

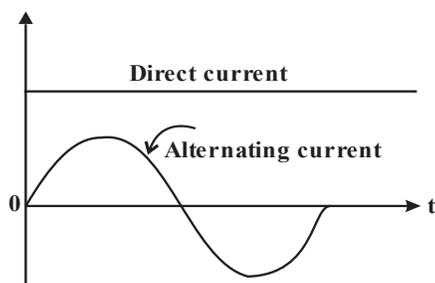
Syllabus

- 13.1 Introduction
- 13.2 AC Generator
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- 13.4 Phasors
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- 13.6 Power in AC circuit
- 13.7 LC Oscillations
- 13.8 Electric Resonance
- 13.9 Sharpness of Resonance : Q-factor
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13.1 Introduction

Alternating current :

- i. An alternating current is that current whose magnitude changes continuously with time and directions reverses periodically. In contrast to it, a direct current is that current which flows with a constant magnitude in the same direction, as shows in figure



- ii. We know that when a coil is rotated in a magnetic field an alternating emf is induced in it, which is given by the relation :

$$e = e_0 \sin \omega t$$

- iii. Suppose this emf is applied to a circuit of resistance R . Then by ohm's law, the current in the circuit will be

$$i = \frac{e}{R} = \frac{e_0}{R} \sin \omega t \quad \text{or} \quad i = i_0 \sin \omega t$$

Thus the current in the circuit varies

sinusoidally with time and is called alternating current. Here

i = instantaneous value of a.c at any instant t

$i_0 = \frac{e_0}{R}$ = Peak or maximum value of a.c and is called current amplitude.

- iv. **Amplitude :** The maximum value attained by an alternating current in either direction is called its amplitude or peak value and is denoted by i_0
- v. **Time Period :** The time taken by an alternating current to complete one cycle of its variations is called its time period and is denoted by T . This time is equal to the time taken by the coil to complete one rotation in the magnetic field. As angular velocity of the coil is ω and its angular displacement in one complete cycle is 2π , so

$$\text{Time period} = \frac{\text{Angular displacement in a complete cycle}}{\text{Angular velocity}}$$

$$T = \frac{2\pi}{\omega}$$

- vi. **Frequency :** The number of cycle completed per second by an alternating current is called its frequency and is denoted by f . The frequency of an alternating current is same as the frequency of rotation of the coil in the magnetic field. Thus

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

So an alternating current can be represented as

$$i = i_0 \sin \omega t = i_0 \sin 2\pi f t = i_0 \sin \frac{2\pi}{T} t$$

- vii. The alternating current supplied to our houses has a frequency of 50 cps or 50 Hz

13.2 AC Generator

Q.1 What is alternating emf and alternating current

Ans :

i. The source of an AC generator produces a time dependent emf (e) given by,

$$e = e_0 \sin \omega t \quad \dots (1)$$

where e_0 = peak value of emf

ω = angular frequency of rotation of the coil in the AC generator

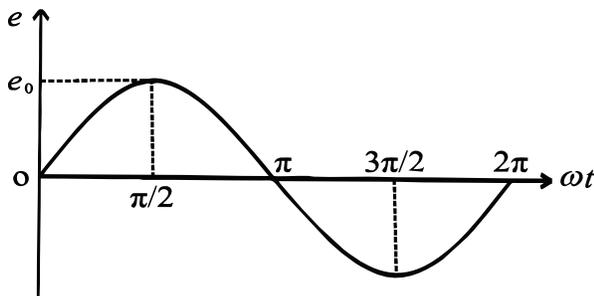
ii. As the time variation of current is similar to that of emf, the current in a circuit is given as,

$$i = i_0 \sin (\omega t + \alpha) \quad \dots (2)$$

Where α = phase difference between the current (i) and the emf (e)

i_0 = peak value of current

iii. For a graph of induced emf v/s ωt the induced emf varies sinusoidally with time.



iv. From the graph can be seen that the direction of the emf is reversed after every half revolution of the coil. This type of emf is called the alternating emf and the corresponding current is called alternation current.

13.3 Average and RMS values

Q.2 Define average value of a.c over half a cycle. Establish the relationship between the average value and the peak value of alternating current.

Ans : Average value of a.c is defined as that value of direct current which sends the same charge in a circuit in the same time as is sent by the given alternating current in its half time period.

Relation between average value and peak value

i. We know that the applied emf is

$$e = e_0 \sin \omega t = e_0 \sin \theta$$

ii. To obtain average value of emf over half cycle

$$e_{av} = \frac{\int_0^\pi e_0 \sin \theta \, d\theta}{\int_0^\pi d\theta}$$

$$= \frac{e_0 [-\cos \theta]_0^\pi}{[\theta]_0^\pi}$$

$$= \frac{e_0 [-\cos \pi - (-\cos 0)]}{[\pi - 0]}$$

$$= \frac{e_0 [-(-1) + 1]}{\pi} = \frac{2e_0}{\pi}$$

$$e_{av} = 0.637e_0$$

iii. Similarly

$$i_{av} = 0.637 i_0$$

Q.3 What is meant by rms value or effective value of alternating current derive relation between rms value and peak value

Ans :

i. **Root mean square or virtual or effective value of a.c.** It is defined as that value of direct current which produces the same heating effect in the given resistor as is produced by the given alternating current when passed for the same time.

ii. Symbol is i_{rms} , i_v or i_{eff}

iii. **Relation between rms value and Peak value**

a. Consider an alternating current of peak value i_0 , flowing through a resistance R. Let H be the heat produced in time t.

b. The same quantity of heat (H) can be produced in the same resistance (R) in the same time (t) by passing a steady current of constant magnitude through it.

c. The value of such steady current is called the effective value or virtual or rms value of the given alternating current and is denoted by i_{rms}

d. The relation between the rms value and

peak value of alternating current is given by,

$$i_{\text{rms}}^2 = \frac{\int_0^{2\pi} i^2 d\theta}{2\pi} = \frac{1}{2\pi} \int_0^{2\pi} i_0^2 \sin^2 \theta d\theta$$

$$\frac{i_0^2}{2\pi} \int_0^{2\pi} \frac{(1 - \cos 2\theta)}{2} d\theta$$

$$\dots (\because \cos 2\theta = 1 - \sin^2 \theta)$$

$$= \frac{i_0^2}{2 \times 2\pi} \left[\left(\theta - \frac{\sin 2\theta}{2} \right) \right]_0^{2\pi}$$

$$\therefore i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = 0.707 i_0$$

iv. Similarly, $e_{\text{rms}} = \frac{e_0}{\sqrt{2}} = 0.707 e_0$

Note :

The heat produced by a sinusoidally varying AC over a complete cycle (for $t = 0$ to $t = T = \frac{2\pi}{\omega}$) will be given by,

$$\therefore H = \int_0^{2\pi/\omega} i^2 R dt$$

$$= R \int_0^{2\pi/\omega} i_0^2 \sin^2(\omega t) dt = \frac{2\pi R i_0^2}{\omega \cdot 2}$$

$$\therefore H = R(i_{\text{rms}})^2 \cdot \frac{2\pi}{\omega}$$

It is the same as the heat produced by a DC current of magnitude i_{rms} for time

$$t = \frac{2\pi}{\omega}$$

Q.4 Explain why ordinary moving coil galvanometer cannot be used to measure A.C

Ans :

i. Ordinary moving coil galvanometer is based on magnetic effect of current which in turn, depends on direction of current. So it cannot be used to measure A.C.

- ii. During one half cycle of A.C, its pointer moves in one direction and during next half cycle, it will move in the opposite direction.
- iii. Now the average value of A.C over a complete cycle is zero.
- iv. Even if we measure an alternating current of low frequency, the pointer, will appear to be stationary at the zero position due to persistence of vision.
- v. We can measure A.C by using a hot-wire ammeter which is based on heating effect of current and this effect is independent of the direction of current.

Key Point

The alternating current and voltages are generally measured and specified in term of their rms values. When we say that the household supply is 220 V A.C., we mean that its rms value is 220 V. The peak value would be

$$V_0 = \sqrt{2} \cdot V_{\text{rms}} = \sqrt{2} \times 220 = 311 \text{ V}$$

Type - I

Numerical based on Average value and rms value

Formulae used

1. $i = i_0 \sin \omega t$ (for R - circuit)
2. $e = e_0 \sin \omega t$
3. $i_{\text{av}} = \frac{2}{\pi} i_0 = 0.637 i_0$
4. $e_{\text{av}} = \frac{2}{\pi} e_0 = 0.637 e_0$
5. $i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = 0.7071 i_0$
6. $e_{\text{rms}} = \frac{e_0}{\sqrt{2}} = 0.7071 e_0$

★ 1) Find the time required for a 50 Hz alternating current to change its value from zero to the r.m.s. value.

Data: $f = 50 \text{ Hz}; i = \frac{i_0}{\sqrt{2}}$

To Find : t

Formula : $i = i_0 \sin \omega t$; $\omega = 2\pi f$

Solution : $i = i_0 \sin \omega t$

$$\frac{i_0}{\sqrt{2}} = i_0 \sin (2\pi \times 50 \times t) \quad \dots (\because \omega = 2\pi f)$$

$$\therefore \frac{1}{\sqrt{2}} = \sin(100 \pi t)$$

$$\therefore 100 \pi t = \sin^{-1} \left(\frac{1}{\sqrt{2}} \right)$$

$$\therefore 100 \pi t = \left(\frac{\pi}{4} \right)$$

$$\therefore t = \frac{1}{400} = 0.25 \times 10^{-2} = 2.5 \times 10^{-3} \text{ sec}$$

Ans : The time required for the alternating current to change its value from zero to rms is 2.5×10^{-3} s.

★ 2) If the effective current in a 50 cycle AC circuit is 5 A, what is the peak value of current? What is the current 1/600 sec. after it was zero?

Data : $f = 50 \text{ Hz}$, $i_{\text{rms}} = 5 \text{ A}$, $t = \frac{1}{600} \text{ s}$

$$\omega = 2\pi f = 2\pi \times 50 = 100\pi \text{ rad / s}$$

To Find: i. i_0
ii. i

Formula: i. $i_0 = \sqrt{2} i_{\text{rms}}$ ii. $i = i_0 \sin (\omega t)$

Solution: i. $i_0 = \sqrt{2} i_{\text{rms}}$

$$\text{ii. } i_0 = \sqrt{2} \times 5 = 1.414 \times 5 = 7.07 \text{ A}$$

$$i = i_0 \sin (\omega t)$$

$$i = 7.07 \sin \left(100\pi \times \frac{1}{600} \right)$$

$$= 7.07 \sin \left(\frac{\pi}{6} \right) = 7.07 \times 0.5 = 3.535 \text{ A}$$

Ans : i. Peak value of current is 7.07 A
ii. Instantaneous value of current is 3.535 A

★ 3) An alternating voltage is given by $e = 6 \sin 314 t$. find
i. the peak value
ii. frequency
iii. time period and

iv. instantaneous value at time $t = 2 \text{ ms}$

Data : $e = 6 \sin 314 t$, $t = 2 \text{ sec}$

To Find : i. e_0 ii. f iii. T iv. e

Formula: i. $\omega = 2\pi f$ ii. $T = \frac{1}{f}$

Solution :

i. The alternating voltage is given as

$$e_0 = 6 \sin 314 t$$

on comparing with $e = e_0 \sin \omega t$ we get

$$e_0 = 6 \text{ V}, \quad \omega = 314 \text{ rad / s} = 100 \pi \text{ rad/s}$$

$$\text{ii. } f = \frac{\omega}{2\pi} = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

$$\text{iii. Time period, } T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ s}$$

iv. At $t = 2 \text{ ms} = 2 \times 10^{-3} \text{ s}$

$$e = 6 \sin (314 \times 2 \times 10^{-3})$$

$$= 6 \sin \left(\frac{100\pi \times 2}{1000} \right) = 6 \sin \left(\frac{\pi}{5} \right) = 6 \sin 36^\circ$$

$$= 6 \times 0.5878 = 3.527 \text{ V}$$

Ans : i. The peak value is 6 v
ii. The frequency is 50 Hz
iii. The time period is 0.02 s
iv. The instantaneous emf is 3.527 V

Problem for Practice

1. The instantaneous emf of an a.c. source is given by $e = 30 \sin 314t$. What is the rms value of the emf?

Ans : 212 V

2. The emf of an a.c. source is given by the expression $e = 300 \sin 614 t$. Write the value of peak voltage and frequency of the source

