = Bv$

**Solution:**  $e = Bv$

$$e = 0.5 \times 10^{-4} \times 20 \times 10$$

$$= 10 \times 10^{-3} \times 1 = 10 \text{mV}$$

**Ans:** Emf induced in wire is 10 mV.

4) A horizontal telegraph wire 10m long oriented along the magnetic east-west direction falls freely under gravity to the ground from the height of 10 m. Find the emf induced in the wire at the instant the wire strikes the ground.

( $B_H = 2.5 \times 10^{-5} \text{Wb/m}^2$ ,  $g = 9.8 \text{m/s}^2$ )

**Data:**  $u = 0$ ,  $a = -g = -9.8 \text{m/s}^2$ ,  $s = -10 \text{m}$ ,  
 $B_H = 2.5 \times 10^{-5} \text{Wb/m}^2$ ,  $l = 10 \text{m}$

**Formula:** i.  $v^2 = u^2 + 2as$

ii.  $e = B_H v$

**Solution:**

i. The speed of wire is

$$v^2 = u^2 + 2as$$

$$= 0 + 2(-9.8) \times (-10)$$

$$v^2 = 196$$

$\therefore v = 14 \text{m/s}$

ii. Induced emf is,

$$e = B_H v$$

$$= 2.5 \times 10^{-5} \times 10 \times 14$$

$$= 3.5 \times 10^{-3} \text{V}$$

$\therefore$  The emf induced in wire is  $3.5 \times 10^{-3} \text{V}$ .

**Ans:** The emf induced in wire is  $3.5 \times 10^{-3} \text{V}$

### Problem for Practice

1. A jet plane is travelling west at  $450 \text{ms}^{-1}$ . If the horizontal component of earth's magnetic field at that place is  $4 \times 10^{-4} \text{tesla}$  and the angle of dip is  $30^\circ$ , find the emf induced between the ends of wings having a span of 30 m.

**Ans: 3.12 V**

2. A railway track running north-south has two parallel rails 1.0 apart. Calculate the value of induced emf between the rails, when a train passes at a speed of  $90 \text{kmh}^{-1}$ . The horizontal component of earth's magnetic field at that place is  $0.3 \times 10^{-4} \text{Wbm}^{-2}$  and angle of dip is  $60^\circ$ .

**Ans:  $1.3 \times 10^{-3} \text{V}$**

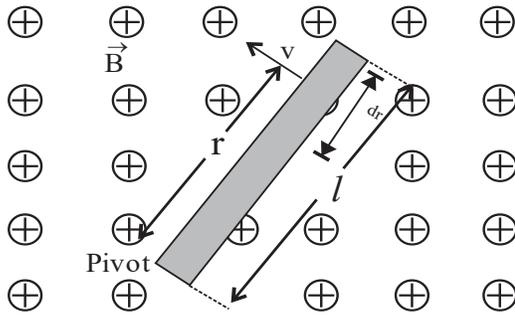
3. A straight conductor 1 meter long moves at right angles to both, its length and a uniform magnetic field. If the speed of the conductor is  $2.0 \text{ms}^{-1}$  and the strength of the magnetic field is  $10^4 \text{gauss}$ , find the value of induced emf in volt.

**Ans: 2V**

4. A metre gauge train is running due north with a constant speed of  $90 \text{kmh}^{-1}$  on a horizontal track. If the vertical component of earth's magnetic field is  $3 \times 10^{-5} \text{Wbm}^{-2}$ , calculate the emf induced across the axle of the train of length 1.25 cm.

**Ans:  $9.375 \times 10^{-6} \text{V}$**

**Q.5 Determine the motional emf induced when rod is rotated in uniform magnetic field**



**Ans:**

i. Consider a conducting bar pivoted at one end and rotating in uniform magnetic field which is perpendicular to the plane of rotation. Let  $l$  be the length of conducting bar.

ii. Consider conducting bar is divided into small segments of length  $dr$ .

Consider one such segment at a distance  $r$  from the pivot

iii. Emf induced in segment is given by  $de = B v dr$

iv. As all segments are arranged in series Therefore total emf of conducting rod is given by

$$v. \quad e = \int_0^l de = \int_0^l Bvdr$$

As  $v = r\omega$

$$\therefore e = \int_0^l B\omega r dr = B\omega \int_0^l r dr = B\omega \frac{l^2}{2}$$

$$\therefore \boxed{e = \frac{1}{2} B\omega l^2} \quad \dots(2)$$

The above equation represent emf induced in a conducting bar rotating in uniform magnetic field.

**Type - III**

**Numerical based on rod rotating in uniform magnetic field**

**Formula used**

$$e = \frac{1}{2} B\omega l^2$$

1) A metal rod  $\frac{1}{\sqrt{\pi}}$  m long rotates about

one of its ends perpendicular to a plane whose magnetic induction is  $4 \times 10^{-3}$  T. Calculate the number of revolutions made by the rod per second if the e.m.f. induced between the ends of the rod is 16 mV.

**Data:**  $l = \frac{1}{\sqrt{\pi}}$  m,  $B = 4 \times 10^{-3}$  T,

$e = 16$  mV =  $16 \times 10^{-3}$  V

**To find:**  $f$

**Formula:**  $e = \frac{1}{2} B\omega l^2$

**Solution:**

$$f = \frac{e}{B\pi l^2} = \frac{16 \times 10^{-3}}{4 \times 10^{-3} \times \pi \times \left(\frac{1}{\sqrt{\pi}}\right)^2} = \frac{16}{4 \times \pi \times \frac{1}{\pi}}$$

$\therefore f = 4$  Hz

**Ans:** The number of revolutions made by rod per second are 4Hz

2) A cycle wheel with 10 spokes each of length 0.5 m is rotated at a speed of 18 km/hr in a plane normal to the earth's magnetic induction of  $3.6 \times 10^{-5}$  T. Calculate the e.m.f. induced between the

- i. axle and the rim of cycle wheel.
- ii. ends of a single spoke and ten spokes.

**Data:**  $r = l = 0.5$  m

$v = 18$  km/hr =  $18 \times \frac{5}{18} = 5$  m/s

$B = 3.6 \times 10^{-5}$  T

**To find:** i. e.m.f. induced between the axle and rim of wheel ( $e_1$ )

ii. e.m.f. induced between the ends of a single

**Formula:**  $e = \frac{1}{2} B\omega l^2 = \frac{1}{2} B(l\omega)l$

$e = \frac{1}{2} Blv$

**Solution :**  $e = \frac{1}{2} Blv$

$e = \frac{1}{2} \times 3.6 \times 10^{-5} \times 0.5 \times 5$

$$e = 1.8 \times 2.5 \times 10^{-5}$$

$$e = 4.5 \times 10^{-5} \text{ v}$$

- Ans :** i. The e.m.f induced between the axle and the rim of the cycle wheel is  $4.5 \times 10^{-5} \text{ V}$   
 ii. The e.m.f induced between the ends of a single spoke and ten spokes is  $4.5 \times 10^{-5} \text{ V}$

**Problem for Practice**

1. A metallic rod of length  $L$  is rotated at an angular speed  $\omega$  normal to a uniform magnetic field  $R$ . Derive expressions for the (i) emf induced in the rod (ii) current induced and (iii) heat dissipation, if the resistance of the rod is  $R$ .

**Ans:** 
$$\frac{1}{4} \frac{B^2 L^4 \omega^2 t}{R}$$

2. A metal rod of length  $1 \text{ m}$  is rotated about one of its ends in a plane right angles to a field of inductance  $2.5 \times 10^{-3} \text{ Wb / m}^2$ . If it makes 1800 revolutions/minute, calculate the induced emf between its ends.

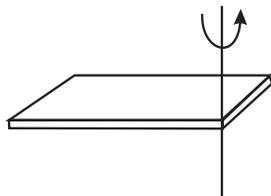
**Ans: 0.471 V**

3. A metal disc of radius  $200 \text{ cm}$  is rotated at a constant angular speed of  $60 \text{ rad s}^{-1}$  in a plane at right angles to an external field of magnetic induction  $0.05 \text{ Wbm}^{-2}$ . Find the emf induced between the centre and a point on the rim.

**Ans: 6V.**

**MULTIPLE CHOICE QUESTIONS**  
**Entrance Corner (Set 2)**

1. A horizontal rod of length  $L$  rotates about a vertical axis with a uniform angular velocity  $\omega$ .

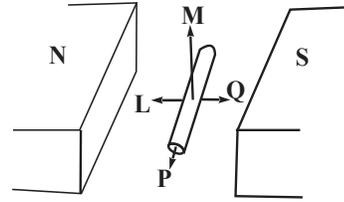


A uniform magnetic field  $B$  exists parallel to the axis of rotation then potential difference between the two ends of the rod is

- a.  $\omega L^2 B$       b.  $\omega^2 L B$

- c.  $\frac{1}{2} \omega L^2 B$       d.  $\frac{1}{2} \omega^2 L B$

2. A potential difference will be induced between the end of the conductor shown in the figure, when the conductor moves along



- a. P                      b. Q  
c. L                      d. M

3. A straight line conductor of length  $0.4 \text{ m}$  is moved with a speed of  $7 \text{ ms}^{-1}$  perpendicular to magnetic field of intensity  $0.9 \text{ Wbm}^{-2}$ . The induced emf across the conductor is

- a.  $1.26 \text{ V}$               b.  $2.52 \text{ V}$   
c.  $5.24 \text{ V}$               d.  $25.2 \text{ V}$

4. A helicopter rises vertically with a speed of  $100 \text{ ms}^{-1}$ . If helicopter has length  $10 \text{ m}$  and horizontal component of earth's magnetic field is  $5 \times 10^{-3} \text{ Wbn}^{-2}$ , then the induced emf between the tip of nose and tail of helicopter is

- a.  $50 \text{ V}$     b.  $0.5 \text{ V}$     c.  $5 \text{ V}$     d.  $25 \text{ V}$

5. A cycle wheel of radius  $0.5 \text{ m}$  is rotated with constant angular velocity of  $10 \text{ rad/s}$  in a region of magnetic field of  $0.1 \text{ T}$  which is perpendicular to the plane of the wheel. The EMF generated between its centre and the rim is

- a. Zero                      b.  $0.25 \text{ V}$   
c.  $0.125 \text{ V}$               d.  $0.5 \text{ V}$