Ans : 300 V, 50 Hz

3. The instantaneous current from an a.c. source is $i = 5 \sin 314 t$ What is the rms value of current?

Ans : 3.54 A

4. An alternating voltage given by $V = 140 \sin 314 t$ is connected across a pure resistor of 50Ω . Find (i) the frequency of the source (ii) the rms current through the resistor

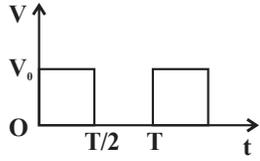
Ans: i. 50 Hz, ii. 1.98 A

MULTIPLE CHOICE QUESTIONS

Entrance Corner (Set 1)

- Alternating current cannot be measured by d.c. ammeter, because
 - a.c. cannot pass through a.c. ammeter
 - a.c. changes direction
 - average value of current of complete cycle is zero.
 - a.c. ammeter will get damaged.
- The peak value of an alternating e.m.f. ε given by $\varepsilon = \varepsilon_0 \cos \omega t$ is 10 volt and its frequency is 50 Hz. At time $t = \frac{1}{600}$ sec, the instantaneous e.m.f. is
 - 1 V
 - 5 V
 - 10 V
 - $5\sqrt{3}$ V
- An a.c. source is rated at 220 V, 50 Hz. The time taken for voltage to change from its peak value to zero is
 - 50 sec
 - 0.02 sec
 - 5 sec
 - 5×10^{-3} sec
- The r.m.s. value of potential difference V shown in the figure is

- $\frac{V_0}{2}$
 - $\frac{V_0}{\sqrt{3}}$



 - V_0
 - $\frac{V_0}{\sqrt{2}}$
- The frequency of an alternating voltage is 50 cycles/sec and its amplitude is 120 V. Then its rms value will be
 - 84.8 V
 - 42.4 V
 - 56.5 V
 - 75.5 V

Try Yourself

- The peak value of a.c. voltage on a 220 V mains is
 - $200\sqrt{2}$ V
 - $230\sqrt{2}$ V
 - $220\sqrt{2}$ V
 - $240\sqrt{2}$ V
- In the AC circuit, the current is expressed as $I = 100 \sin 200 \pi t$. In the circuit the current

rises from zero to peak value in time

- $\frac{1}{300}$ s
- $\frac{1}{400}$ s
- $\frac{1}{100}$ s
- $\frac{1}{200}$ s

- In an a.c. generator, a coil with N turns, all the same area A total resistance R , rotates with frequency ω in a magnetic field B . The maximum value of emf generated in the coil is
 - $NABR$
 - $NAB \omega$
 - $NABR \omega$
 - NAB
- The average emf during the positive half cycle of an AC supply of peak value E_0 is
 - $\frac{E_0}{\pi}$
 - $\frac{E_0}{\sqrt{2\pi}}$
 - $\frac{E_0}{2\pi}$
 - $\frac{2E_0}{\pi}$
- An alternating emf given by $V = V_0 \sin \omega t$ has peak value 10 volt and frequency 50 Hz. The instantaneous emf at $t = \frac{1}{600}$ s is
 - 10 V
 - $5\sqrt{3}$ V
 - 5 V
 - 1 V

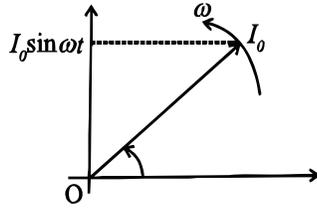
13.4 Phasors

Q. 5 Explain phasor and phasor diagram for alternating current or emf.

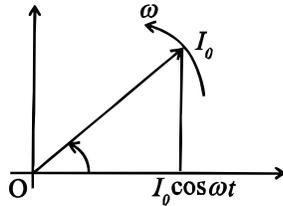
Ans :

- A rotating vector that represents a quantity varying sinusoidally with time is called a **phasor**. The diagram representing a phasor is called **phasor diagram**
- The phasor for alternating emf and alternating current are inclined to the horizontal axis at angle ωt or $\omega t + \alpha$ and rotate in anticlockwise direction.
- The length of the arrow represent the maximum value of the quantity (i_0 and e_0)
The projection of the vector on fixed axis given the instantaneous value of alternating current and alternating emf.

- iv. In sine form $i = i_0 \sin \omega t$ and $e = e_0 \sin \omega t$ projection is taken on Y-axis.



- v. In cosine form $i = i_0 \cos t \omega$ and $e = e_0 \cos \omega t$ projection is taken on X - axis.



13.5 Different types of AC circuits

Pure R - circuit

Q.6 Explain the theory of an AC circuit with resistor.

OR

Show that the voltage and current vary in the same phase in an a.c. circuit containing resistance only

Ans:

- i. Consider a resistor R connected to AC source with instantaneous value e which is given by $e = e_0 \sin \omega t$... (1)

Where e_0 be the peak value of voltage and ω is its angular frequency.

- ii. **Expression for current :**

According to ohm's law $e = iR$

- ∴ From equation (1) we can write $iR = e_0 \sin \omega t$

∴ $i = \frac{e_0}{R} \sin \omega t$

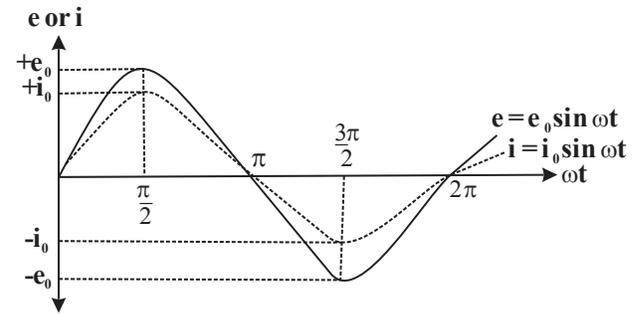
∴ $i = i_0 \sin \omega t$... (2)

Where

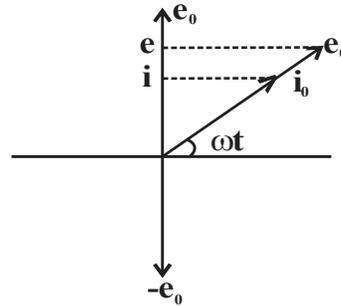
$i = \frac{e_0}{R} =$ maximum or peak value of a.c.

- iii. From equation (1) and (2) we note that both e and i are function of $\sin \omega t$. Hence e and i are in same phase

Wave diagram



Phasors diagram



Key Point

For AC circuit with pure Resistor

- i. Applied emf $e = e_0 \sin \omega t$
- ii. $i = i_0 \sin \omega t$
- iii. Opposition to current is resistance

$$R = \frac{e}{i} = \frac{e_0}{i_0} = \frac{e_{rms}}{i_{rms}}$$

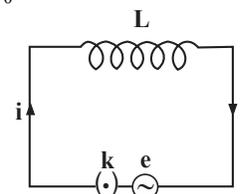
- iv. Both e and i are in same phase. This means that e and i attain their zero, minimum and maximum values at the same times.

Pure L - circuit

Q.7 Show that in an AC circuit containing a pure inductor, the voltage is ahead of current by $\pi / 2$ in phase.

Ans: AC circuit with inductor:

- i. Consider an alternating e.m.f 'e' applied across a pure inductor of self-inductance L
- ii. Let alternating emf supplied be represented by $e = e_0 \sin \omega t$... (1)



iii. According to Faraday's law, when the key k is closed, current i begins to grow in the inductor because magnetic flux linked with it changes and induced emf is produced which opposes the applied emf.

iv. According to Lenz's law,

$$e' = -L \frac{di}{dt} \quad \dots(2)$$

Where e' is the induced emf and $\frac{di}{dt}$ is the rate of change of current.

v. To maintain the flow of current in the circuit, applied emf (e) must be equal and opposite to the induced emf (e').

vi. According to Kirchhoff's voltage law as there is no resistance in the circuit
 $e = -e'$

$$e = -\left(-L \frac{di}{dt}\right) \quad \dots[\text{from equation (2)}]$$

$$e = L \frac{di}{dt}$$

$$di = \frac{e}{L} dt$$

Integrating the above equation on both the sides, we get,

$$\int di = \int \frac{e}{L} dt$$

$$i = \int \frac{e_0 \sin \omega t}{L} dt \quad \dots (\because e = e_0 \sin \omega t)$$

$$i = \frac{e_0}{L} \left[\frac{-\cos \omega t}{\omega} \right] + \text{Constant}$$

vii. Constant of integration is time independent and has the dimensions of current. As the emf oscillates about zero, current also oscillates about zero so that there cannot be any component of current which is time independent. Thus, the integration constant is zero.

$$\text{viii. } i = \frac{-e}{\omega L} \sin\left(\frac{\pi}{2} - \omega t\right) \dots \left[\because \cos \omega t = \sin\left(\frac{\pi}{2} + \omega t\right)\right]$$

$$i = \frac{e_0}{\omega L} \sin\left[\omega t - \frac{\pi}{2}\right] \dots (\because -\sin \theta = \sin(-\theta))$$

$$\text{When } \sin\left(\omega t - \frac{\pi}{2}\right) = 1$$

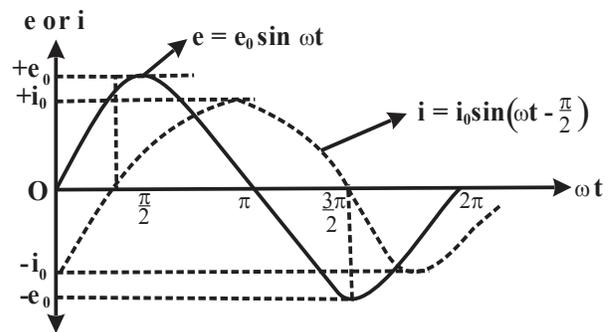
$$\text{then } i = i_0 \text{ (peak)}$$

$$\therefore i_0 = \frac{e_0}{\omega L} = \text{peak value of current.}$$

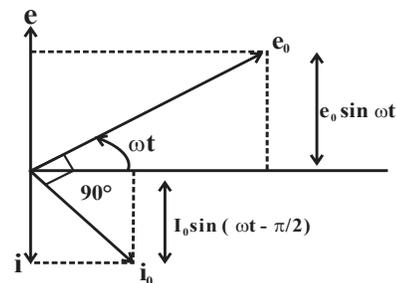
$$\therefore i = i_0 \sin\left[\omega t - \frac{\pi}{2}\right] \quad \dots(3)$$

The above equation gives the alternating current developed in a purely inductive circuit when connected to a source of alternating emf. Comparing equations (1) and (3), we find that the alternating current i lags behind the alternating voltage emf e by a phase angle of $\pi/2$ radians (90°) or the voltage across L leads the current by a phase angle of $\pi/2$ radians (90°).

xi. **Wave diagram**



xii. **Phasor diagram**



Q.8 Explain the term inductive reactance (X_L).

Ans:

i. The opposite nature of an inductor to the flow of alternating current is called inductive reactance.

ii. In an inductive circuit,

$$i_0 = \frac{e_0}{\omega L}$$

- iii. According to Ohm's law, opposition to current is
- $$\frac{e_0}{i_0} = \omega L = X_L$$
- iv. Hence, the effective resistance X_L offered by the inductance L is called inductive reactance and is given as,
- $$X_L = \omega L = 2\pi fL \quad \dots (\because \omega = 2\pi f)$$
- Where f = frequency of the AC supply.
- v. X_L is directly proportional to the inductance (L) and the frequency (f) of the alternating current.
- vi. In DC circuits, $f = 0$
- $\therefore X_L = 0$
- It implies that a pure inductor offer zero resistance to DC, i.e., it cannot reduce DC. Thus, it passes DC and blocks AC of very high frequency.
- vii. In an inductive circuit, the self-induced emf opposes the growth as well as decay of current.
- viii. The dimensions of inductive reactance is the same as those of resistance and its SI unit is ohm (Ω).

Key Points

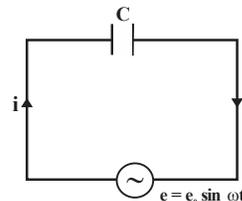
1. **Applied emf**
 $e = e_0 \sin \omega t$
2. **Current in L - circuit**
 $i = i_0 \sin \left(\omega t - \frac{\pi}{2} \right)$
3. **Relation between e and i**
i lags behind e by $\frac{\pi}{2}$
4. **Opposition to current is inductive reactance**
 $X_L = L\omega$
5. Inductor blocks AC and allows DC

Pure C - Circuit

★ Q.9 An AC source generating a voltage $e = e_0 \sin \omega t$ is connected to a capacitor of capacitance C . Find the expression for the current i flowing through it. Plot a graph of e and i versus ωt .