6. Two parallel rails of a railway track insulated from each other and with the ground are connected to a millivoltmeter. The distance between the rails is one metre. A train is traveling with a velocity of  $72 \text{ kmph}$  along the track. The reading of the millivoltmeter ( in  $\text{mV}$  ) is : (Vertical component of the earth's magnetic induction is  $2 \times 10^{-5} \text{ T}$ )

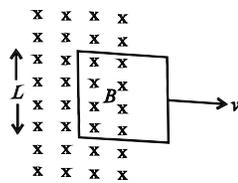
- a.  $144$     b.  $0.72$     c.  $0.4$     d.  $0.2$

7. A wheel has three spokes and is in a uniform magnetic field perpendicular to its plane, with the axis of rotation of the wheel parallel to the magnetic field. When the wheel rotates with a uniform angular velocity  $\omega$ , the emf induced between the center and rim of the wheel is 'e'. If another wheel having same radius but with six spokes is kept in the same field and rotated with a uniform angular velocity ' $\omega/2$ ', the emf induced between the center and the rim will be

a. e    b. e/2    c. 2e    d. e/4

8. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is

- a. zero  
b.  $RvB$   
c.  $vBL/R$   
d.  $vBL$



9. A train is moving towards north with a speed of 180 kilometers per hour. If the vertical component of the earth's magnetic field is  $0.2 \times 10^{-4}$  T, the emf induced in the axle of length 1.5 m is

- a. 1.5 mV                      b. 15 mV  
c. 54 mV                        d. 5.4 mV

10. A metal conductor of length 1m rotates vertically about one of its ends at angular velocity 5 radians per second. If the horizontal component of earth's magnetic field is  $0.2 \times 10^{-4}$ T, then the e.m.f.developed between the two ends of the conductor is

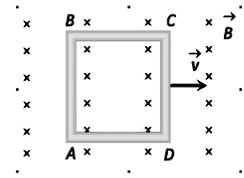
- a. 5 mV                              b.  $5 \times 10^{-4}$ V  
c. 50 mV                            d. 50  $\mu$  V

**Try yourself**

11. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A

magnetic induction B constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere. The current induced in the loop is

- a.  $\frac{Blv}{R}$  clockwise  
b.  $\frac{Blv}{R}$  anticlockwise  
c.  $\frac{2Blv}{R}$  anticlockwise      d. Zero



12. A straight conductor of length 0.4 m is moved with a speed of 7 m/s perpendicular to the magnetic field of intensity of 0.9 Wb/m<sup>2</sup>. The induced e.m.f. across the conductor will be

- a. 7.25 V                              b. 3.75 V  
c. 1.25 V                              d. 2.52 V

13. A rod of length 20 cm is rotating with angular speed of 100 rps in a magnetic field of strength 0.5 T about it's one end. What is the potential difference between two ends of the rod

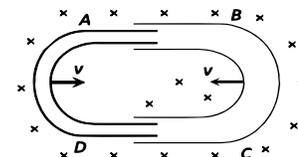
- a. 2.28 V                              b. 4.28 V  
c. 6.28 V                              d. 2.5 V

14. A wheel with ten metallic spokes each 0.50 m long is rotated with a speed of 120 rev/min in a plane normal to the earth's magnetic field at the place. If the magnitude of the field is 0.4 Gauss, the induced e.m.f. between the axle and the rim of the wheel is equal to

- a.  $1.256 \times 10^{-3}$  V                      b.  $6.28 \times 10^{-4}$  V  
c.  $1.256 \times 10^{-4}$  V                      d.  $6.28 \times 10^{-5}$  V

15. One conducting U tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v then the emf induced in the circuit in terms of B, l and v where l is the width of each tube, will be

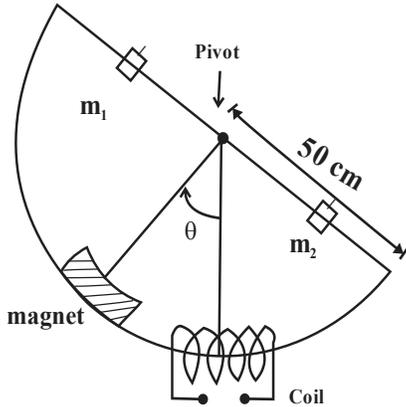
- a. Zero  
b.  $2Blv$   
c.  $B/v$   
d.  $-B/v$



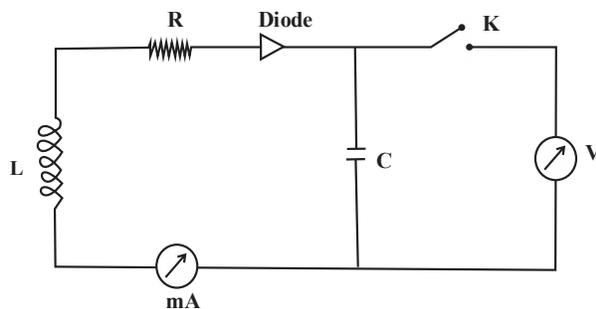
**12.6 Induced EMF in a Stationary Coil in a Changing Magnetic Field**

**Q.6** Show that the peak induced emf  $e_0$  is directly proportional to angular amplitude and inversely proportional to time period (T).

**Ans: Construction:**



- i. In a magnet-coil system, a permanent bar magnet is mounted on an arc of a semicircle of radius 50 cm.
- ii. The arc is a part of a rigid frame of aluminum and is suspended at the centre of arc so that the whole system can oscillate freely in its plane.
- iii. A coil of about 10,000 turns of copper wire loop the arc so that the bar magnet can pass through the coil freely
- iv. When the magnet moves through the coil, the magnetic flux through the coil changes.
- v. In order to measure the induced emf, a capacitor (C) and diode (D) are connected across the coil.

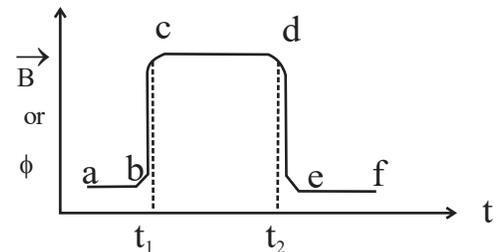


- vi. The induced emf produced in the coil is used for charging a capacitor through a diode. The voltage developed across the capacitor is measured.

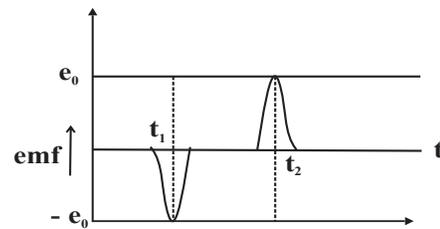
- vii. the time constant (RC) being larger than the time during which the emf in the coil is generated, the capacitor requires a few oscillations to change to the peak value which is measured by the ammeter (mA).

**Working:**

- i. As the magnet, kept in the middle of the arc, start far away from the coil, moves through it and recedes, the magnetic field (magnetic flux) through the coil changes from a small value, increases to its maximum and becomes small again thus inducing an emf.
- ii. The speed of the magnet is largest when it approaches the coil (Placed at the mean position of the oscillation).
- iii. Thus the magnetic field changes quite slowly with time when the magnet is far away and changes rapidly when it approaches the coil.
- iv. The variation of magnetic field B-(at the coil in mean position) with time is shown in figure



- v. Now, the induced emf is proportional to  $\frac{d\phi}{dt}$ , that is to the slope of the curve in the figure
- vi. As the slope of the curve is largest at times  $t_1$  and  $t_2$ , the magnitude of induced emf will be largest at these times. But as per Lenz's law emf (e) is 'negative' when f is increasing at time  $t_1$  and emf (e) is 'positive' when f is decrease in at time  $t_2$  as is shown in figure.



- vii. Thus, one 'negative' and positive occurs during just half a cycle of motion of the magnet. During the return swing, the negative pulse is

- viii. The diode will conduct only during the 'positive' pulse. At the first half swing, the capacitor will charge up to a potential ( $e_1$ ). During the next half swing, the diode will be cut off until 'positive' pulse is produced and then the capacitor will charge upto a slightly higher potential ( $e_2$ ). This will continue for a few oscillations till the capacitor charges upto its peak values  $e_0$  by the voltage/emf pulse. At this stage, ammeter will show no kick (further increase) in the current of the circuit.

- ix. The equation for induced emf can be written as

$$|e| = \left| \frac{d\phi}{dt} \right| = \left| \frac{d\phi}{d\theta} \right| \cdot \left| \frac{d\theta}{dt} \right| \quad \dots(1)$$

- x. From the oscillation equation, we have,  $q = q_0 \sin 2\pi vt$ ,  $q_0$  being the amplitude of oscillating magnet.

$$\therefore \text{frequency}(v) = \frac{1}{\text{time period}(T)}$$

$$\therefore \theta = \theta_0 \sin \frac{2\pi}{T} t$$

$$\therefore \frac{d\theta}{dt} = \theta_0 \left( \frac{2\pi}{T} \right) \cos \frac{2\pi}{T} t$$

$$\frac{d\theta}{dt} = \frac{2\pi\theta_0}{T} \cos \frac{2\pi t}{T} \quad \dots(2)$$

- xi. The peak voltage (emf)  $e_0$  in the induced emf

pulse corresponds to  $\left( \frac{d\phi}{dt} \right)_{\max}$

It can be seen from the figure that  $\left( \frac{d\phi}{dt} \right)_{\max}$

occurs at positions near the mean position.

- xii. In equation(2), the cosine term does not differ much from unity for very small angles (close to zero).

$$\therefore |e_0| = \left( \frac{d\phi}{dt} \right)_{\max} \approx \left( \frac{d\phi}{dt} \right)_{\max} \left( \frac{2\pi\theta_0}{T} \right) \quad \dots(3)$$

For given magnet-coil system, the peak induced emf  $e_0$  is directly proportional to angular amplitude ( $q_0$ ) and inversely proportional to time period (T).