Ans: A.C circuit with capacitance:

- i. Let us consider a capacitor with capacitance C connected across AC source of emf



- ii. Let 'q' be the magnitude of charge on any one plate of a capacitor at any instant. The potential difference across its plates at that instant is given by,

$$e = \frac{q}{C} \Rightarrow q = Ce$$

$$q = C (e_0 \sin \omega t) \dots (2)$$

- iii. Differentiating both sides of equation (2) w.r.t 't', we get,

$$\frac{dq}{dt} = \frac{d}{dt} (C e_0 \sin \omega t) = \omega C e_0 \cos \omega t$$

$$i = C e_0 \omega \cos \omega t$$

$$\therefore i = C e_0 \omega \sin \left(\omega t + \frac{\pi}{2} \right) \dots (3)$$

$$\text{When } \sin \left(\omega t + \frac{\pi}{2} \right) = 1$$

$$i = i_0 \text{ (peak)}$$

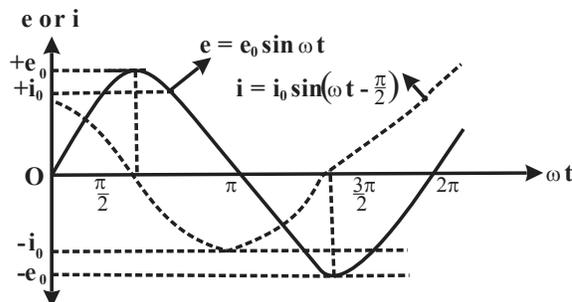
$$\therefore i_0 = C e_0 \omega \dots (4)$$

Substituting in equation (3)

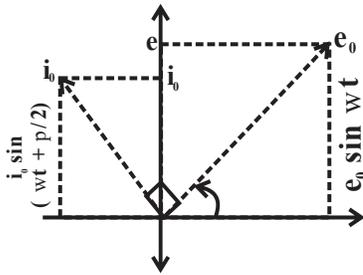
$$\therefore i = i_0 \sin \left(\omega t + \frac{\pi}{2} \right) \dots (5)$$

- iv. Comparing equation (1) and equation (5) we find that in an AC circuit containing a capacitor only the, alternating current i leads the alternating emf e by phase angle of $\pi/2$ radian.

vi. **Wave diagram**



vii. **Phasor diagram**



Q. 10 Explain the term capacitive reactance.

Ans:

i. We know that $i_0 = e_0 C \omega$
According to ohm's law opposition to current is

$$\frac{e_0}{i_0} = \frac{1}{C\omega} = X_C$$

Where X_C is capacitive reactance

ii. The function of capacitive reactance in a purely capacitive circuit is to limit the amplitude of the current similar to the resistance in a purely resistive circuit.

iii. For DC, $\omega = 0$

$\therefore X_C$ is maximum
and for AC, $\omega =$ maximum

$\therefore X_C$ is minimum
From this we can conclude capacitor blocks DC and allows AC

iv. The dimensions of capacitive reactance is the same as those of resistance and its SI unit is ohm (Ω).

Key point

1. **Applied emf**

$$e = e_0 \sin \omega t$$

2. **Expression for current**

$$i = i_0 \sin \left(\omega t + \frac{\pi}{2} \right)$$

3. **Relation between e and i**

i leads e by phase angle of $\frac{\pi}{2}$ radian

4. **Opposition to current is capacitive reactance**

$$X_C = \frac{1}{C\omega}$$

5. Capacitor allows AC and blocks DC.

★ **Q.11** A device Y is connected across an AC source of emf $e = e_0 \sin \omega t$. The current

$$\text{through Y is given as } i = i_0 \sin \left(\omega t + \frac{\pi}{2} \right)$$

i. Identify the device Y and write the expression for its reactance.

ii. Draw graphs showing variation of emf and current with time over one cycle of AC for Y.

iii. How does the reactance of the device Y vary with the frequency of the AC? Show graphically.

iv. Draw the phasor diagram for the device Y.

Ans :

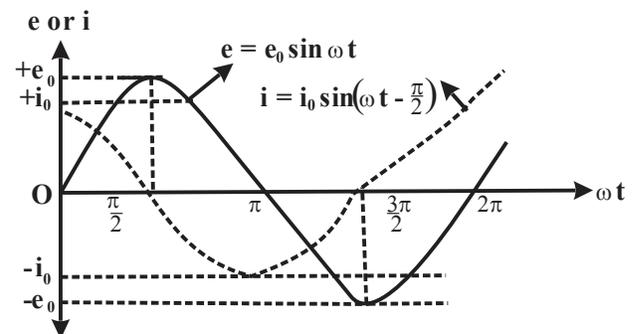
i. From the given equations of emf and current, it can be concluded that the current is leading the emf by 90°

\therefore The device Y is a capacitor

The expression for reactance of a capacitor is given as

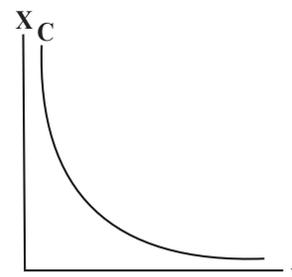
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

ii. Wave diagram

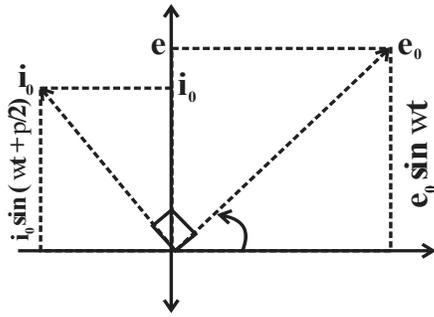


iii. Capacitive reactance $X_C = \frac{1}{2\pi f C}$

$$\therefore X_C \propto \frac{1}{f}$$



iv. For phasor diagram :



★ Q.12 For very high frequency AC supply, a capacitor behaves like a pure conductor why?

Ans:

i. Capacitive reactance is given as

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \Rightarrow X_C \propto \frac{1}{f}$$

ii. As frequency is increased, capacitive reactance (X_C) decreases.

iii. Therefore, at very high frequencies, X_C will be nearly zero and the capacitor behaves as a pure conductor.

★ Q.13 Compare resistance and reactance.

Ans :

Sr. No	Resistance	Reactance
i.	In the resistance circuit voltage and current in same phase. i.e. phase difference is 0 degree	In reactive circuit voltage and current are not in the same phase. It may be leading by 90° or lagging by 90°.
ii.	Resistance is independent of the frequency of input signal	Reactance is a quantity which depends on the frequency of input signal.
iii.	Resistance measure the opposition to a flow of current	Reactant measure the opposition to a change in current.

Type - II

Numerical based on inductive reactance and capacitive reactance

Formulae Used

1. For an a.c. circuit containing inductor only
i. Inductive reactance, $X_L = \omega L = 2\pi f L$

ii. Current amplitude, $i_0 = \frac{e_0}{X_L} = \frac{e_0}{\omega L}$

iii. Effective current,

$$i_{rms} = \frac{e_{rms}}{X_L} = \frac{e_{rms}}{\omega L} = \frac{e_0}{\sqrt{2} \cdot \omega L}$$

2. For an a.c circuit containing capacitor only

i. Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

ii. Current amplitude, $i_0 = \frac{e_0}{X_C} = \frac{e_0}{1/\omega C}$

iii. Effective current,

$$i_{rms} = \frac{e_{rms}}{X_C} = \frac{e_{rms}}{1/\omega C} = \frac{e_0}{\sqrt{2} \cdot 1/\omega C}$$

★ 1) An alternating voltage given by $e = 140 \sin 3142 t$ is connected across a pure resistor of 50Ω . Find

i. the frequency of the source

ii. the rms current through the resistor.

Data : $e = 140 \sin 3142 t$

$$R = 50 \Omega$$

To Find :
i. (e_0)
ii. (f)
ii. (i_{rms})

Formula : i. $f = \frac{\omega}{2\pi}$ ii. $i_{rms} = \frac{e_{rms}}{R} = \frac{e_0}{\sqrt{2} \times R}$

Solution:

i. The alternating voltage is, $e = 140 \sin 3142 t$ on comparing it with standard equation

$$e_0 = 140 \text{ V}$$

$$\therefore \omega = 3142 \text{ rad/s} = 1000 \pi \text{ rad/s}$$

$$\text{ii. } f = \frac{\omega}{2\pi} = \frac{1000\pi}{2\pi} = 500 \text{ Hz}$$

$$\begin{aligned} \text{iii. } i_{rms} &= \frac{e_{rms}}{R} = \frac{e_0}{\sqrt{2} \times R} \\ &= \frac{140}{\sqrt{2} \times 50} = \frac{14}{1.414 \times 5} \\ &= \frac{14}{7.07} \approx 1.98 \text{ A} \end{aligned}$$

Ans : i. The frequency of the source is 500 Hz
ii. The rms current through the resistor is 1.98 A

2) **An inductor of inductance 200 mH is connected to an AC source of peak emf 210 V and frequency 50 Hz. Calculate the peak current. What is the instantaneous voltage of the source when the current is at its peak value?**

Data : $L = 200 \text{ mH} = 200 \times 10^{-3} \text{ H} = 0.2 \text{ H}$
 $e_0 = 210 \text{ V}$, $F = 50 \text{ Hz}$

To Find : i. Peak current (i_0)
ii. Instantaneous voltage at peak value of current (e)

Formula : i. $X_L = 2\pi fL$ ii. $i_0 = \frac{e_0}{X_L}$

Solution :

i. $X_L = 2\pi fL$
 $X_L = 2 \times 3.142 \times 50 \times 0.2 = 62.84 \Omega$

ii. $i_0 = \frac{e_0}{X_L}$
 $i_0 = \frac{210}{62.84} = 3.341 \text{ A}$

As in an inductive AC circuit, current lags behind the emf by $\frac{\pi}{2}$, so the voltage is zero when the current is at its peak value.

Ans : i. The peak current is 3.341 A
ii. The instantaneous voltage of the current peak is zero

3) **Alternating emf of $e = 220 \sin 100 \pi t$ is applied to a circuit containing an inductance of $\left(\frac{1}{\pi}\right)$ henry. Write an equation for instantaneous current through the circuit. What will be the reading of the AC galvanometer connected in the circuit?**

Data : $e = 220 \sin 100 \pi t$

$$L = \frac{1}{\pi} \text{ H}$$

To Find : i. Instantaneous current (i)
ii. (i_{rms})

Formula: i. $X_L = \omega L$

ii. $i = \frac{e_0}{X_L} \sin\left(100\pi t - \frac{\pi}{2}\right)$

iii. $i_{\text{rms}} = \frac{i_0}{\sqrt{2}}$

Solution : The alternating emf $e = 220 \sin 100 \pi t$
On comparing with general equation of alternating emf, i.e., $e = e_0 \sin(\omega t)$

$\therefore e_0 = 220 \text{ V}$ and $\omega = 100 \pi$

i. $X_L = \omega L = 100\pi \times \frac{1}{\pi} = 100 \Omega$

Also in inductive circuit, current lags behind by $\frac{\pi}{2}$

ii. $i = \frac{e_0}{X_L} \sin\left(100 \pi t - \frac{\pi}{2}\right)$
 $= \frac{220}{100} \sin\left(100 \pi t - \frac{\pi}{2}\right)$
 $= 2.2 \sin\left(100 \pi t - \frac{\pi}{2}\right) \quad \dots (1)$

Reading in galvanometer is of i_{rms}

From equation (1)

we get $I_0 = 2.2$

iii. $i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = 2.2 \times 0.707 = 1.554 \text{ A}$

Ans : i. Equation of instantaneous current through the circuit is $2.2 \sin\left(100 \pi t - \frac{\pi}{2}\right)$
ii. The rms current is 1.554 A

4) **An AC circuit consists of only an inductor of inductance 2 H. If the current is represented by a sine wave of amplitude 0.25 A and frequency 60 Hz, calculate the effective potential difference across the inductor ($\pi = 3.142$)**

Data : $L = 2 \text{ H}$, $i_0 = 0.25 \text{ A}$, $f = 60 \text{ Hz}$

To Find : Effective potential difference (e_{rms})

Formula : i. $i_{\text{rms}} = \frac{i_0}{\sqrt{2}}$ ii. $X_L = 2\pi fL$
 iii. $e_{\text{rms}} = i_{\text{rms}} \times X_L$

Solution :

i. $i_{\text{rms}} = \frac{i_0}{\sqrt{2}}$
 $i_{\text{rms}} = \frac{0.25}{\sqrt{2}}$
 $= 0.25 \times 0.707 = 0.1768 \text{ A}$

ii. $X_L = 2\pi fL$
 $X_L = 2 \times 3.142 \times 60 \times 2 = 754.08 \Omega$

iii. $e_{\text{rms}} = i_{\text{rms}} \times X_L$
 $e_{\text{rms}} = 0.1768 \times 754.08$
 $= 1.334 \times 10^2 = 133.4 \text{ V}$

Ans : The effective potential difference across the inductor is 133.4 V

5) **A capacitor of $2 \mu\text{F}$ is connected to an AC source of emf $e = 250 \sin 100\pi t$. Write an equation for instantaneous current through the circuit and give reading of AC ammeter connected in the circuit.**

Data : $C = 2 \mu\text{F} = 2 \times 10^{-6} \text{ F}$
 $e_0 = 250\text{V}, \omega = 100\pi$

To Find : i. Instantaneous current (i)
 ii. (i_{rms})

Formula : i. $i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ ii. $i_{\text{rms}} = \frac{i_0}{\sqrt{2}}$

Solution :

As current in capacitive circuit leads emf

by $\frac{\pi}{2}$

i. $i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$
 $\Rightarrow \omega C e_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad [\because i_0 = \omega C e_0]$
 $= 100 \times 3.142 \times 2 \times 10^{-6} \times 250 \sin\left(100\pi t + \frac{\pi}{2}\right)$
 $= 500 \times 100 \times 3.142 \times 10^{-6} \sin\left(100\pi t + \frac{\pi}{2}\right)$
 $= 0.1571 \sin\left(100\pi t + \frac{\pi}{2}\right)$

$i = 0.1571 \text{ A}$

ii. $i_{\text{rms}} = \frac{i}{\sqrt{2}} = 0.1571 \times 0.707 = 0.111 \text{ A}$

Ans : i. The equation of instantaneous current is $0.1571 \sin\left(100\pi t + \frac{\pi}{2}\right)$
 ii. The rms current is 0.11 A.

Problem for Practice

1. What is the inductive reactance of a coil if current through it is 800 mA and the voltage across it is 40 V?

Ans : 50 Ω

2. Find the value of current through an inductance of 2.0 H and negligible resistance, when connected to an a.c. source of 150 V and 50 Hz

Ans : 0.239 A

3. An inductance of negligible resistance, whose reactance is 22Ω at 220 Hz is connected to a 220V, 50 Hz power line. What is the value of inductance and reactance?

Ans : 0.0175 H, 5.5 Ω

4. A coil of self-inductance has inductive reactance of 88Ω . Calculate the self-inductance of the coil if the frequency is 50 Hz.

Ans : 0.28 H

5. Find the maximum current through an inductance of 2H connected to an a.c. source of 150 V, 50 Hz

Ans : 0.337 A

6. Calculate the frequency at which the inductive reactance of 0.7 H inductor is 220Ω .

Ans : 50 HZ

7. What is the capacitive reactance of a $5 \mu\text{F}$ capacitor when it is a part of a circuit whose frequency is (i) 50 Hz (ii) 10^6 Hz ?

Ans : 636.6 Ω, $3.18 \times 10^{-2} \Omega$

8. A capacitor has a capacitance of $\frac{1}{\pi} \mu\text{F}$. Find its reactance for a frequency of (i) 50 Hz and (ii) 10^6 Hz

Ans : 10 Ω, 0.5 Ω

MULTIPLE CHOICE QUESTIONS

Entrance Corner (Set 2)

- A capacitor of 50 mF is connected to a supply of 220 V and regular frequency 50 rad/s. The value of rms current in the circuit is
a. 0.45 A b. 0.50 A
c. 0.55 A d. 0.60 A
- In an ac circuit an alternating voltage $e = 200\sqrt{2} \sin 100 t$ volts is connected to a capacitor of capacity 1mf. The rms value of the current in the circuit is
a. 10 mA b. 100 mA
c. 200 mA d. 20 mA
- In an A.C. circuit, a capacitor of $10\mu\text{F}$ is connected with source of 240 volts and 60Hz. The effective value of current in ampere will be
a.1.9 b. 0.28 c. 0.9 d. 0.14
- An inductor of 1 henry is connected across a 220 v, 50 Hz supply. The peak value of the current is approximately
a. 0.5 A b. 0.7 A
c. 1 A d. 1.4 A
- A condenser of capacity 1pF is connected to an A.C source of 220V and 50Hz frequency. The current flowing in the circuit will be
a. $6.9 \times 10^{-8}\text{A}$ b. 6.9A
c. $6.9 \times 10^{-6}\text{A}$ d. zero
- The instantaneous value of emf and current in an A.C. circuit are;
$$E = 1.414\text{Sin}\left(100\pi t - \frac{\pi}{4}\right)$$

$$I = 0.707\text{Sin}(100\pi t)$$
. The admittance of the circuit will be
a. 1Ω b. 2Ω
c. $\sqrt{2} \Omega$ d. $\frac{1}{2} \Omega$
- The capacitive reactance at 1600Hz is 81Ω . When the frequency is doubled then the capacitive reactance will be
a. 40.5Ω b. 81Ω
c. 162Ω d. zero

- A capacitor of $2\mu\text{F}$ draws a current of 4mA when connected across an a.c. of 300Hz. The voltage drop across capacitor is
a. 1.06V b. 1.5V
c. 2.1V d. 6.6V

Try Yourself

- Phase difference between voltage and current in a capacitor in a. c. circuit is
a. π b. $\pi/2$
c. 0 d. $\pi/3$
- For high frequency, capacitor offers
a. more reactance
b. zero reactance
c. less reactance
d. none of these
- In a circuit, the frequency is $f = \frac{1000}{2\pi}$ Hz and the inductance is 2 henry, then the reactance will be
a. 200Ω b. $200\mu\Omega$
c. 2000Ω d. $2000\mu\Omega$
- The instantaneous value of emf and current in an A.C. circuit are;
$$E = 1.414\text{Sin}\left(100\pi t - \frac{\pi}{4}\right)$$

$$I = 0.707\text{Sin}(100\pi t)$$
. RMS value of current will be
a. 1A b. $\frac{1}{\sqrt{2}}$ A
c. $\sqrt{2}$ A d. $\frac{1}{2}$ A
- The capacitive reactance of $50\mu\text{F}$ capacitance at a frequency of $2 \times 10^3\text{Hz}$ will be _____ Ω
a. $\frac{2}{\pi}$ b. $\frac{3}{\pi}$
c. $\frac{4}{\pi}$ d. $\frac{5}{\pi}$
- A pure inductor of self inductance 1 H is connected across an alternating voltage of 115 V and frequency 60 HZ. Then the X_L ; peak

current

- a. $37.1\Omega, 0.43\text{ A}$ b. $337.1\Omega, 0.43\text{ A}$
c. $377.1\Omega, 0.3\text{ A}$ d. $3.7\Omega, 0.42\text{ A}$

7. The source frequency for which a $5\mu\text{F}$ capacitor has a reactance of 1000Ω is

- a. $\frac{100}{\pi}\text{ Hz}$ b. $\frac{1000}{\pi}\text{ Hz}$
c. 200 Hz d. 5000 Hz

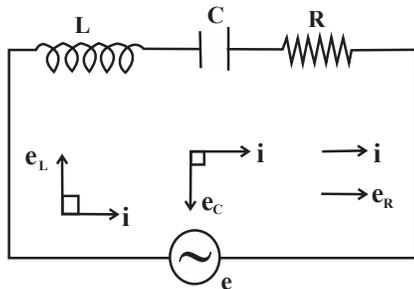
LCR Circuit

Q.14 Derive an expression for the impedance of an LCR circuit connected to an AC supply

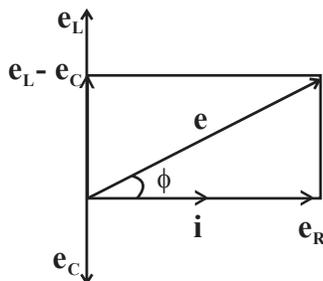
OR

Obtain the expression for a current voltage relationship or impedance in an L-C-R circuit.

Ans:



- i. Suppose an AC emf $e = e_0 \sin(\omega t)$ is applied to a series combination of inductance L , resistance R , capacitance C .
- ii. Let, e_L, e_C and e_R - r.m.s. voltages across inductor, capacitor and resistor respectively.
 i - r.m.s. current flowing through the circuit. In this circuit, current is same.
- iii. Phasor diagram is as shown in fig. below



In the diagram, e_R is in phase with current i , e_L leads the current by $\frac{\pi}{2}$ radian and e_C lags

the current i by $\frac{\pi}{2}$ radian,

iv. The voltages e_L, e_C and e_R are given by, $e_R = iR$, $e_L = iX_L$ and $e_C = iX_C$... (1)

v. **If $e_L > e_C$:** The resultant e.m.f. $e_L - e_C$, is in the direction of e_L as shown in phasor diagram.

Resultant applied e.m.f. is given by

From phasor diagram,

$$e^2 = e_R^2 + (e_L - e_C)^2 \quad \text{from equ.(1)}$$

$$= (iR)^2 + (iX_L - iX_C)^2$$

$$= i^2[R^2 + (X_L - X_C)^2]$$

$$\therefore e = i \sqrt{R^2 + (X_L - X_C)^2} \quad \dots(2)$$

$$\therefore e = iZ$$

vi. **Impedance:**

Comparing eq (2) with Ohms law.

The quantity $Z = \sqrt{R^2 + (X_L - X_C)^2}$ represents impedance.

It can be defined as the ratio of r.m.s voltage to r.m.s value of current. It is given by,

$$Z = \frac{e_{\text{rms}}}{i_{\text{rms}}} \quad \dots(3)$$

Unit : Ohm (Ω)

vii. **Phase angle between emf and current:**

From phasor diagram,

$$\tan \phi = \frac{e_L - e_C}{e_R}$$

$$= \frac{iX_L - iX_C}{iR}$$

$$= \frac{X_L - X_C}{R}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \quad \dots(4)$$

The equation for current is given by,

$$i = i_0 \sin(\omega t - \phi)$$

viii. **If $e_C > e_L$:** The resultant e.m.f. $e_C - e_L$, is in the direction of e_C .

The equation for current is given by,

$$i = i_0 \sin(\omega t + \phi)$$

ix. In general the equations for resultant emf and current can be expressed as,

$$e = e_0 \sin(\omega t \pm \phi) \text{ and}$$

$$i = i_0 \sin(\omega t \pm \phi)$$

Kye Point

- i. Impedance for LR series circuit
i.e capacitor is absent $Z = \sqrt{R^2 + X_L^2}$
- ii. Impedance for RC series circuit
i.e inductor is absent $Z = \sqrt{R^2 + X_C^2}$
- iii. When $X_L = X_C$ then $\tan \phi = 0$.
Hence voltage and current are in phase.
Thus the AC circuit is non inductive.
- iv. When $X_L > X_C$, $\tan \phi$ is positive
 $\therefore \phi$ is positive.
Hence voltage leads the current by a phase angle ϕ The AC circuit is inductance dominated circuit.
- v. When $X_L < X_C$, $\tan \phi$ is negative
 $\therefore \phi$ is negative.
Hence voltage lags the current by a phase angle ϕ . The AC circuit is capacitance dominated circuit.

Q.15 Define impedance and admittance

Ans :

- i. The ratio of rms voltage to the rms value of current is called impedance. The reciprocal of impedance of an AC circuit is called admittance.
- ii. The SI unit of impedance is ohm (Ω).
- iii. $Z = \frac{e_{rms}}{i_{rms}}$
 $Y = \frac{1}{Z}$
- vi. The SI unit of admittance is ohm⁻¹.

★ Q.16 The total impedance of circuit decrease when a capacitor is added in series with L and R. Explain why?

Ans :

- i. The total impedance of LCR series circuit is,
 $Z_{LCR} = \sqrt{R^2 + (X_L - X_C)^2}$... (1)
- ii. When C is absent the impedance of series LR circuit will be,
 $Z_{LR} = \sqrt{R^2 + X_L^2}$... (2)

- iii. Comparing equations (1) and (2), it can be concluded that,
 $Z_{LR} > Z_{LCR}$
 \therefore The capacitor impedance of a circuit decrease when a capacitor is added in series with L and R.

★ Q.17 An electric lamp is connected in series with a capacitor and an AC source is glowing with a certain brightness. How does the brightness of the lamp change on increasing the capacitance?