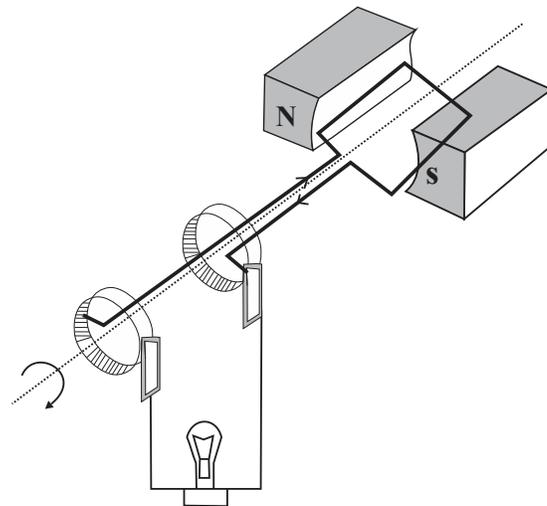
## 12.7 Generators

### Electric Generator :

AC generator is a machine that converts mechanical energy into electrical energy. The AC Generator's input supply is mechanical energy supplied by steam turbines, gas turbines and combustion engines. The output is alternating electrical power in the form of alternating voltage and current.

AC generators work on the principle of **Faraday's law of electromagnetic induction**, which states that electromotive force – EMF or voltage – is generated in a current-carrying conductor that cuts a uniform magnetic field. This can either be achieved by rotating a conducting coil in a static magnetic field or rotating the magnetic field containing the stationary conductor. The preferred arrangement is to keep the coil stationary because it is easier to draw induced alternating current from a stationary armature coil than a rotating coil.

**The generated EMF depends on the number of armature coil turns, magnetic field strength, and the speed of the rotating field.**



### ○ AC Generator Parts and Function

The various parts of an AC generator are:

- |                |               |
|----------------|---------------|
| 1. Field       | 2. Armature   |
| 3. Prime Mover | 4. Rotor      |
| 5. Stator      | 6. Slip Rings |

The following are the functions of each of these components of an AC generator.

- **Field**  
The field consists of coils of conductors that receive a voltage from the source and produce magnetic flux. The magnetic flux in the field cuts the armature to produce a voltage. This voltage is the output voltage of the AC generator.
- **Armature**  
The part of an AC generator in which the voltage is produced is known as an armature. This component primarily consists of coils of wire that are large enough to carry the full-load current of the generator.
- **Prime Mover**  
The component used to drive the AC generator is known as a prime mover. The prime mover could either be a diesel engine, a steam turbine, or a motor.
- **Rotor**  
The rotating component of the generator is known as a rotor. The generator's prime mover drives the rotor.
- **Stator**  
The stator is the stationary part of an AC generator. The stator core comprises a lamination of steel alloys or magnetic iron to minimise the eddy current losses.
- **Slip Rings**  
Slip rings are electrical connections used to transfer power to and from the rotor of an AC generator. They are typically designed to conduct the flow of current from a stationary device to a rotating one.
- **Working of an AC Generator**  
When the armature rotates between the poles of the magnet upon an axis perpendicular to the magnetic field, the flux linkage of the armature changes continuously. As a result, an electric current flows through the galvanometer and the slip rings and brushes. The galvanometer swings between positive and negative values. This indicates that there is an alternating current flowing through the

galvanometer. The direction of the induced current can be identified using **Fleming's Right Hand Rule.**

**Related Articles:**

- [AC Motor](#)
- [DC Motor](#)
- [Universal Motor](#)
- [Difference Between AC Generator and DC Generator](#)

**Advantages of AC Generators over DC Generators**

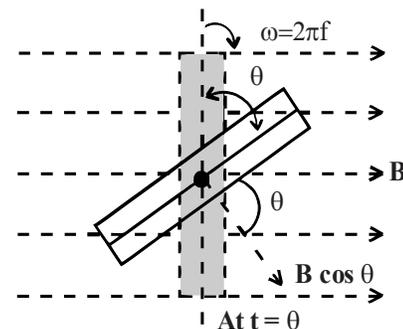
Following are a few advantages of AC generators over DC generators:

- i. AC generators can be easily stepped up and stepped down through transformers.
- ii. Transmission link size in AC Generators is thinner because of the step-up feature.
- iii. Losses in AC generators are relatively lesser than in DC machine
- iv. The size of an AC generator is smaller than a DC generator

Most of us begin our study with Direct Current, but eventually, we learn that direct current is not the only type of current we come across. There are sources of electricity that produce voltages and currents which are alternating in nature. This type of current is called an alternating current or an AC. The video will help you explore the differences between an alternating current and a direct current.

**Q.7 Derive an expression the emf induced in a coil rotating in uniform magnetic field. Represent graphically the induced emf.**

**Ans:**



- i. Consider a conducting coil rotating with an uniform angular velocity  $\omega$ .

- ii. The axis of rotation is in the plane of the coil and perpendicular to the uniform magnetic field of induction B.
- iii. Let, n - number of turns A - area of the coil.
- iv. Initially, the plane of the coil is perpendicular to the magnetic induction. The magnetic flux passing through the coil is  $\phi = nAB$
- v. After time t, the plane of the coil rotates through an angle  $\theta$   
The magnetic flux passing through the coil at time t is,

$$\phi = nAB \cos \theta = nAB \cos \omega t$$

where,

$\theta = \omega t$ . the angle between normal to plane of the coil and magnetic induction B.

- vi. As time t changes, the magnetic flux goes on changing,.

The induced emf in the coil is

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(nAB \cos \omega t)$$

$$= -nAB \frac{d}{dt}(\cos \omega t) = nAB \omega \sin(\omega t)$$

$$\therefore e = 2\pi f nAB \sin(\omega t) \quad \dots(1)$$

This gives instantaneous e.m.f.

- vii. The maximum value of induced e.m.f. is called peak value i.e. when  $\sin \omega t = 1$

$$e_0 = nAB \omega$$

$$= nAB \times 2\pi f \quad \dots(2)$$

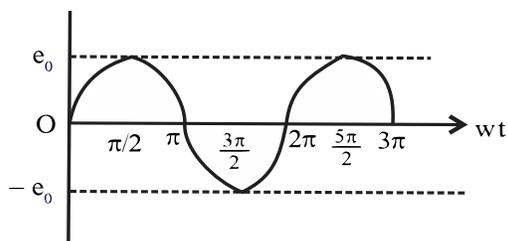
- viii. The instantaneous e.m.f. is given by,

$$\therefore e = e_0 \sin(\omega t) \quad \dots(3)$$

As induced emf is not constant but varies with time or  $\sin \omega t$ , hence the emf is called **sinusoidal emf**.

- ix. The variation of the induced emf with  $\theta = \omega t$  is a sine curve.

$\theta = \omega t$	0	$\pi/2$	$\pi$	$3\pi/2$	$2\pi$	$5\pi/2$	$3\pi$
e	0	$e_0$	0	$-e_0$	0	$e_0$	0



- x. The e.m.f. induced is positive during first half

cycle and negative during the next half cycle. Therefore, the induced e.m.f. generated in the coil is called an **alternating emf**.

**Note :**

- i. When plane of coil is parallel to the direction of the magnetic field, maximum emf is induced.
- ii. When plane of coil is perpendicular to the direction of the magnetic field, minimum emf is induced.
- iii. The symbol for AC source is  $\ominus$

### 12.8 Back emf and Back Torque

#### Q.8 What is back emf

**Ans :**

- i. When the armature of a DC motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence emf is induced in it as in a generator
- ii. The induced emf acts in opposite direction to the applied voltage V and is known as Back emf
- iii. The back emf is always less than the applied voltage.

#### Q.9 Give reason : The Current through motor is larger in the beginning than when the motor is running at full speed.

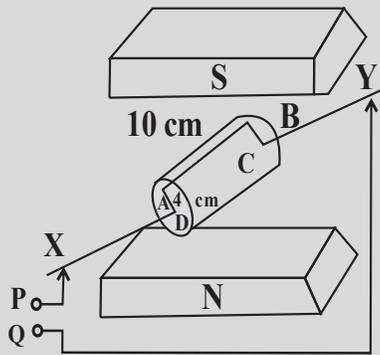
**Ans :**

- i. Initially, when a motor is just starting up, its armature is not turning and hence it is not producing any back emf.
- ii. As the motor starts speeding up the back emf increases and armature current decreases.
- iii. This explains the reason as to why the current through a motor is larger in the beginning than when the motor is running at full speed.

### INTEXT QUESTION

**A rotating armature of a simple generator consists of a loop ABCD to which connections are made through sliding contacts. The armature is rotated at 1500 rpm in the magnetic field B of 0.5 N/Am. Determine the**

induced emf between the terminals P and Q of the generator at the instant shown in the adjoining figure.



**Data:**  $r = 4 \text{ cm} = 0.04 \text{ m}$ ,  $l = 10 \text{ cm} = 0.1 \text{ m}$ ,  
 $B = 0.5 \text{ N/Am}$ ,

$$f = 1500 \text{ rpm} = \frac{1500}{60} \text{ rps} = 25 \text{ Hz}$$

$$\omega = 2\pi f = 2\pi \times 25 = 50\pi \text{ rad/s}$$

**To find:**  $|e|$

**Formulae:** i.  $v = \omega r$  ii.  $e = Blv \sin \theta$

**Solution:**

i. For wire AB

$$\begin{aligned} v &= \omega r = 0.04 \times 50\pi \\ &= 0.04 \times 50 \times 3.14 \\ &= 6.284 \text{ m/s} \end{aligned}$$

ii. The magnetic field is acting vertically upward from North to south pole.

Now wire AB is perpendicular to magnetic field

Therefore emf induced in wire AB is

$$e = Blv \sin \theta = 0.5 \times 0.1 \times 6.28 \times \sin 90$$

$$e_{AB} = 0.3142 \text{ V} = 314.2 \text{ mV}$$

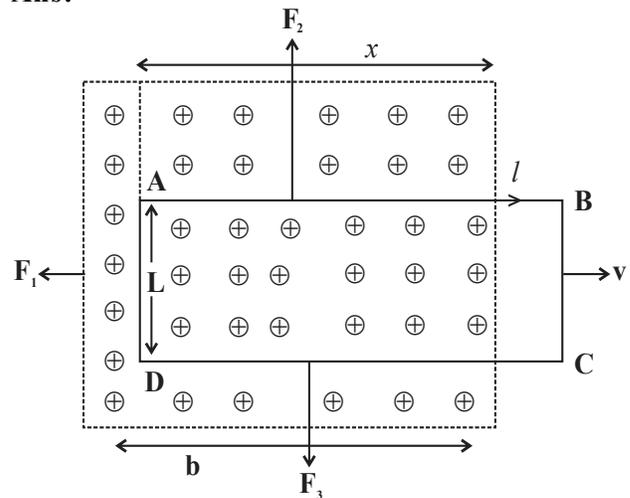
**Ans:** The induced emf between the terminals P and Q of the generator is 314.2 mV.

### 12.9 Induction and energy transfer

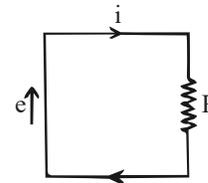
**Q.10** Obtain expression for rate of production of heat energy when wire loop is pulled out through the magnetic field  
**OR**

Show that rate of doing mechanical work is exactly same as the rate of production of heat energy in the circuit

**Ans:**



- i. Consider a loop ABCD moving with constant velocity  $\vec{v}$  in a constant magnetic field  $\vec{B}$ .
- ii. When loop is moved with constant velocity current is induced in loop. The segments which are within magnetic field experience force due to Lorentz force.
- iii. To pull the loop at a constant velocity towards right, it is required to apply an external force on the loop so as to overcome the magnetic force of equal magnitude but acting in opposite direction.



iv. The rate of doing work on the loop is,

$$P = \frac{\text{Work}(W)}{\text{time}(t)} = \frac{\text{Force}(F) \times \text{displacement}(d)}{\text{time}(t)}$$

$$P = \text{Force}(F) \times \text{velocity}(v)$$

$$\therefore P = F.v \quad \dots(1)$$

v. Magnitude of magnetic flux through the loop is,

$$\phi_B = B.A = B.Lx \quad \dots(2)$$

vi. As the loop is moved to the right, the area lying within the magnetic field decreases, thus causing a decrease in the magnetic flux linked with the moving loop. As per Lenz's law, the

decreasing magnetic flux induces current in the loop.

vii. Induced current  $i$  is given as

$$|e| = \left| \frac{d\phi}{dt} \right| = \frac{d}{dt}(BLx) = BL \cdot \frac{dx}{dt} = BLv \quad \dots(3)$$

viii. The magnitude of induced current  $I$  can be written using equation (3) as

$$i = \frac{|e|}{R} = \frac{BLv}{R} \quad \dots(4)$$

ix. The three segments of the current carrying loop experience the deflecting forces  $\vec{F}_1$ ,  $\vec{F}_2$  and  $\vec{F}_3$  in magnetic field in accordance with equation,  $\vec{F} = i\vec{L} \times \vec{B}$

x. From the symmetry, the forces  $\vec{F}_2$  and  $\vec{F}_3$  being equal and opposite, cancel each other. The remaining force  $\vec{F}_1$  is directed opposite to the external force  $\vec{F}$  on the loop,  $\vec{F} = -\vec{F}_1$

xi. The magnitude of  $|\vec{F}_1|$  can be written as

$$|\vec{F}_1| = iLB \sin 90 = iLB = |\vec{F}| \quad \dots(5)$$

xii. From equation (4) and (5),

$$|\vec{F}| = |\vec{F}_1| = iLB = \frac{BLv}{R} \cdot LB \quad (\text{From 4})$$

xiii. From equation(1) and (6), the rate of doing mechanical work (power), is given as,

$$\therefore |\vec{F}| = \frac{B^2 L^2 v}{R}$$

$$P = \vec{F} \cdot \vec{v} = \frac{B^2 L^2 v}{R} \cdot v = \frac{B^2 L^2 v^2}{R} \quad \dots(7)$$

xiv. If current  $i$  is flowing in the closed circuit with collective resistance  $R$ , the rate of production of heat energy in the loop can be written as  $P = i^2 R$  (From 4)

$$P = \left( \frac{BLv}{R} \right)^2 R$$

$$P = \frac{B^2 L^2 v^2}{R} \quad \dots(8)$$

xvi. Comparing equation (7) and equation (8), it can be found that the rate of doing mechanical work is exactly same as the rate of production of heat energy in the circuit.

Thus the work done in pulling the loop through the magnetic field appears as heat energy in the loop.

### 12.10 Eddy current

#### ★ Q.11 What are eddy currents?

##### State applications of eddy current

**Ans:**

##### Eddy currents :

- i. The circulating currents generated in a metallic conductor, placed in a changing magnetic field, are called eddy currents.
- ii. Eddy currents are also called Foucault's current.
- iii. According to Lenz's law, eddy currents oppose the change in the magnetic flux.

##### Properties :

- i. When eddy currents are developed large amount of heat is produced.
- ii. Eddy currents opposes the motion of metallic conductor in magnetic field.

##### Applications of eddy currents :

##### A) Dead beat galvanometer :

- i. When an electric current flows through a galvanometer, the coil of the galvanometer gets deflected. When the current stops, the coil tends to go on oscillating.
- ii. In order to avoid these unwanted oscillations the coil is mounted on a copper frame.
- iii. The eddy currents produced in the conducting frame then oppose the oscillations of the coil. Such a galvanometer is called dead beat galvanometer.

##### B) Induction furnace (Induction heating):

- i. In induction furnace, the heat generated by the eddy currents is used for melting the metals.
- ii. The metal is kept in rapidly changing magnetic field. Large eddy currents are generated in the metal, and the metal gets heated.

##### C) Electric brakes (Magnetic Induction):

- i. When a train in motion is to be suddenly stopped, the current supply to the motor to

rotate the axle is switched off. This produces a powerful magnetic field across the metal drum fixed co-axially on the axle of wheels.

- ii. An induced emf is then produced such that it opposes the rotation of the drum, and brings the train to rest.

**D) Speedometer :**

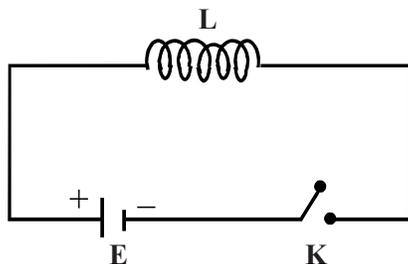
- i. Eddy currents are used to know the speed of any vehicle. A pointer shows the speed on a calibrated scale. Speedometer consists of a strong magnet, kept rotating according to the speed of the vehicle.
- ii. A magnet rotates in an aluminum drum, pivoted by means of spring. Eddy currents are produced in the drum. The drum turns in the direction of the rotating magnet.
- iii. A pointer attached to the drum indicates the speed of the vehicle on a calibrated scale.

**12.11 Self inductance**

**Q.12 Explain the phenomenon of self induction? Define coefficient of self inductance of a coil and state its SI unit and dimensions.**

**Ans : Self induction :**

- i. The phenomenon of production of an induced emf in a coil due to the changes in the current in the same coil is called self induction.
- ii. Consider a coil of negligible resistance connected with source E and key K in series as shown in fig.



- iii. When the key K is closed, The current in the coil increases from zero value, produces emf in the same coil.
- iv. During growth or decay of current in the coil an opposing emf called back emf. is induced

in the coil.

- v. The magnetic flux ( $\phi$ ) linked with the coil is directly proportional to the current I.

i.e.,  $\phi \propto I$

$\therefore \phi = LI,$

- vi. The induced emf is given by

$$e = -\frac{d\phi}{dt} = -\frac{d(LI)}{dt} = -L \frac{dI}{dt}$$

-ve sign shows that 'e' is opposite to the rate of change of current.

where, L - coefficient of self induction or self inductance.

- vii. **Coefficient of Self Induction (L) :** It is defined as the ratio e.m.f. induced in the coil to the rate of change of current in the same coil.

$$L = \left| \frac{e}{dI/dt} \right|$$

**SI unit :** henry (H).

**Dimensions:**  $[L] = [L^2M^{-1}T^{-2}I^{-2}]$

**Q.13 Show that self inductance of a circuit is numerically equal to twice the work done in establishing the magnetic flux associated with unit current in the circuit.**

**Ans:**

- i. When a current increases in the circuit, an induced emf acts opposite to it. Consequently, work will have to be done in order to establish the magnetic flux associated with a steady current  $i_0$  in the circuit.
- ii. Work done in time dt is given as,

$$\begin{aligned} dW &= e \cdot Idt \\ &= \left( -L \frac{dI}{dt} \right) \cdot (I) \cdot dt \quad \dots \left[ \because e = -L \frac{dI}{dt} \right] \\ &= -L \frac{dI}{dt} \cdot dt = -LI \cdot dI \end{aligned}$$

$\therefore$  Total work,  $W = \int dW = \int_0^{i_0} -LI dI$

$\therefore W = -L \frac{I_0^2}{2} \dots(1)$

$W = \frac{1}{2} LI_0^2$  (In magnitude)

- iii. Now if  $I_0 = 1$ ,  
Then  $W = L \cdot \frac{1}{2}$   
 $L = +2W$  (numerically) ... (2)  
Hence self-inductance of a circuit is numerically equal to twice the work done in establishing the magnetic flux associated with unit current in the circuit.

**Note**

- i. This work done  $W$ , will represent the energy of the circuit.

$$\text{Energy of the circuit} = \frac{1}{2} LI_0^2$$

- ii. The mechanical energy is expressed in terms of kinetic energy is given as,

$$K.E. = \frac{1}{2} mv^2$$

Comparing both the equations, it can be found that self-inductance ( $L$ ) of an electrical circuit plays the same role (electrical inertia) as played by mass (inertia) in mechanical motion.

**Q.14 Obtain expression for inductance of a solenoid.**

**Ans:**

- i. Consider a current  $I$  established in the windings (turns) of a long solenoid. The current produces a magnetic flux  $\phi_B$  through the central region.  
ii. The inductance of the solenoid is given by,

$$L = \frac{N\phi_B}{I}, \quad \dots(1)$$

Where  $N$  = the number of turns,  
 $\phi_B$  = magnetic flux linkage.

- iii. The flux linkage for length  $l$  near the middle of the solenoid is  
 $N\phi_B = (n/)(\vec{B} \cdot \vec{A}) = n/BA \cos \theta = n/BA \cos 0$   
 $N\phi_B = n/BA$   
iv. Substituting  $N\phi_B$  in equation (1) we get,

$$L = \frac{n/BA}{I} \quad \dots(2)$$

- v. At the centre of solenoid,  
 $B = \mu_0 nI$   
vi.  $\therefore$  From equation (2) we can write  
 $L = \frac{n/(\mu_0 nI)A}{I}$   
 $\therefore L = \mu_0 n^2 lA$   
vii. Inductance per unit length of solenoid is

$$\frac{L}{l} = \mu_0 n^2 A = \mu_0 n^2 \left( \frac{\pi d^2}{4} \right)$$

where  $d$  is diameter of solenoid.