Ans

- i. When an electric lamp is connected in series with a capacitor and an AC source, the capacitor offers capacitive reactance,
 $X_C = \frac{1}{\omega C}$
- ii. When capacitance is increased, capacitive reactance X_C decreases, hence impedance of the circuit $Z = \sqrt{R^2 + X_C^2}$ decreases.
- iii. In turn, the current $i = \frac{e}{Z}$ increases.
- iv. As the brightness of the bulb depends on the current passed through the circuit, it increases.

Type - III
Numerical based on L.C.R

Formulae used

In LCR circuit

1. $Z = \frac{e_{rms}}{i_{rms}} = \sqrt{R^2 + (X_L - X_C)^2}$
2. LR circuit $Z = \sqrt{R^2 + X_L^2}$
3. CR circuit $Z = \sqrt{R^2 + X_C^2}$

- 1) **A coil of 0.01 H inductance and 1 Ω resistance is connected to 200 V, 50 Hz AC supply. Find the impedance of the circuit and time lag between maximum alternating voltage and current.**

Data : L = 0.01 H, R = 1 Ω $e_0 = 200$ V,
f = 50 Hz

To Find : i. Impedance of circuit (Z)

ii. Time lag (Δt)

Formula : i. $X_L = 2\pi fL$ ii. $Z = \sqrt{R^2 + X_L^2}$

iii. $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$ iv. $\phi = \omega\Delta t$

Solution :

i. $X_L = 2\pi fL$
 $X_L = 2 \times 3.142 \times 50 \times 0.01 = 3.142 \Omega$

ii. $Z = \sqrt{R^2 + X_L^2}$
 $Z = \sqrt{1^2 + (3.142)^2} = \sqrt{1 + 9.872}$
 $= \sqrt{10.872} = 3.297 \Omega$

iii. $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$
 $\phi = \tan^{-1}\left(\frac{3.142}{1}\right) = 72.35^\circ$
 $= \left(72.35^\circ \times \frac{\pi}{180}\right) \text{rad}$

iv. $\phi = \omega\Delta t$
 $72.35^\circ \times \frac{\pi}{180} = 2 \times \pi \times 50 \times \Delta t$
 $\dots (\because \omega = 2\pi f)$
 $\Delta t = \frac{72.35}{180 \times 2 \times 50} = \frac{0.7235}{180}$
 $= 4.019 \times 10^{-3} = 0.004 \text{ s}$

Ans : i. The impedance of circuit is 3.297Ω
ii. The time lag between maximum alternating voltage and current is 0.004 s

2) **A capacitor of $100 \mu\text{F}$, a coil of resistance 50Ω and an inductance 0.5 H are connected in series with a $110 \text{ V} - 50 \text{ Hz}$ source. Calculate the rms value of current in the circuit.**

Data : $C = 100 \mu\text{F} = 100 \times 10^{-6} \text{ F} = 10^{-4} \text{ F}$
 $R = 50 \Omega$, $L = 0.5 \text{ H}$
 $e_{\text{rms}} = 110 \text{ V}$, $f = 50 \text{ Hz}$

To Find : rms current (i_{rms})

Formula : i. $X_C = \frac{1}{2\pi fC}$ ii. $X_L = 2\pi fL$

iii. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

iv. $i_{\text{rms}} = \frac{e_{\text{rms}}}{Z}$

Solution :

i. $X_C = \frac{1}{2\pi fC}$

$X_C = \frac{1}{2 \times 3.142 \times 50 \times 10^{-4}} = \frac{100}{3.142}$
 $= 31.83 \Omega$

ii. $X_L = 2\pi fL$
 $X_L = 2 \times 3.142 \times 50 \times 0.5 = 157.1 \Omega$

iii. $Z = \sqrt{R^2 + (X_L - X_C)^2}$
 $Z = \sqrt{(50)^2 + (157.1 - 31.83)^2}$
 $= \sqrt{(50)^2 + (125.27)^2}$
 $= \sqrt{2500 + 15692.6}$
 $= \sqrt{18192.6} = 134.88 \Omega$

iv. $i_{\text{rms}} = \frac{e_{\text{rms}}}{Z}$
 $i_{\text{rms}} = \frac{110}{134.88} \approx \frac{110}{134.9}$
 $= 8.155 \times 10^{-1} = 0.816 \text{ A}$

Ans : The rms value of current in the circuit is 0.816 A

★ 3) A $150 \mu\text{F}$ capacitor is connected to a $220 \text{ V} - 50 \text{ Hz}$ source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is double, what will happen to the capacitive reactance and the current?

Data : $C = 150 \mu\text{F} = 15 \times 10^{-6} \text{ F}$
 $e_{\text{rms}} = 220 \text{ volt}$, $f_1 = 50 \text{ Hz}$,
 $f_2 = 2f_1 = 2 \times 50 = 100 \text{ Hz}$

To Find : i. Capacitive reactance (X_{C_1})
ii. (i_{rms})
iii. Peak current (i_0)
iv. Capacitive reactance when frequency is doubled (X_{C_2})
v. Peak current when frequency is doubled (i_{0_2})

Formula : i. $X_C = \frac{1}{2\pi fC}$ ii. $i_{\text{rms}} = \frac{e_{\text{rms}}}{X_C}$

iii. $i_0 = \sqrt{2} i_{\text{rms}}$

Solution :

i. $X_C = \frac{1}{2\pi fC}$

$$X_C = \frac{1}{2 \times 3.142 \times 50 \times 15 \times 10^{-6}}$$

$$= \frac{10^4}{3.142 \times 15} = 2.121 \times 10^2 = 212.1 \Omega$$

ii. $i_{\text{rms}} = \frac{e_{\text{rms}}}{X_C} = \frac{220}{212.1} = 1.04 \text{ A}$

iii. $i_0 = \sqrt{2} i_{\text{rms}}$
 $i_0 = \sqrt{2} \times 1.04$
 $= 1.414 \times 1.04 = 1.470 \text{ A}$

Now, $X_C = \frac{1}{2\pi fC}$

$\therefore X_C \propto \frac{1}{f}$

$\therefore \frac{(X_C)_1}{(X_C)_2} = \frac{2f}{f}$

$\therefore (X_C)_2 = \frac{1}{2} (X_C)_1$

If frequency is doubled then capacitive reactance will be halved For capacitive circuit

$i = 2\pi fC \times e$

$\therefore i \propto f$

$\therefore \frac{i_1}{i_2} = \frac{f}{2f} \quad \therefore i_2 = 2i_1$

\therefore If frequency is doubled, then current will also get doubled.

Ans : When the frequency is 50 Hz

i. The capacitive reactance in the circuit is 212Ω

ii. The rms current in the circuit is 1.04 A

iii. The peak current in the circuit is 1.47 A

iv. When the frequency is doubled the capacitive reactance becomes half and the current becomes double

★ 5) A $25 \mu\text{F}$ capacitor, a 0.10 H inductor and a 25Ω resistor are connected in series with an AC source whose emf is given by $e = 310 \sin 314 t$ (volt). What is the frequency, reactance, impedance, current and phase angle of the circuit?

Data : $C = 25 \mu\text{F} = 25 \times 10^{-6} \text{ F}$, $L = 0.10 \text{ H}$,
 $R = 25 \Omega$, $e = 310 \sin 314 t$,

To Find : i. (f)
 ii. Reactance of circuit $|X_L - X_C|$
 iii. (Z)
 iv. Current (i)
 v. Phase Angle (ϕ)

Formula: i. $e = e_0 \sin \omega t$ ii. $X_C = \frac{1}{\omega C}$

iii. $X_L = \omega L$

iv. $Z = \sqrt{R^2 + (X_C - X_L)^2}$

v. $i_{\text{rms}} = \frac{e_{\text{rms}}}{Z}$

vi. $\phi = \tan^{-1} \left(\frac{X_C - X_L}{R} \right)$

Solution :

i. On comparing $e = 310 \sin 314 t$
 $e_0 = 310 \text{ V}$ and $\omega = 314 \text{ rad/s}$

$$f = \frac{\omega}{2\pi} = \frac{314}{2 \times 3.14} = 50 \text{ Hz}$$

ii. $X_C = \frac{1}{\omega C}$

$$X_C = \frac{1}{3.14 \times 25 \times 10^{-6}}$$

$$= \frac{100}{314 \times 25} \times 10^4 = 127.4 \Omega$$

iii. $X_L = \omega L$

$$X_L = 314 \times 0.10 = 31.4 \Omega$$

Reactance of the circuit due to inductor and capacitor

$$|X_L - X_C| = |31.4 - 127.4| = 96 \Omega$$

iv. $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$$Z = \sqrt{(25)^2 + (96)^2} = \sqrt{625 + 9216}$$

$$= \sqrt{9841} = 99.2 \Omega$$

v.
$$i_{\text{rms}} = \frac{e_{\text{rms}}}{Z}$$

$$i_{\text{rms}} = \frac{310 \times 0.707}{99.2}$$

$$\dots \left(\because e_{\text{rms}} = \frac{e_0}{\sqrt{2}} = 0.707e_0 \right)$$

$$= 2.20 \text{ A}$$

vi.
$$\phi = \tan^{-1} \left(\frac{X_C - X_L}{R} \right)$$

$$\tan \phi = \frac{96}{25} = 3.84$$

$$\phi = \tan^{-1}(3.84) = 75.4^\circ$$

$$= \frac{75.4 \times 3.142}{180} = 1.316 \text{ rad}$$

- Ans :** For the given LCR series circuit
- i. The frequency of the circuit is 50 Hz
 - ii. The reactance of the circuit is 96 Ω
 - iii. The impedance of the circuit is 99.2 Ω
 - iv. The rms current of the circuit is 2.21 A
 - v. The phase angle of the circuit is 1.316 rad.

Problem for Practice

1. When an inductor L and a resistor R in series are connected across a 12 V, 50 Hz supply, a current of 0.5 A flows in the circuit. The current differs in phase from applied voltage by $\frac{\pi}{3}$ radian. Calculate the value of R.
Ans : 12 Ω
2. A bulb of resistance 10 Ω connected to an inductor of inductance L, is in series with an a.c. source marked 100 V, 50 Hz. If the phase angle between the voltage and current is $\frac{\pi}{4}$ radian, calculate the value of L .
Ans : 0.0318 H
3. A 60 – 10 W electric lamp is to be run on 100 V – 60 Hz mains (i) Calculate the inductance of the choke coil required. (ii) If a resistor is to be used in place of choke coil to achieve the

same result, calculate its value.

Ans : 1.273H

4. When an alternating voltage of 220 V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current flows through the circuit but it leads the applied voltage by $\frac{\pi}{2}$ radian. (i) Name the devices X and Y (ii) Calculate the current flowing in the circuit, when same voltage is applied across the series combination of X and Y

Ans : 0.35 A

5. An alternating current of 1.5 mA rms and angular frequency $\omega = 100 \text{ rad s}^{-1}$ flows through a 10 kΩ resistor and 0.50 μF capacitor in series. Calculate the value of rms voltage across the capacitor and the impedance of the circuit.

Ans : 30 V, 1.2 × 10⁴ Ω

6. A resistor of 50 ohm an inductor of $\left(\frac{20}{\pi}\right)$ H and a capacitor of $\left(\frac{5}{\pi}\right)$ μF are connected in series to a voltage source 230 V, 50 Hz. Find the impedance of the circuit.