**INTEXT QUESTION**

- Q.15 i. Derive an expression for the self-inductance of a toroid of circular cross-section of radius  $r$  and major radius  $R$ .**  
**ii. Calculate the self-inductance ( $L$ ) of toroid for major radius ( $R$ ) = 15 cm, cross-section of toroid having radius ( $r$ ) = 2.0 cm and the number of turns ( $n$ ) = 1200.**

**Ans:**

- i. The magnetic field inside a toroid,  $B = \frac{\mu_0 Ni}{2\pi r}$   
Where  $N$  = number of turns and  $r$  is the distance from the toroid axis.  
ii. As  $r \ll R$ , magnetic field ( $B$ ) in the cavity of toroid is uniform and can be written as,  
 $B = \frac{\mu_0 Ni}{2\pi R} \quad \dots(1)$   
iii. The magnetic flux  $\phi$  passing through cavity toroid that links each turns is,  
 $\phi = BA = \frac{\mu_0 Ni}{2\pi R} (\pi r^2) = \frac{\mu_0 Nir^2}{2R}$   
iv. When the current  $i$  varies with time, the induced emf  $e$  across the terminals of toroid is given by Faraday's law.

$$e = -\frac{Nd\phi}{dt} = -N \frac{d}{dt} \left( \frac{\mu_0 Nir^2}{2R} \right)$$

$$e = -N \left( \frac{\mu_0 N r^2}{2R} \right) \frac{di}{dt}$$

Comparing with  $e = -L \frac{di}{dt}$

We get,  $L = \frac{\mu_0 N^2 r^2}{2R} \dots (r \ll R) (2)$

- v. **Data:**  $N = 1200, r = 2.0 \text{ cm}$   
 $= 2 \times 10^{-2} \text{ m},$   
 $R = 15 \text{ cm} = 15 \times 10^{-2} \text{ m},$

**To find:**  $L$

**Formula:**  $L = \frac{\mu_0 N^2 r^2}{2R}$

**Solution:**

$$L = \frac{4 \times 3.142 \times 10^{-7} \times (1200)^2 \times (2 \times 10^{-2})^2}{2 \times 15 \times 10^{-2}}$$

$$= 2.41 \times 10^{-3} \text{ H}$$

The self-inductance of the toroid is

**2.41 mH.**

**Note:**

*Induced in series and parallel.*

*If several inductances are connected in series or in parallel, then the total inductance is determined by using following relations:*

$$L_{\text{Total}} = L_1 + L_2 + L_3 + \dots \text{ (Series combination)}$$

$$\frac{1}{L_{\text{Total}}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots \text{ (Parallel combination)}$$

### Type - IV

#### Numerical based on self inductance

#### Formula used

1.  $\phi = LI$

2.  $e = -L \frac{dI}{dt}$

3.  $W = \frac{1}{2} LI^2$

4. For solenoid

a.  $B = \mu_0 nI$       b.  $L = \frac{N\phi}{I}$

c.  $\frac{L}{l} = \mu_0 n^2 A$

- 1) The self-inductance of a closely wound coil of 200 turns is 10 mH. Determine the value of magnetic flux through the cross-section of the coil when the current passing through the coil is 4 mA.

**Data:**  $L = 10 \text{ mH} = 10 \times 10^{-3} \text{ H}, N = 200,$   
 $I = 4 \text{ mA} = 4 \times 10^{-3} \text{ A}$

**To find:**  $\phi/N$  [flux through cross section]

**Formulae:**  $\frac{\phi}{N} = \frac{LI}{N}$

**Solution:**  $\frac{\phi}{N} = \frac{LI}{N}$

$$\phi = \frac{10^{-2} \times 4 \times 10^{-3}}{200}$$

$$= 2 \times 10^{-7} \text{ Wb}$$

**Ans:** Magnetic flux through cross section is  $2 \times 10^{-7} \text{ Wb}$

- ★ 2) Consider a uniformly wound solenoid having  $N$  turns and length  $l$ . The core of the solenoid is air. Find the inductance of the solenoid of  $N = 200, l = 20 \text{ cm}$  and induced emf  $e_L$ , if the current flowing through the solenoid decreases at a rate of 60 A/s.

**Data:**  $l = 20 \text{ cm} = 0.2 \text{ m}, N = 200,$   
 $A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$

$$\frac{dI}{dt} = -60 \text{ A/s}$$

**To find:** i.  $L$     ii.  $e$

**Formula:** i.  $n = \frac{N}{l}$     ii.  $L = \mu_0 n^2 l A$

iii.  $e = -L \frac{dI}{dt}$

**Solution:**

i.  $n = \frac{N}{l} = \frac{200}{0.2} = 10^3 \text{ turns/m}$

ii.  $L = \mu_0 n^2 l A$   
 $= 4\pi \times 10^{-7} \times (10^3)^2 \times 0.2 \times 5 \times 10^{-4}$   
 $= 4 \times 3.14 \times 0.2 \times 5 \times 10^{-5}$   
 $= 12.56 \times 10^{-5} \text{ H}$

$$= 0.1256 \times 10^{-3} \text{ H}$$

$$= 0.1256 \text{ mH}$$

$$\text{iii. } e = -L \frac{dI}{dt} = -(0.1256 \times 10^{-3}) \times (-60)$$

$$= 7.56 \times 10^{-3} \text{ V} = 7.56 \text{ mV}$$

**Ans:** i. Self inductance of the coil is 0.1256 mH.  
ii. Induced emf is 7.56 mV.

- ★ 3) A toroidal ring, made from a bar of length ( $l$ ) 1 m and diameter ( $d$ ) 1 cm, is bent into a circle. It is wound tightly with 100 turns per cm. If the permeability of bar is equal to that of free space ( $\mu_0$ ), calculate the magnetic field inside the bar ( $B$ ) when the current ( $i$ ) circulating through the turns is 100 A. Also determine the self-inductance ( $L$ ) of the coil.

**Data:**  $l = 1 \text{ m}$ ,  $d = 1 \text{ cm} = 10^{-2} \text{ m}$ ,  
 $n = 100 \text{ turns/cm} = 10000 \text{ turns/m}$   
 $= 10^4 \text{ turns/m}$   
 $i = 100 \text{ A}$

**To find:** i.  $B$     ii.  $L$

**Formulae:** i.  $B = \mu_0 nI$     ii.  $L = \frac{N\phi}{I}$

**Solution :**

i.  $B = \mu_0 nI$   
 $B = 4 \times 3.142 \times 10^{-7} \times 10000 \times 100$   
 $= 1.256 \text{ T}$

ii.  $L = \frac{N\phi}{i} = \frac{NBA}{i} = \frac{(nl)BA}{i}$   
 $L = \frac{nIB}{i} \left( \frac{\pi d^2}{4} \right)$   
 $= \frac{10^4 \times 1 \times 1.256 \times 3.14 \times (10^{-2})^2}{100 \times 4}$   
 $= \frac{3.944}{4} \times 10^{-2} = 0.98 \times 10^{-2} \text{ H}$   
 $= 9.8 \times 10^{-3} \text{ H} = 9.8 \text{ mH}$

**Ans:** i. Magnetic field is 1.256 T.  
ii. Self inductance is 9.8 mH.

- 4) What is the self-inductance of a coil, in which magnetic flux of 40 milliweber is produced when 2 A current flows through it?

**Data:**  $\phi = 40 \text{ mwb} = 40 \times 10^{-3} \text{ wb}$ ,  $I = 2 \text{ A}$

**To find:**  $L$

**Formula:**

$$\phi = LI$$

$$\therefore L = \frac{\phi}{I}$$

**Solution:**

$$L = \frac{40 \times 10^{-3}}{2} = 20 \times 10^{-3} = 20 \times 10^{-2} \text{ H}$$

**Ans:** Self inductance of coil is  $2 \times 10^{-2} \text{ H}$

- 5) If a rate of change of current of  $4 \text{ As}^{-1}$  induces an emf of 20 mV in a solenoid, what is the self-inductance of the solenoid?

**Data:**  $\frac{dI}{dt} = 4 \text{ As}^{-1}$ ,  
 $e = 20 \text{ mV} = 20 \times 10^{-3} \text{ V}$

**To find:**  $L$

**Formula:**  $|e| = L \frac{dI}{dt}$

**Solution:**

$$L = \frac{|e|}{dI/dt} = \frac{20 \times 10^{-3}}{4}$$

$$= 5 \times 10^{-3} \text{ H} = 5 \text{ mH}$$

**Ans:** Self inductance of Solenoid is 5 mH

**Problem for Practice**

- An inductor of 5H inductance carries a steady current of 2A. How can a 50 V self-induced emf be made to appear in the inductor?  
**Ans: 0.2s**
- A toroidal solenoid with an air-core has an average radius of 15 cm, area of cross-section  $12 \text{ cm}^2$  and 1200 turns. Obtain the self-inductance of the toroid. Ignore field variation across the cross-section of the toroid.  
**Ans: 2.3mH**

3. Magnetic flux of 5 microweber is linked with a coil, when a current of 1 mA flows through it. What is the self-inductance of the coil?

**Ans: 5mH**

4. The self inductance of an inductor coil having 100 turns is 20mH. Calculate the magnetic flux through the cross-section of the coil corresponding to a current of 4mA. Also, find the total flux.

**Ans:  $8 \times 10^{-7}$  Wb,  $8 \times 10^{-5}$  Wb**

5. An average induced emf of 0.4 V appears in a coil when the current in it is changed from 10A in one direction to 10A in opposite in 0.40 second. Find the coefficient of self inductance of the coil.

**Ans: 8 mH**

**12.12 Energy stored in a magnetic field**

**Q.16 Obtain expression for the energy stored in magnetic field**

**Ans.**

- i. Changing magnetic flux in a coil causes an induced emf.
- ii. The induced emf is given as,  $e = -L \frac{dI}{dt}$ .
- iii. The induced emf so produced opposes the change and hence the energy has to be spent to overcome it to build up the magnetic field.
- iv. The work done in moving a charge  $dq$  against this emf is,

$$dW = -e \cdot dq = L \frac{dI}{dt} \cdot dq$$

$$\therefore dW = L \cdot I \cdot dI \quad \dots \left[ \because \frac{dq}{dt} = I \right]$$

Therefore total work,

$$W = \int dW = \int_0^I LI dI = \frac{1}{2} LI^2 = U_B \quad \dots(1)$$

The above equation represent energy stored in magnetic field.