Ans : 50 Ω

7. A 0.3 H inductor 60 μF capacitor and a 50 Ω resistor are connected in series with a 120 V, 60 Hz supply. Calculate (i) impedance of the circuit (ii) current flowing in the circuit

Ans : 1.41 A

8. A 2 μF capacitor 100 Ω resistor and 8H inductor are connected in series with an a.c source. What should be the frequency of the a.c source, for which the current drawn in the circuit is maximum? If the peak value of emf of the source is 200V, Find for maximum current: (i) the inductive and capacitive reactances of the circuit (ii) total impedance of the circuit (iii) peak value of current in the

circuit. (iv) the phase difference between voltages across inductor and resistor and (v) the phase difference between voltages across inductor and capacitor

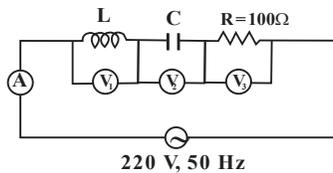
Ans : 39.8 Hz

MULTIPLE CHOICE QUESTIONS
Entrance Corner (Set 3)

1. An ac voltage is applied to a resistance R an inductor L in series. If R and the inductive reactance are both equal to 3Ω , the phase difference between the applied voltage and the current in the circuit is
- a. $\frac{\pi}{6}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{2}$ d. zero

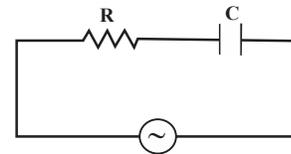
2. A coil has resistance 30 ohm and inductive reactance 20 ohm at 50 Hz frequency. If an ac source of 200 volt, 100 Hz, is connected across the coil, the current in the coil will be
- a. $\frac{20}{\sqrt{13}}$ A b. 2.0 A
c. 4.0 A d. 8.0 A

3. In the given circuit the readings of voltmeter V_1 and V_2 are 300 volts each. The readings of the voltmeter V_3 and ammeter A are respectively



- a. 150 V, 2.2 A b. 220 V, 2.2 A
c. 220 V, 2.0 A d. 100 V, 2.0 A
4. In an LCR-series a.c. circuit, the voltage across each of the components. L,C and R is 50 V. The voltage across the LC-combination will be
- a. 50V b. $50\sqrt{2}$ V
c. 100V d. Zero
5. A circuit has a resistance of 12 ohm and an impedance of 15ohm. The power factor of the circuit will be
- a. 0.8 b. 0.4 c. 1.25 d. 0.125

6. In an a.c. circuit the voltage applied is $\xi = \xi_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin(\omega t - \pi/2)$. The power consumption in the circuit is given by
- a. $p = \sqrt{2}\xi_0 I_0$ b. $p = \frac{\xi_0 I_0}{\sqrt{2}}$
c. $p = 0$ d. $p = \frac{\xi_0 I_0}{2}$
7. A capacitor in an ideal LCR-circuit is fully charged by a DC source. Then it is disconnected from DC source, the current in the circuit
- a. becomes zero instantaneously
b. grows monotonically
c. decays monotonically
d. oscillates infinitely.
8. A 50 Hz a.c. source of 20 V is connected across R and C as shown in figure. The voltage across R is 12 V



- The voltage across C is
- a. 8V b. 16 V c. 10 V
d. not possible to determine, unless values of R and C are given.
9. In a series LCR circuit, the voltage across the resistance, capacitance and inductance is 10 V each. If the capacitance is short circuited the voltage across the inductance will be
- a. 10 V b. $10\sqrt{2}$ V
c. $10/\sqrt{2}$ V d. 20 V
10. In a series LR-circuit $X_L = 3R$. Now a capacitor with $X_C = 2R$ is added in series. Ratio of new to old power factor is
- a. $\sqrt{2}$ b. $1/\sqrt{2}$
c. 2 d. 1

Try Yourself

11. In a LCR-series circuit, the potential difference between the terminals of the inductance is 60 V, between the terminals of

the capacitor is 30 V and that across the resistance is 40 V. Then, supply voltage will be equal to

- a. 50 V b. 70 V c. 130 V d. 10 V

12. In circuit L, C and R are connected in series with an alternating voltage source of frequency f . The current leads the voltage by 45° . The value of C is

- a. $\frac{1}{2\pi f(2\pi fL + R)}$ b. $\frac{1}{\pi f(2\pi fL + R)}$
c. $\frac{1}{2\pi f(2\pi fL - R)}$ d. $\frac{1}{\pi f(2\pi fL - R)}$

13. In an a.c. series circuit, the instantaneous current is maximum when the instantaneous voltage is maximum. The circuit element connected to the source will be

- a. pure inductor b. pure capacitor
c. pure resistor
d. combination of a capacitor and an inductor

14. In an a.c. circuit, the potential differences across an inductance and resistance joined in series are respectively 16 V and 20 V. The total potential difference of the source is

- a. 20.0 V b. 25.6 V
c. 31.9 V d. 53.5 V

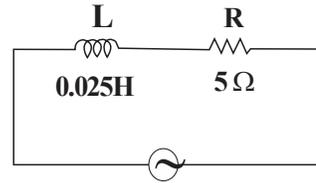
15. A current $I = I_0 \sin(\omega t + \pi/2)$ flows in a circuit across which an alternating potential $\xi = \xi \sin \omega t$ is applied. The power consumed in the circuit is

- a. $\xi_0 I_0 / 2$ b. $\xi_0 I_0$
c. ξ_0 d. zero

16. In an a.c. circuit, with phase voltage V and current I, the power dissipated is

- a. $\frac{1}{2}VI$ b. $\frac{1}{\sqrt{2}}VI$ c. VI
d. depends on the phase angle between V and I.

17. For the LR-circuit shown in figure the phase angle, if frequency is $f = 100/\pi$, is



- a. 60° b. 45° c. 30° d. 90°

18. If resistance of 100Ω , inductance of 0.5 H and capacitance of 10×10^{-6} F are connected in series through 50 Hertz AC supply, the impedance will be

- a. 1.87Ω b. 101.3Ω
c. 18.7Ω d. 189.7Ω

19. In an LR-circuit; $L = \frac{0.4}{\pi}$ H and $R = 30\Omega$. If

the circuit has an alternating emf of 220 volt 50 cycles per sec, the impedance and current in the circuit will be.

- a. $40.4\Omega, 4.4$ A b. $50.4\Omega, 4.4$ A
c. $3.07\Omega, 6.0$ A d. $11.4\Omega, 17.5$ A

20. A coil of self - inductance $\left(\frac{1}{\pi}\right)$ H is connected in series with a 300Ω resistance. A voltage of 200V at frequency 200Hz is applied to this combination. The phase difference between the voltage and the current will be

- a. $\tan^{-1}\left(\frac{4}{3}\right)$ b. $\tan^{-1}\left(\frac{3}{4}\right)$
c. $\tan^{-1}\left(\frac{1}{4}\right)$ d. $\tan^{-1}\left(\frac{5}{4}\right)$

21. The frequency at which the inductive reactance of 2H inductance will be equal to the capacitive reactance of $2\mu\text{F}$ capacitance (nearly)

- a. 80Hz b. 40 Hz c. 60Hz d. 20Hz

22. In an LCR series circuit, the capacitor is changed from C to 4C. For the same resonant frequency, the inductance should be changed from L to

- a. 2 L b. L/2 c. L/4 d. 4L

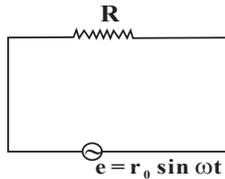
13.6 Power in AC circuit

- Q.18 Find the average power consumed by a resistance connected to an ac voltage

source.

Ans:

- i. Let $e = e_0 \sin \omega t$ be applied e.m.f. across a resistor of resistance R as shown in fig.



The instantaneous current is given by,

$$i = i_0 \sin \omega t$$

- ii. The power at time t is given by,

$$P = e i = e_0 \sin \omega t \cdot i_0 \sin \omega t \\ = e_0 i_0 \sin^2 (\omega t)$$

- iii. Therefore, the average power is

$$\therefore P_{av} = \frac{\left[\begin{array}{c} \text{Work done by the emf on the} \\ \text{charges in one cycle} \end{array} \right]}{\text{time for one cycle}}$$

$$= \frac{\int_0^T P dt}{T} = \frac{\int_0^T e_0 i_0 \sin^2 \omega t dt}{T} \\ = \frac{e_0 i_0}{T} \int_0^T \sin^2 \omega t dt \\ = \frac{e_0 i_0}{T} \left(\frac{T}{2} \right) \left[\because \int_0^T \sin^2 \omega t dt = \frac{T}{2} \right] \\ = \frac{e_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}}$$

$$\therefore P = P_{av} = e_{rms} \times i_{rms}$$

Q.19 When an AC source is connected to an ideal inductor show that average power supplied by the source over a complete cycle is zero.

Ans:

- i. In an purely inductive circuit, the current lags behind the voltage by a phase angle of $\pi/2$. i.e., when $e = e_0 \sin \omega t$ then $i = i_0 \sin(\omega t - \pi/2)$

- ii. Instantaneous power in the circuit is given by,

$$P = e i \\ P = (e_0 \sin \omega t)(i_0 \sin(\omega t - \pi/2)) \\ = -e_0 i_0 \sin \omega t \cos \omega t$$

$$\therefore P_{av} = \frac{\text{Work done in one cycle}}{\text{time for one cycle}}$$

$$= \frac{\int_0^T P dt}{T} = \frac{\int_0^T -e_0 i_0 \sin \omega t \cos \omega t dt}{T} \\ = -\frac{e_0 i_0}{2} \frac{\int_0^T 2 \sin \omega t \cos \omega t dt}{T} \\ = -\frac{e_0 i_0}{2} \frac{\int_0^T \sin 2\omega t dt}{T} = -\frac{e_0 i_0}{2T} \left[\frac{-\cos 2\omega t}{2\omega} \right]_0^T$$

$$\therefore P_{av} = 0$$

\therefore Average power over a complete cycle of AC through an ideal inductor is zero.

Q. 20 Obtain an expression for average power dissipated in a purely capacitive circuit.

OR

★ Prove that an ideal capacitor in an AC circuit does not dissipate power.

Ans:

- i. In a purely capacitive circuit, the current leads the emf by a phase angle of $\pi/2$ i.e., when

$$e = e_0 \sin \omega t$$

$$i = i_0 \sin (\omega t + \pi/2)$$

$$\therefore i = i_0 \cos \omega t$$

- ii. Instantaneous power in the given circuit is given by,

$$P = e i \\ = (e_0 \sin \omega t)(i_0 \cos \omega t) = e_0 i_0 \sin \omega t \cos \omega t$$

$$\therefore P_{av} = \frac{\text{Work done in one cycle}}{\text{time for one cycle}}$$

$$\therefore P_{av} = \frac{\int_0^T P dt}{T} = \frac{\int_0^T e_0 i_0 \sin \omega t \cos \omega t dt}{T}$$

$$= \frac{e_0 i_0}{2} \frac{\int_0^T 2 \sin \omega t \cos \omega t dt}{T}$$

$$= \frac{e_0 i_0}{2T} \int_0^T \sin 2 \omega t dt$$

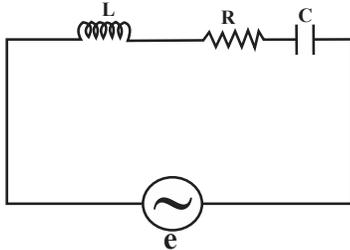
$$= \frac{e_0 i_0}{2T} \left[-\frac{\cos 2\omega t}{2\omega} \right]_0^T$$

∴ $P_{av} = \text{zero}$.

Q.21 Find an expression for power dissipated in the LCR circuit. Explain power factor.

Ans:

i. Consider an AC circuit containing a resistance R, an inductance L and a capacitance C in series as shown in fig.



ii. Let, an emf $e = e_0 \sin \omega t$ be applied to the circuit.

iii. The instantaneous current is given by,
 $i = i_0 \sin (\omega t \pm \phi)$

ϕ is positive for capacitive reactance, and negative for the inductive reactance.

iv. The instantaneous power is given by,
 $P = e i = e_0 \sin (\omega t) i_0 \sin (\omega t \pm \phi)$
 $= e_0 i_0 \sin \omega t (\sin \omega t \cos \phi \pm \cos \omega t \sin \phi)$
 $= e_0 i_0 \sin^2 \omega t \cos \phi \pm e_0 i_0 \sin \omega t \cos \omega t \sin \phi$

v. Therefore average power dissipated is given by

$$P_{ave} = \int_0^T \frac{P_{inst}}{T} dt$$

$$P_{ave} = \int_0^T \frac{(e_0 i_0 \sin^2 \omega t \cos \phi \pm e_0 i_0 \sin \omega t \cos \omega t \sin \phi)}{T} dt$$

$$= e_0 i_0 \cos \phi \int_0^T \frac{\sin^2(\omega t) dt}{T} \pm e_0 i_0 \sin \phi \int_0^T \frac{\sin(\omega t) \cos(\omega t) dt}{T}$$

since $\int_0^T \frac{\sin^2(\omega t) dt}{T} = \frac{1}{2}$

and $\int_0^T \frac{\sin(\omega t) \cos(\omega t) dt}{T} = 0$

$$= \frac{e_0 i_0 \cos \phi}{2} \pm 0$$

$$= \frac{e_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \cos \phi$$

$$= e_{rms} i_{rms} \cos \phi$$

vi. $\cos \phi$ is called a power factor of the circuit. If Z is the impedance of the circuit, then the power factor $\cos \phi = \frac{R}{Z}$.

vii. The term $e_{rms} i_{rms}$ is called the apparent power. This is the value calculated from the ammeter and the voltmeter readings.

viii. The true power dissipated in the circuit is
 $P_{ave} = \text{apparent power} \times \text{power factor}$.

$$\text{Power factor} = \frac{\text{True power}}{\text{apparent power}}$$

Key point

1. Pure resistive circuit . Here the voltage and current are in same phase, i.e $\phi = 0$ and $\cos \phi = 1$

$$\therefore P_{av} = e_{rms} \cdot I_{rms} \times 1 = e_{rms} \cdot I_{rms} = \frac{e_{rms}^2}{R}$$

2. Pure inductive circuit. Here the voltage leads the current in phase by $\frac{\pi}{2}$, i.e. $\phi = \frac{\pi}{2}$

$$\therefore P_{av} = e_{rms} \cdot I_{rms} \cos \frac{\pi}{2} = 0$$

Thus the average power consumed in an inductive circuit over a complete cycle is zero.

3. Pure capacitive circuit Here the voltage lags behind the current in Phase by $\frac{\pi}{2}$, i.e. $\phi = \frac{\pi}{2}$

$$\therefore P_{av} = e_{rms} \cdot I_{rms} \cos \left(-\frac{\pi}{2} \right) = 0$$

Thus the average power consumed in a capacitive circuit over a complete cycle is also zero.

4. **Series LCR - circuit** For a series LCR-circuit

$$P_{av} = e_{rms} \cdot I_{rms} \cos \phi \text{ where}$$

$$\phi = \tan^{-1} \frac{X_L - X_C}{R}$$

sometimes, ϕ may have a non-zero value for series LR- LC - and LCR-circuits. So power is consumed in such circuits, but only in the resistor R.

Q.22 What do you mean by power factor of an a.c. circuit? write an expression for it. When is the value of the power factor of an a.c. circuit minimum and maximum?

Ans:

- i. The average power of an a.c circuit is given by $P_{av} = e_{rms} \cdot I_{rms} \cos\phi$
- ii. Average power = rms emf \times rms current $\times \cos\phi$
- iii. The product $e_{rms} \cdot i_{rms}$ does not give the actual power and is called apparent power. It give the actual true power only when multiplied by factor $\cos\phi$. The factor $\cos\phi$ is called the power factor of an a.c. circuit
- iv. True power = Apparent power \times power factor
- v. Thus power factor may be defined as the ratio of the true power to the apparent power of an A.C circuit.
- vi. Its value varies from 0 to 1.
- vii. The power factor of a series LCR-circuit is given by

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L - \omega C)^2}}$$

Case :1 For a purely inductive or capacitive circuit $\phi = 90^\circ$

$$\text{Power factor} = \cos 90^\circ = 0$$

Thus the power factor assumes the minimum value for a purely inductive or capacitive circuit.

Case 2 : For a purely resistive circuit $\phi = 0^\circ$

$$\text{Power factor} = \cos 0^\circ = 1$$

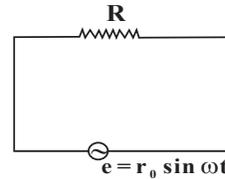
Thus the power factor assumes the maximum value for a purely resistive circuit.

Q.23 An emf $e_0 \sin \omega t$ applied to a series L - C - R circuit derives a current $i = i_0 \sin \omega t$ in the circuit. Deduce the expression for the average power dissipated in the circuit.

Ans:

- i. For given alternating emf and current, the phase difference ϕ is zero.

- $\therefore \cos\phi = 1$
- $Z = R$
- ii. Therefore, the series LCR circuit can be considered as a purely resistive circuit.
- iii. Let $e = e_0 \sin \omega t$ be applied e.m.f. across a resistor of resistance R as shown in fig.



- The instantaneous current is given by,
 $i = i_0 \sin \omega t$
- iv. The power at time t is given by ,
 $P = e i = e_0 \sin \omega t \cdot i_0 \sin \omega t$
 $= e_0 i_0 \sin^2 (\omega t)$
 - v. Therefore, the average power is

$$\therefore P_{av} = \frac{\left[\begin{array}{l} \text{Work done by the emf on the} \\ \text{charges in one cycle} \end{array} \right]}{\text{time for one cycle}}$$

$$\begin{aligned} &= \frac{\int_0^T P dt}{T} = \frac{\int_0^T e_0 i_0 \sin^2 \omega t dt}{T} \\ &= \frac{e_0 i_0}{T} \int_0^T \sin^2 \omega t dt \\ &= \frac{e_0 i_0}{T} \left(\frac{T}{2} \right) \quad \left[\because \int_0^T \sin^2 \omega t dt = \frac{T}{2} \right] \\ &= \frac{e_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \end{aligned}$$

$$\therefore P = P_{av} = e_{rms} \times i_{rms}$$

Q. 24 For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

Ans:

- i. Power in the circuits used for transporting electric power is given as,
 $P = e_{rms} i_{rms} \cos\phi \quad \dots(1)$
Where $\cos\phi =$ power factor.
- ii. As per equation(1), to transmit a given power P at a given voltage e_{rms} ; if $\cos\phi$ is small, current (i_{rms}) has to increase accordingly.
- iii. Therefore, power loss = $i_{rms}^2 R$ in transmission also increases.

Q.25 What is watt less current?

Ans: Current through pure inductor or ideal capacitor which consumes no power for its maintenances, in the circuit is called idle current or watt less current.

Q. 26 In a series LR circuit $X_L = R$ and power factor of the circuit is P_1 . When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2 . Calculate P_1/P_2 .

Ans:

i. In the series LR circuit,
 $X_L = R$

$$\therefore \text{Power factor } P_1 = \frac{\text{Resistance}(R)}{\text{impedance}(Z)}$$

$$\therefore P_1 = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{2R^2}}$$

$$\therefore P_1 = \frac{1}{\sqrt{2}} \quad \dots(1)$$

ii. When capacitor of capacitance C is put in series,

$$\therefore \text{Power factor } P_2 = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Given that, $X_L = X_C$

$$\therefore P_2 = \frac{R}{\sqrt{R^2}} = \frac{R}{R} = 1 \quad \dots(2)$$

iii. From (1) and (2), we have,

$$\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$$

Type - IV

Numerical based on Power in AC circuit

Formula used

Average power in AC circuit with resistance

$$1. \quad P_{av} = \frac{e_0 i_0}{2} = \frac{e_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} = e_{rms} \times i_{rms}$$

Average power dissipated in A.C circuit with LCR in series.

$$2. \quad P_{av} = e_{rms} i_{rms} \cos \phi$$

Power factor

$$3. \quad \cos \phi = \frac{R}{Z} = \frac{\text{True power}}{\text{Apparent power}}$$

★ 1) A 100 Ω resistor is connected to a 220 V, 50 Hz supply

i. What is the rms value of current in the circuit?

ii. What is the net power consumed over a full cycle?

Data: $R = 100\Omega, e_{rms} = 220 \text{ V}, f = 50 \text{ Hz}$

To find: i. rms current (i_{rms})
ii. Net power consumed (P_{av})

Formulae: i. $i_{rms} = \frac{e_{rms}}{R}$ ii. $P_{av} = e_{rms} i_{rms}$

Solution: $i_{rms} = \frac{e_{rms}}{R} = \frac{220}{100} = 2.2 \text{ A}$

$$P_{av} = e_{rms} i_{rms} = 220 \times 2.2 = 484 \text{ W}$$

Ans : i. The rms current in the circuit is 2.2 A
ii. Net power consumed over a full cycle is 484 W.

★ 2) A light bulb is rated 100 W for 220V AC supply of 50 Hz. Calculate

i. resistance of the bulb

ii. the rms current through the bulb.

Data: $P_{av} = 100 \text{ W}, e_{rms} = 220 \text{ V}, f = 50 \text{ Hz}$

To find: i. (R)
ii. rms current (i_{rms})

Formulae: i. $P_{av} = e_{rms} i_{rms}$

ii. $i_{rms} = \frac{e_{rms}}{R}$

Solution:

$$P_{av} = e_{rms} i_{rms}$$

$$i_{rms} = \frac{P_{av}}{e_{rms}}$$

$$\therefore i_{rms} = \frac{100}{220} = \frac{10}{22} = 0.4545 \text{ A}$$

$$i_{rms} = \frac{e_{rms}}{R}$$

$$R = \frac{e_{rms}}{i_{rms}} = \frac{220}{0.4545} = 484 \Omega$$

Ans : i. Resistance of the bulb is 484 Ω
ii. The rms current through the bulb is 0.4545 A.

- ★ 3) Find the capacity of a capacitor which when put in series with a 10Ω resistor makes the power factor equal to 0.5.

Assume an 80 V-100 Hz AC supply.

Data: $R = 10\Omega$, $\cos \phi = 0.5$, $V = 80V$,
 $f = 100\text{Hz}$

To find: Capacitance (C)

Formulae: i. $\cos \phi = \frac{R}{Z}$ ii. $Z^2 = R^2 + X_C^2$

Solution:

$$\cos \phi = \frac{R}{Z} \Rightarrow 0.5 = \frac{10}{Z}$$

$$\therefore Z = \frac{10}{0.5} = 20\Omega$$

$$Z^2 = R^2 + X_C^2$$

$$(20)^2 = (10)^2 + X_C^2$$

$$\therefore X_C^2 = 400 - 100$$

$$\therefore X_C = \sqrt{300} = 10\sqrt{3}$$

$$\therefore \frac{1}{\omega C} = 10\sqrt{3} \quad \dots \left(\because X_C = \frac{1}{\omega C} \right)$$

$$\therefore C = \frac{1}{10\sqrt{3} \times 2\pi \times 100} \quad \dots (\because \omega = 2\pi f)$$

$$= \frac{\sqrt{3}}{3 \times 2 \times 3.142} \times 10^{-3} = \frac{1.732}{18.852} \times 10^{-3}$$

$$= 9.187 \times 10^{-2} \times 10^{-3} = 9.2 \times 10^{-5} \text{ F}$$

Ans: The capacity of a capacitor is $9.2 \times 10^{-5} \text{ F}$.

- ★ 4) A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which $R = 3\Omega$, $L = 25.48 \text{ mH}$ and $C = 796 \mu\text{F}$. Find.

- The impedance of the circuit
- The phase difference between the voltage across source and the currents
- The power factor
- The power dissipated in the circuit.

Data: $e_0 = 283V$, $f = 50\text{Hz}$, $R = 3\Omega$
 $L = 25.48\text{mH} = 25.48 \times 10^{-3}\text{H}$,
 $C = 796 \mu\text{F} = 796 \times 10^{-6}\text{F}$

To find: i. (Z)

- Phase difference (ϕ)
- Power factor ($\cos \phi$)
- (P_{av})

Formulae: i. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\text{ii. } \phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

iii. Power factor = $\cos \phi$

$$\text{iv. } P_{av} = e_{rms} i_{rms} \cos \phi$$

Solution:

$$X_L = \omega L$$

$$= 2\pi fL \quad \dots (\because \omega = 2\pi f)$$

$$= 2 \times 3.142 \times 50 \times 25.48 \times 10^{-3}$$

$$= 3.142 \times 2.548 = 8.005 \approx 8\Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{2 \times 3.142 \times 50 \times 796 \times 10^{-6}}$$

$$= \frac{10^4}{3.142 \times 796} = 3.998 \approx 4\Omega$$

$$\text{i. } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{3^2 + (8 - 4)^2}$$

$$= \sqrt{3^2 + (4)^2} = \sqrt{25} = 5\Omega$$

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

$$\phi = \tan^{-1} \left(\frac{8 - 4}{3} \right)$$

$$= \tan^{-1} \frac{4}{3} = 53.1^\circ$$

ii. Power factor = $\cos \phi$

iii. Power factor = $\cos (53.1^\circ) = 0.6$

$$\text{iv. } P_{av} = e_{rms} i_{rms} \cos \phi$$

$$P_{av} = \frac{e_0}{\sqrt{2}} \times \frac{e_0}{\sqrt{2} \times Z} \times 0.6$$

$$\dots \left[\because e_{rms} = \frac{e_0}{\sqrt{2}}; i_{rms} = \frac{e_{rms}}{Z} \right]$$

$$= \frac{283}{\sqrt{2}} \times \frac{283}{\sqrt{2} \times 5} \times 0.6 = 283 \times 283 \times 0.6$$

= 4805.34 W

- Ans :**
- The impedance of the circuit is 5Ω .
 - The phase difference between the voltage across the source and the current is 53.1° .
 - The power factor of the circuit is 0.6.
 - The power dissipated in the circuit is 4805.34W

Problem for Practice

- A light bulb is rated at 100 W for a 220 V supply of 50 Hz. Find (a) the resistance of the bulb; (b) the peak voltage of the source; and (c) the rms current through the bulb

Ans : (a) 484Ω (b) 311 (c) 0.45A

- A capacitor and a resistor are connected in series with an a.c. source. If the potential differences across C, R are 120 V, 90V respectively and if the r.m.s. current of the circuit is 3 A, calculate the (i) impedance, (ii) power factor of the circuit.