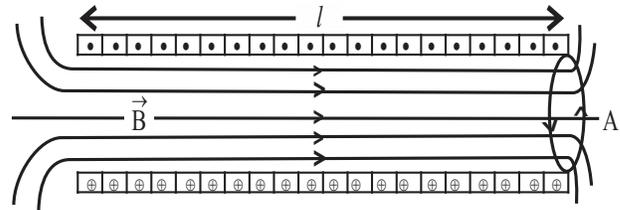
**12.13 Energy Density of a Magnetic Field**

**Q.18 Obtain an expression for energy density**

**stored at any point in a magnetic field.**

**Ans:**

- i. Consider a long solenoid having length  $l$  near the middle, cross-sectional area  $\bar{A}$  and carrying a current  $i$  through it.
- ii. The volume associated with long  $l$  will be  $Al$ .



- iii. The energy  $U_B$  stored by the length  $l$  of the solenoid must lie entirely within volume  $Al$ , because the magnetic field outside the solenoid is almost zero.
- iv. The energy stored will be uniformly distributed within the volume as the magnetic field  $\bar{B}$  is uniform everywhere inside the solenoid.
- v. Thus, the energy stored per unit volume ( $u_B$ ) in the magnetic field is,

$$u_B = \frac{U_B}{Al} \quad \dots(1)$$

- vi. But,  $U_B = \frac{1}{2} LI^2$

$$\therefore u_B = \frac{1}{2} LI^2 \cdot \frac{1}{Al} = \left( \frac{L}{l} \right) \cdot \frac{I^2}{2A} \quad \dots(2)$$

- vii. For a long solenoid, the inductance ( $L$ ) per unit length is given as.

$$\left( \frac{L}{l} \right) = \mu_0 n^2 A \quad \dots(3)$$

- viii. substituting equation(3) in equation (2), we get,

$$\therefore u_B = \frac{\mu_0 n^2 I^2}{2}$$

$$u_B = \frac{1}{2} \mu_0 n^2 I^2$$

- ix. For a solenoid the magnetic field at interior points is given as,

$$B = \mu_0 nI$$

- x. Therefore, the expression for energy density ( $u_B$ ) stored in magnetic field can be

written as

$$\therefore \mu_B = \frac{B^2}{2\mu_0}$$

This equation gives the density of stored energy at any point where magnetic field is B.

**INTEXT QUESTION**

**Calculate the self-inductance of a coaxial cable of length  $l$  and carrying a current  $I$ . The current flows down the inner cylinder with radius  $a$ , and flows out the outer cylinder with radius  $b$ .**

**Ans.**

- i. Consider point between two cylinders at a distance  $r$  from the axis.  
According to Ampere law, magnetic field at this point is given by

$$B = \frac{\mu_0 I}{2\pi r} \quad \dots(1)$$

- ii. Magnetic field is zero elsewhere  
iii. Magnetic energy density is given as

$$u_B = \frac{B^2}{2\mu_0}$$

Using equation(1) we get,

$$U_B = \frac{1}{2\mu_0} \left( \frac{\mu_0 I}{2\pi r} \right)^2 = \frac{\mu_0 \times I^2}{2\mu_0 \times 4\pi^2 r^2}$$

$$U_B = \frac{\mu_0 I^2}{8\pi^2 r^2}$$

- iv. Consider a cylindrical shell of length ' $l$ ', radius ' $r$ ' and thickness ' $dr$ '  
Energy stored in this cylindrical shell is

Energy stored =  $u_B \times$  volume of Cylindrical shell

$$U_s = \frac{\mu_0 I^2}{8\pi^2 r^2} \times 2\pi r l dr$$

$$U_s = \frac{\mu_0 I^2 l dr}{4\pi r} \quad \dots(2)$$

- v. Energy stored between two cylinders

$$U = \int_a^b U_s = \frac{\mu_0 I^2 l}{4\pi} \int_a^b \frac{1}{r} dr = \frac{\mu_0 I^2 l}{4\pi} [\ln r]_a^b$$

$$U = \frac{\mu_0 I^2 l}{4\pi} \ln \left[ \frac{b}{a} \right]$$

$$\therefore U = \frac{\mu_0 l}{4\pi} \ln \left[ \frac{b}{a} \right] \cdot I^2$$

- vi. Magnetic energy confined in an inductor (L) carrying current I can be written as

$$U = \frac{1}{2} LI^2$$

- vii. By comparing we get

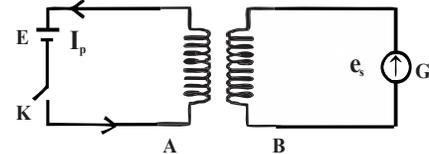
$$L = \frac{\mu_0}{4\pi} \cdot l \cdot \ln \left[ \frac{b}{a} \right]$$

**12.14 Mutual Inductance (M)**

**Q.19 Explain the phenomenon of mutual induction? Define coefficient of mutual inductance of a coil and state its SI unit and dimensions.**

**Ans :** Mutual induction :

- i. The phenomenon of production of an induced emf in a coil due to the changes of current in a neighboring coil, is called mutual induction.  
ii. Consider two coils 1 and 2 placed near each other. The coil 1 is connected in series with the source E and key K. The coil 2 is connected to sensitive galvanometer



- iii. Let  $\phi_{21}$  be the flux linked with coil-2 due to current in coil 1

$$\therefore \phi_{21} \propto I_1$$

$$\therefore \phi_{21} = M_{21} I_1$$

Where  $M_{21}$  is mutual inductance of coil-2 with respect to coil-1

- iv. emf induced in coil-2 is given by

$$e_{21} = - \frac{d\phi_{21}}{dt}$$

$$e_{21} = -M_{21} \frac{dI_1}{dt}$$

- v. Similarly flux linked with coil-1 due to current in coil-2

$$\phi_{12} \propto I_2$$

$$\therefore \phi_{12} = M_{12} I_2$$

where  $M_{12}$  is coefficient of mutual inductance of coil-1 w.r.t. coil-2

$\therefore$  emf induced in coil-1 is given by

$$e_{12} = \frac{-d\phi_{12}}{dt}$$

$$\therefore \boxed{e_{12} = -M_{12} \frac{dI_2}{dt}}$$

vi. **Coefficient of Mutual Induction or Mutual Inductance :** emf induced in secondary coil is due to current flowing in primary coil

$$\therefore e_s = -M \frac{dI_p}{dt}$$

**Q.20 State and define the SI unit of mutual inductance.**

i. The unit of mutual inductance is henry (H)

$$\text{henry} = \frac{\text{volt}}{\text{As}} = \text{ohm.s}$$

$$1 \text{ henry} = 1 \text{ ohm.s}$$

ii. If corresponding to 1 A/s rate of change of current in the primary circuit, the induced emf produced in the secondary circuit is 1 volt, then the mutual inductance (M) of the two circuits is 1 H.

★ **Q.21 A long solenoid of length  $l$ , cross sectional area  $A$  and having  $N_1$  turns ( primary coil), has a small coil of  $N_2$  turns (secondary coil) wound about its centre. Determine the Mutual inductance (M) of the two coils.**

**Ans:**

i. Due to current  $I$ , there is a magnetic field in coil 1,

$$B_1 = \mu_0 n_1 I_1 = \mu_0 \frac{N_1}{L} I_1$$

ii. Due to this field, a flux linkage is in coil 2

$$\phi_2 = N_2 B_1 A$$

$$= N_2 \times \mu_0 \frac{N_1}{L} I_1 \times A = \frac{\mu_0 N_1 N_2 A}{L} \times I_1 = M I_1$$

$$\therefore M = \frac{\mu_0 N_1 N_2 A}{L}$$

**Q.22 Explain coefficient of coupling between two circuits.**

**Ans :**

i. The coefficient of coupling (K) is a measure of the portion of flux that reaches coil 2 which is in the vicinity of coil 1.

ii. Greater the coefficient of coupling, greater will be the mutual inductance (M)

iii. Inductance of any circuit is proportional to the induced voltage it can develop. This is equally true for mutual inductance.

$$\therefore M \propto e_{21} \quad \dots(1)$$

Where,

$e_{21}$  = induced emf developed in coil 2 due to the portion of the flux coil reaching coil 2.

iv. We have, induced emf  $e_{21} = K \phi_1 \dots(2)$

v. But induced emf is also proportional to the number of turns in the coil,

$$\therefore e_{21} \propto N_2 (K \phi_1) \quad \dots (3)$$

$$\therefore e_{21} \propto N_2 (K N_1) \text{ (Since, } \phi_1 \propto N_1) \dots(4)$$

vi. Also  $L \propto N^2$  or  $N \propto \sqrt{L}$

$$\therefore N_1 N_2 \propto \sqrt{L_1} \sqrt{L_2} = \sqrt{L_1 L_2} \quad \dots(5)$$

vii. Substituting equation (5) and replacing  $e_{21}$  with M in equation (4), we have,

$$M = K \sqrt{L_1 L_2} \quad \dots(6)$$

where,

K = coefficient of coupling and is usually less than unity.

viii. If  $K = 1$ , the two coils will be perfectly coupled and  $M = \sqrt{L_1 L_2}$

ix. If  $K > 0.5$ , the two coils are tightly coupled.

x. If  $K < 0.5$  the coils are loosely coupled.

xi. If  $L_1 = L_2$ , then a coil with self-inductance L is coupled to itself with mutual inductance

$$M = \sqrt{L_1 L_2} = \sqrt{L^2} = L$$

**Q.23 State the factors on which coefficient of coupling depends on.**

**Ans :**

i. If two coils are wound on a common iron core, the coefficient of coupling (K) can be considered as unity.

ii. For two air-core coils or two coils in separate iron cores, the coefficient of coupling depends on the distance between two coils and the angle between the axes of the two coils.

iii. when the coils are parallel (and in line), the coefficient K is maximum.

iv. If the axes of the coils are at right angles (and in line) K is minimum.

- v. In order to prevent interaction between the coils, the coils should be oriented at right angle to each other and be kept as far apart as possible.
- vi. K-value for radio coils (Radio frequency, intermediate frequency transformers) lies between 0.001 to 0.05.

INTEXT QUESTION

**What is mutual inductance of the wireless charging system?**

**Ans:**

- i. In a wireless battery charger, the base unit can be imagined as a solenoid (coil B) of length  $l$  with  $N_B$  turns, carrying a current  $i_B$  and having a cross-section area  $A$ .
- ii. The handle coil (coil H) has  $N_H$  turns and surrounds the base solenoid (coil B completely).
- iii. The base unit is designed to hold the handle of the charging unit. The handle has a cylindrical hole so that it fits loosely over a matching cylinder on the base, unit.
- iv. When the handle is placed on the base, the current flowing in coil B induces a current in the coil H. This, induced current in coil H is used to charge the battery housed in the handle.
- v. In Coil B, number of turns per unit length,

$$n = \frac{N_B}{l}$$

- vi. Magnetic field due to solenoid coil B,

$$B = \mu_0 n i = \mu_0 \times \frac{N_B}{l} \times i$$

- vii. Magnetic flux through coil H caused by the magnetic field B due to solenoid coil B,

$$\phi_H = BA$$

$$\text{Flux linkage} = N_H \phi_H$$

- viii. Mutual inductance of the wireless system,

$$M = \frac{\text{Flux linkage}}{\text{Electric current}} = \frac{N_H BA}{i}$$

$$= \frac{\mu_0 \left( \frac{N_B}{l} \right) i \times A \times N_H}{i}$$

$$= \mu_0 \left( \frac{N_B N_H}{l} \right) \times A$$

**Type - V**

**Numerical based on mutual inductance**

**Formula used**

1.  $\phi = MI$
2.  $e = -M \frac{dI}{dt}$
3.  $M = \frac{\mu_0 N_1 N_2 A}{L}$
4.  $M = K \sqrt{L_1 L_2}$

- ★ 1). A pair of adjacent coils has a mutual inductance of 1.5H. If the current in one coil changes from 0 to 10A in 0.2s, what is the rate of change of flux linkage with the other coil?

**Data:**  $M = 1.5\text{H}$ ,  $I_2 = 10\text{ A}$ ,  $I_1 = 0\text{ A}$ ,  $\Delta t = 0.2\text{ s}$

**To find:**  $\left( \frac{\Delta\phi}{\Delta t} \right)$

**Formula:**  $\phi = MI$

$$\therefore \frac{\Delta\phi}{\Delta t} = M \frac{\Delta I}{\Delta t}$$

**Solution:**

i.  $\Delta I = I_2 - I_1 = 10 - 0 = 10\text{ A}$

ii.  $\frac{\Delta\phi}{\Delta t} = M \frac{\Delta I}{\Delta t} = 1.5 \times \frac{10}{0.2} = 75\text{ Wb/s}$

**Ans:** Rate of change of flux linkage is 75Wb/s.

- ★ 2). An emf of 96.0 mV is induced in the windings of a coil when the current in a nearby coil is increasing at the rate of 1.20 A/s. What is the mutual inductance (M) of the two coils?

**Solution:**

**Data:**  $e = 96\text{ mv} = 96 \times 10^{-3}\text{ V}$ .

$$\frac{dI}{dt} = 1.20\text{ A/s}$$

**To find:**  $M$

**Formula:**  $e = -M \frac{dI}{dt}$

**Solution :**  $M = \frac{|e|}{\left| \frac{dI}{dt} \right|}$

$$M = \frac{96 \times 10^{-3}}{1.2} = 80 \times 10^{-3} = 80 \text{mH}$$

**Ans :** Mutual Inductance of the two coil is 80mH

- ★ 3). The value of mutual inductance of two coils is 10 mH. If the current in one of the coil changes from 5A to 1A in 0.2s, calculate the value of emf induced in the other coil. Also calculate the value of induced charge passing through the coil if its resistance is 5 ohm.

**Data:**  $M = 10 \text{mH} = 10 \times 10^{-3} \text{ H}$   
 $I_1 = 5 \text{A}, I_2 = 1 \text{A},$   
 $\Delta t = 0.2 \text{s}, R = 5 \Omega$

**To find:**

- e
- q

**Formulae:** i.  $\Delta \phi = M \Delta I$

ii.  $e = \left| \frac{\Delta \phi}{\Delta t} \right|$

iii.  $q = \frac{e}{R} \times \Delta t = \frac{\Delta \phi}{R}$

**Solution:**

i.  $\Delta \phi = M \Delta I$

$$\Delta \phi = 10 \times 10^{-3} \times [1 - 5] = -4 \times 10^{-2} \text{ Wb}$$

ii.  $e = \left| \frac{\Delta \phi}{\Delta t} \right|$

$$e = \left| \frac{-4 \times 10^{-2}}{5} \right| = 0.2 \text{ V}$$

iii.  $q = \frac{e}{R} \times \Delta t = \frac{\Delta \phi}{R}$

$$q = \frac{4 \times 10^{-2}}{5} = 8 \times 10^{-3} = 8 \text{mC}$$

**Ans :** Emf induced in coil is 0.2 V and induced charge is 8 mC

- ★ 4). Two coils having self inductances  $L_1 = 75 \text{mH}$  and  $L_2 = 55 \text{mH}$  are coupled

with each other. The coefficient of coupling (K) is 0.75, calculate the mutual inductance (M) of the two coils.

**Data:**  $L_1 = 75 \text{mH}, L_2 = 55 \text{mH}, K = 0.75$

**To find:** M

**Formula:**  $M = K \sqrt{L_1 L_2}$

**Solution:**  $M = K \sqrt{L_1 L_2}$

$$M = 0.75 \sqrt{75 \times 55} = 48.11 \text{ mH}$$

**Ans :** Mutual inductance of coil is 48.11mH.

- ★ 5). The mutual inductance (M) of two coils is given as 1.5H. The self inductance of the coil are :  $L_1 = 5 \text{ H}, L_2 = 4 \text{ H}$ . Find the coefficient of coupling between the coils.

**Data:**  $L_1 = 5 \text{ H}, L_2 = 4 \text{ H}, M = 1.5 \text{ H}$ .

**To find:** K

**Formula:**  $M = K \sqrt{L_1 L_2}$

**Solution:**

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{1.5}{\sqrt{5 \times 4}} = \frac{1.5}{2 \times \sqrt{5}} = \frac{0.75}{2.24} = 0.3349 \times 10^2 = 33.5 \%$$

**Ans :** Coefficient of coupling between coils is 33.5%.

### Problem for Practice

1. A large circular coil, of radius R and a small circular coil, of radius r, are put in vicinity of each other. If the coefficient of mutual inductance, for this pair, equals 1 mH, what would be the flux linked with the larger coil when a current of 0.5 A flows through the smaller coil?

When the current in the smaller coil falls to zero, what would be its effects in the larger coil?

**Ans:**  $5 \times 10^{-4} \text{ Wb}$

2. An emf of 0.5 V is developed in the secondary coil, when current in primary coil changes from 5.0A to 2.0 A in 300 millisecond. Calculate the mutual inductance of the

two coils.

**Ans: 0.05 H**

3. A solenoidal coil has 50 turns per centimeter along its length and a cross-sectional area of  $4 \times 10^{-4} \text{m}^2$ . 200 turns of another wire are wound round the first solenoid coaxially. The two coils are electrically insulated from each other. Calculate the mutual inductance between the two coils. Given

$$\mu_0 = 4\pi \times 10^{-7} \text{NA}^{-2}$$

**Ans:  $5.027 \times 10^{-4} \text{ H}$**

4. Two coils having self inductances  $L_1 = 37.5 \text{mH}$  and  $L_2 = 27.5 \text{mH}$  are coupled with each other. The coefficient of coupling (K) is 0.37, calculate the mutual inductance (M) of the two coils.

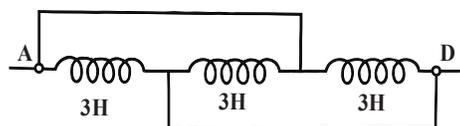
**Ans: 11.88mH**

5. The mutual inductance (M) of two coils is given as 3 H. The self inductance of the coil are :  $L_1 = 7.5 \text{ H}$ ,  $L_2 = 6.5 \text{ H}$ . Find the coefficient of coupling between the coils.

**Ans: 42.9%**

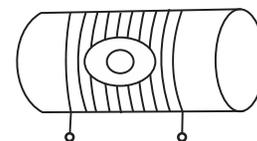
**MULTIPLE CHOICE QUESTIONS**  
**Entrance Corner (Set 3)**

1. When the current changes from +2A to -2A in 0.05 s, an emf of 8 V is induced in the coil. The coefficient of self-induction of the coil is  
a. 0.2 H                      b. 0.4 H  
c. 0.8 H                      d. 0.1 H
2. Two coils are placed closed to each other. The mutual inductance of the pair of coils depend upon  
a. the rates at which currents are changing in the two coils.  
b. relative position and orientation of the two coils.  
c. the material of the wires of the coils.  
d. the current in the two coils.
3. The inductance between A and D is



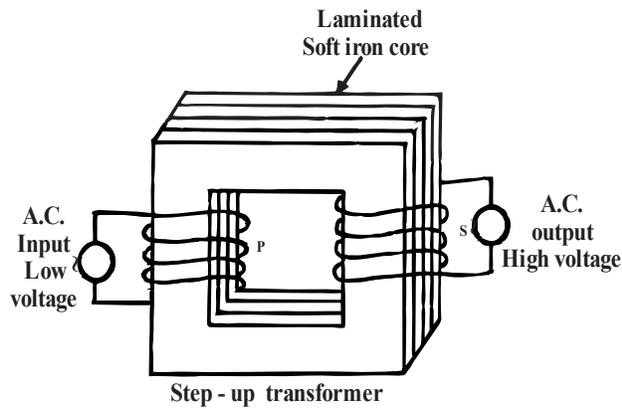
- a. 3.66 H                      b. 9 H

- c. 0.66 H                      d. 1 H
4. A coil of 100 turns carries a current of 5 mA and creates a magnetic flux of  $10^{-5}$  weber. The inductance is  
a. 0.2 mH                      b. 2.0 mH  
c. 0.02 mH                      d. none of these
5. Two coils of self-inductance  $L_1$  and  $L_2$  are placed so closed to each other so that effective flux in one coil is completely linked with other. If M is mutual inductance between them,  $M =$   
a.  $L_1 L_2$                       b.  $L_1 / L_2$   
c.  $(L_1 L_2)$                       d.  $\sqrt{L_1 L_2}$
6. Two identical induction coils each of inductance L joined in series are placed very close to each other such that the winding direction of one is exactly opposite to that of the other, What is the inductance?  
a.  $L^2$                       b.  $L_1 / L_2$   
c.  $L/2$                       d. zero
7. coefficient of coupling between two coils of self-inductances  $L_1$  and  $L_2$  is unity. It means  
a. 50% of  $L_1$  is linked with  $L_2$   
b. 100% flux of  $L_1$  is linked with  $L_2$   
c.  $\sqrt{L_1}$  time of flux of  $L_1$  is linked with  $L_2$   
d. none of the above.
8. A coil of wire of a certain radius has 100 turns and a self inductance of 15 mH. The self inductance of a second similar coil of 500 turns will be  
a. 75 mH                      b. 375 mH  
c. 15 mH                      d. none of these
9. A circular coil with a cross-sectional area of  $4 \text{ cm}^2$  has 10 turns. It is placed at the centre of a long solenoid



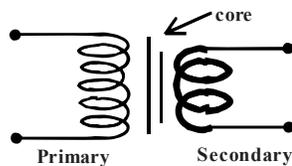
that has 15 turn/cm and a cross sectional area of  $10 \text{cm}^2$ , as shown in the figure. The axis of the coil of emf coincides with the axis of





- i) It consists of two coils of conducting wire, primary and secondary coil wound on laminated soft iron core.
- ii) The a.c. in put is applied across the primary coil and an a.c. output is obtained across the secondary coil.

**Symbol :**



**Working :**

- i) When alternating e.m.f. is applied across the primary coil, the current through the goes on changing. The magnetic flux through the core also changes.
- ii) The changing magnetic flux is linked with both the coil, an e.m.f. is induced in each coil.
- iii) The magnetic flux linked with two coils depend upon number of turns of the coil.

Let,  $N_p$ - no. of turns of primary coil  
 $N_s$ - no. of turns of secondary coil  
 $N_p\phi$  = flux linked with primary coil  
 $N_s\phi$  = flux linked with secondary coil

$$\text{i.e. } \frac{\phi_s}{\phi_p} = \frac{N_s}{N_p}$$

- iv) Induced e.m.f. produced in the coils,

$$e_p = -\frac{d\phi_p}{dt} = -N_p \frac{d\phi}{dt} \text{ and}$$

$$e_s = -\frac{d\phi_s}{dt} = -N_s \frac{d\phi}{dt}$$

$$\therefore \frac{e_s}{e_p} = \frac{N_s}{N_p} \quad \dots(i)$$

This is called equation of transformer.

- v) The ratio  $\frac{N_s}{N_p} = K$  is called turn ratio or transformer ratio.
- vi) For an ideal transformer

Inst. output power = Inst. input power

$$e_s i_s = e_p i_p$$

$$\therefore \frac{e_s}{e_p} = \frac{i_p}{i_s} \quad \dots(ii)$$

From eq (i) and (ii)

$$\frac{e_s}{e_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s}$$

**Key point**

**Efficiency of transformer :**

- i) The ratio of out put power to the input power is called efficiency.  
 % efficiency is given by  

$$\% \eta = \frac{\text{output power}}{\text{input power}} \times 100$$

$$= \frac{e_s i_s}{e_p i_p} \times 100 \%$$
- ii) Due to different types of energy losses output power of transformer is always less than input power i.e.  $e_s I_s < e_p I_p$ .
- iii) Hence, in practice, the efficiency of transformer is never 100%.
- iv) Frequency of output a.c. voltage / current is same as that of frequency of input signal.

**21. What are types of transformer? Explain.**

**Ans: Transformers are of two types :**

**A) Step up transformer :**

- i) It convert a low voltage at high current into high voltage at low current.
- ii) The primary coil is made of a thick insulated copper wire while the secondary coil is made of thin insulated wire.

In this transformer,  $N_s > N_p$ .

- iii) In step up transformer,  $e_s > e_p$   
 $\therefore i_p > i_s$

**B) Step-down transformer :**

- i) It converts high voltage at low current into a low voltage at high current.
- ii) The primary coil consists of large number of turns of thin wire while secondary coil is made of thick wire.

In this transformer,  $N_s < N_p$ .

- iii) In step down transformer,  $e_s < e_p$   
 $\therefore i_p < i_s$ .

**Type - V**

**Numerical based on Transformer**

**Formula used**

$$\frac{e_s}{e_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s}$$

- 1). A step down transformer converts a voltage of 2200 V into 220 V in the transmission line. Number of turns in primary coil is 5000. Efficiency of the transformer is 90% and its output power is 8 kW. Calculate the
- i) number of turns in the secondary coil
  - ii) input power.

Data :  $e_p = 2200$  V,  $e_s = 220$  V,  $n_p = 5000$ ,

$$\eta = 90\% = \frac{90}{100}, P_s = 8 \text{ kW}$$

To Find : i.  $n_s$

ii.  $P_i$

$$i) \frac{n_s}{n_p} = \frac{e_s}{e_p} = \frac{220}{2200} = \frac{1}{10}$$

$$n_s = \frac{n_p}{10} = \frac{5000}{10} = 500$$

$$ii) \eta = \frac{e_s I_s}{e_p I_p}$$

$$e_p I_p = \frac{e_s I_s}{\eta} = \frac{P_s}{\eta} = \frac{8}{9/10} = \frac{80}{9} = 8.888 \text{ kW}$$

- ★ 2). The primary and secondary coil of a transformer each have an inductance of  $200 \times 10^{-6}$  H. The mutual inductance(M) between the windings is  $4 \times 10^{-6}$  H. What percentage of the flux from one coil reaches the other?

Data:  $L_1 = L_2 = 2 \times 10^{-4}$  H  
 $M = 4 \times 10^{-6}$  H

To find: K

$$M = K \sqrt{L_1 L_2}$$

Formula:

Solution:

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{4 \times 10^{-6}}{\sqrt{(2 \times 10^{-4})^2}} = \frac{4 \times 10^{-6}}{2 \times 10^{-4}} = 2 \times 10^{-2} \times 10^2 = 2\%$$

**Ans :** 2% of the flux from one coil reaches the other.

**Problem for practice**

1. The primary coil of an ideal step-up transformers has 100 turns and the transformation ratio is also 100. The input voltage and the power are 220 V and 1100 W respectively. Calculate:
  - (i). number of turns in the secondary
  - (ii). the current in the primary
  - (iii). voltage across the secondary
  - (iv). the current in the secondary
  - (v). power in the secondary

**Ans: 10.000, 5A, 22,000V, 0.05A**
2. How much current is drawn by the primary of a transformer which steps down 220 V to 22 V to operate a device with an impedance of 220 Ω.?  

**Ans: 0.01A**
3. Calculate the current drawn by the primary of a transformer, which steps down 200 V to 20 V to operate a device of resistance 20 Ω. Assume the efficiency of the transformer to be 80%.  

**Ans: 0.125 A**

**MULTIPLE CHOICE QUESTIONS**

**Entrance Corner (Set 4)**

1. The core of a transformer is laminated, because
  - a. rusting of the core may be prevented
  - b. energy losses due to eddy currents may be minimised
  - c. ratio of voltage in primary and secondary

may be increased

- d. the weight of the transformer may be reduced.
2. The primary winding of a transformer has 500 turns, whereas its secondary has 5,000 turns. The primary is connected to an a.c. supply 20 V-50 Hz. The secondary will have an output of
- a. 200 V – 50 Hz      b. 200 V – 500 Hz  
c. 2 V – 50 Hz      d. 2 V – 5 Hz
3. The primary and secondary coils of a transformer have 50 and 1500 turns respectively. If the magnetic flux  $\phi$  linked with the primary coil is given by  $\phi = \phi_0 + 4t$ , where 0 is in weber, t is time in second and  $\phi_0$  is a constant, the output voltage across the secondary coil is
- a. 90 V      b. 120 V  
c. 220 V      d. 30 V
4. A transformer is used to light a 100 W and 110 V lamp from a 220 V mains. If the main current is 0.5 A, the efficiency of the transformer is approximately
- a. 30%      b. 50%  
c. 90%      d. 10%
5. A 220 volts input is supplied to a transformer. The output circuit draws a current of 2.0 ampere at 440 volts. If the efficiency of the transformer is 80%, the current drawn by the primary windings of the transformer is
- a. 3.6 ampere      b. 2.8 ampere  
c. 2.5 ampere      d. 5.0 ampere

**Try yourself**

6. A step-up transformer operates on a 230 V line and supplies a current of 2 A. The ratio of primary and secondary windings is 1 : 25. The primary current is
- a. 12.5 A      b. 50 A  
c. 8.8 A      d. 25 A
7. To manufacture the core of a transformer, the best material is
- a. stainless steel      b. hard steel  
c. mild steel      d. soft iron
8. The primary of a step-down transformer used for ringing door bell has 2000 turns of fine wire and the secondary has 100 turns. This transformer when connected to a 110 V A.C. source will deliver at its secondary a potential difference of
- a. 220 V      b. 11 V  
c. 55 V      d. 5.5 V
9. If primary winding of a transformer were connected to a battery, the current in it will
- a. increase  
b. remain constant  
c. decrease  
d. first (a) then (c)
10. A transformer having efficiency of 90% is working on 200V and 3kW power supply. If the current in the secondary coil is 6 A, the voltage across the secondary coil and the current in the primary coil respectively are:
- a. 300V, 15A      b. 450V, 15A  
c. 450V, 13.5A      d. 600V, 15A

**Answer Key**

**Set - 1 ( MCQ)**

1	b	2	d	3	a	4	b	5	a
6	a	7	b	8	c	9	d	10	d

**Try Yourself**

11	c	12	a	13	a	14	d	15	b
16	d	17	b	18	b	19	d	20	b

**Set - 2 ( MCQ)**

1	c	2	d	3	b	4	c	5	d
6	c	7	b	8	a	9	a	10	d

**Try Yourself**

11	d	12	d	13	c	14	d	15	b
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**Set - 3 ( MCQ)**

1	d	2	b	3	b	4	b	5	d
6	d	7	d	8	b	9	a	10	c

**Try Yourself**

11	a	12	a	13	c	14	a	15	b
16	d	17	b	18	d	19	b	20	b

**Set - 4 ( MCQ)**

1	b	2	a	3	b	4	c	5	d
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**Try Yourself**

6	b	7	d	8	d	9	b	10	b
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