Ans: (i) 50Ω , (ii) 0.6

- An alternating voltage $e = 200 \sin 300 t$ is applied across a series combination of $R = 10\Omega$ and an inductor of 800 mH. Calculate:

- impedance of the circuit
- Peak value of current in the circuit
- Power factor of the circuit.

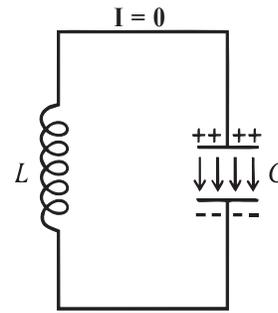
Ans : (i) 240.2Ω , (ii) $0.832A$, (c) 0.041

13.7 LC Oscillations

Q.27 What are LC Oscillations? Explain qualitatively how these oscillations are produced.

Ans: LC Oscillations: When a charge capacitor is allowed to discharge through a non-resistive inductor, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called LC-Oscillations.

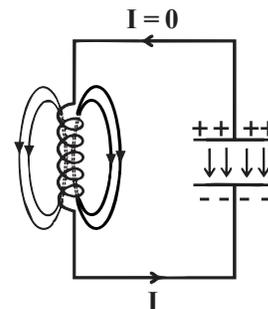
- The parallel combination of an inductor of self-inductance 'L' and capacitor of capacitance C produces electrical oscillations of desired frequency.
- Consider a parallel combination of L and C as shown in figure (a).



- Let a capacitor with initial charge q_0 at ($t = 0$) be connected to an ideal inductor (zero resistance). The electrical energy stored in the Dielectric medium between the plates of the

capacitor is $U_e = \frac{1}{2} \frac{q_0^2}{c}$ since there is no the current is the circuit the energy stored in the magnetic field of the inductor is zero.

- As the circuit is closed, the capacitor begins to discharge through the inductor giving rise to a current (I) in the circuit. As the current (I) increases, it builds up a magnetic field around the inductor as shown in fig. (b).

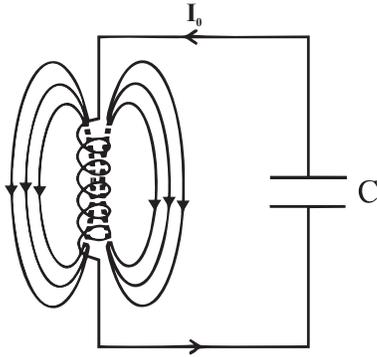


A part of the electrical energy of the capacitor gets stored in the inductor in the form of magnetic energy,

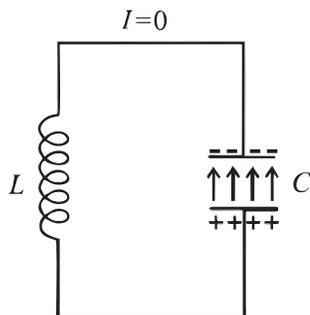
$$U_B = \frac{1}{2} LI^2$$

- At a later instant the capacitor gets fully discharged and the potential difference across its plates become zero [figure (c)]. The current reaches its maximum value I_0 , the energy in

the magnetic field is energy $\frac{1}{2} LI_0^2$ Thus the entire electrostatic energy of the capacitor has been converted into the magnetic field energy of the inductor.



- vi. As per Lenz's law, after the discharge of the capacitor is complete, the magnetic flux linked with the inductor decreases inducing a current in the same direction as the earlier current in the same direction as the earlier current. The current thus persists but with decreasing magnitude and charges the capacitor in the opposite direction. The magnetic energy of the inductor begins to change into the electrostatic energy of the capacitor.
- vii. The process continues till the capacitor is full charged with a polarity which is opposite to that in its initial state [figure(d)]. Thus the entire energy is again stored as $\frac{1}{2} \frac{q_0^2}{k}$ in the electric energy of the capacitor. The capacitor begins to discharge again sending current in opposite direction.
- viii. The energy is once again transferred to the magnetic field of the inductor. Thus the process repeats itself in the opposite direction.



- ix. The circuit eventually returns to the initial state.
- x. Thus the energy of the system continuously surges back and forth between the electric field of the capacitors and magnetic field of the inductor. This produces electrical

oscillations of a definite frequency. These are called LC oscillations.

- xi. If there is no loss of energy, the amplitude of the oscillation remain constant and the oscillations are undamped.

Q.28 Explain why LC oscillations are usually damped?

Ans: LC oscillations are damped because:

- Every inductor has some resistance. This causes energy loss as heat. The amplitude of oscillations goes on decreasing and they finally die out.
- Even if the resistance were zero, total energy of the system would not remain constant. It is radiated away in the form of electromagnetic waves.

Type - V

Numerical Based on LC Oscillations

Formulae used

- $U_E = \frac{1}{2} \frac{q_0^2}{c} = \frac{1}{2} CV^2$
- $U_B = \frac{1}{2} LI^2$

★ 1) A 10 μF capacitor is charged to a 25 volt of potential. The battery is disconnected and a pure 100 mH coil is connected across the capacitor so that LC oscillations are set up. Calculate the maximum current in the coil.

Data: $C = 10 \mu F = 10 \times 10^{-6} F = 10^{-5} F$,
 $V = 25 V$,
 $L = 100mH = 100 \times 10^{-3} = 10^{-1} H$

To find: Maximum current in the coil (I)

Formulae: i. $U_E = \frac{1}{2} \frac{q_0^2}{C} = \frac{1}{2} CV^2$

ii. $U_B = \frac{1}{2} LI^2$

Solution: From energy conservation,
Energy stored in capacitor
= Energy stored in inductor

$$\therefore \frac{1}{2} CV^2 = \frac{1}{2} LI^2$$

$$\therefore I^2 = \frac{CV^2}{L}$$

$$\therefore I = \sqrt{\frac{CV^2}{L}} = \sqrt{\frac{10^{-5} \times 25^2}{10^{-1}}}$$

$$= \sqrt{10^{-4} \times 25^2} = 25 \times 10^{-2} = 0.25 \text{ A}$$

Ans: The maximum current in the coil is 0.25 A.

- ★ 2) A 100 μ F capacitor is charged with a 50 V source supply. Then source supply is removed and the capacitor is connected across an inductance, as a result of which 5 A current flows through the inductance. Calculate the value of the inductance.

Data: $C = 100\mu\text{F} = 100 \times 10^{-6} \text{ F} = 10^{-4} \text{ F}$
 $V = 50 \text{ V}, I = 5 \text{ A}$

To find: Inductance (L)

Formulae: i. $U_E = \frac{1}{2} \frac{q_0^2}{C} = \frac{1}{2} CV^2$

ii. $U_B = \frac{1}{2} LI^2$

Solution: From energy conservation,
Energy stored in capacitor
= Energy stored in inductor

$$U_E = \frac{1}{2} \frac{q_0^2}{C} = \frac{1}{2} CV^2 \text{ and}$$

$$U_B = \frac{1}{2} LI^2$$

$$\frac{1}{2} LI^2 = \frac{1}{2} CV^2$$

$$\therefore L = \frac{CV^2}{I^2} = \frac{10^{-4} \times (50)^2}{5^2}$$

$$= \frac{10^{-4} \times 5^2 \times 10^2}{5^2} = 10^{-2} = 0.01 \text{ H}$$

Ans: The value of Inductance is 0.01 H.

Problem for Practice

1. A 10 μ F capacitor is charged to a 20 V potential. The battery is disconnected and a pure 40 mH coil is connected across the capacitor so that LC oscillations are set up. the maximum current in the coil.

Ans : 0.4 A

2. A capacitor of 10 μ F is first charged with a 100 V supply connected across it and then after the supply is removed it connected across an inductor. As a result maximum of 2A flows through the inductance. What is the value of the inductance.

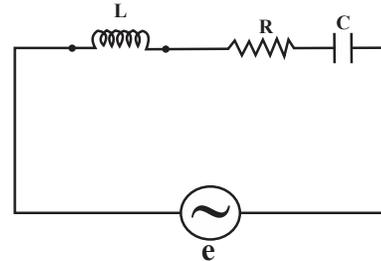
Ans : 125 mH

13.8 Electric Resonance

- Q.29** What is series resonant circuit. State conditions for series resonance. Obtain an expression for the resonant frequency.

Ans:

- i. Consider an AC circuit containing a resistance R, an inductance L and a capacitance C in series as shown in fig.



- ii. Let, e - applied rms emf
The rms current through circuit is given by,

$$i = \frac{e}{\sqrt{R^2 + (X_L - X_C)^2}}$$

- iii. The impedance of the circuit is given by,

$$Z = \frac{e}{i} = \sqrt{R^2 + (X_L - X_C)^2}$$

But, $X_L = \omega L$, and $X_C = \frac{1}{\omega C}$.

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

- i.e. Impedance and current of the circuit varies with the frequency of the applied emf.
- iii. If $X_L = X_C$, then $Z_{\min} = R$ i.e. the circuit acts as a purely resistive circuit.
For $Z = Z_{\min}$ then $I = I_{\max}$ and current and the emf are in phase.
This condition is called a series resonance of the circuit.
- iv. **Expression for resonant frequency (f_r):**

The frequency at which the resonance takes place and the maximum current flows through the circuit is called the resonating frequency.

At resonance,

$$X_L = X_C$$

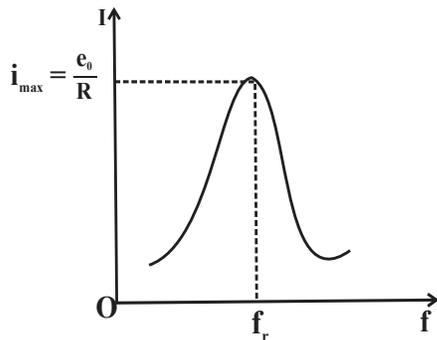
$$\therefore \omega L = \frac{1}{\omega C} \quad \text{and} \quad \omega^2 = \frac{1}{LC}$$

$$\therefore \omega = \frac{1}{\sqrt{LC}}$$

$$\omega = 2\pi f_r$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}},$$

vi. The variation of the current with frequency for a series LCR circuit is given below:

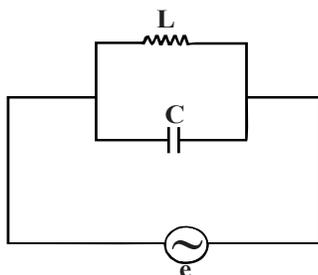


vii. This circuit is called an acceptor circuit because it accept only current of the resonant frequency and rejects currents of other frequencies.

viii. This circuit use in radio receivers or T.V. receivers for tuning the signal from a desired transmitting station or channel.

Q.30 Explain the performance of a parallel resonance circuit. Derive an expression for the resonating frequency. Give the current frequency graph.

Ans:



i. Suppose an AC emf, $e = e_0 \sin \omega t$ is applied to a parallel combination of an inductance L and a capacitor C.

ii. The current in the inductance lags behind the

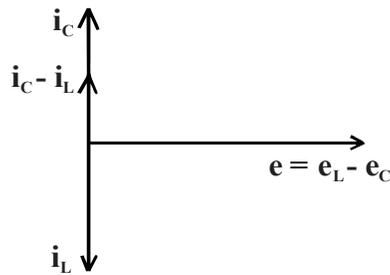
emf by $\frac{\pi}{2}$. Therefore, the current through the inductance

$$i_L = \frac{e_0}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right) \\ = -\frac{e_0}{X_L} \cos \omega t$$

iii. The current through the capacitor leads the emf by $\frac{\pi}{2}$.

$$i_C = \frac{e_0}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{e_0}{X_C} \cos \omega t$$

where X_L, X_C are the inductive and capacitive reactances respectively.



iv. Then the total current in the circuit is

$$i = i_L + i_C \\ = -\frac{e_0}{X_L} \cos \omega t + \frac{e_0}{X_C} \cos \omega t \\ = \left(\frac{e_0}{X_C} - \frac{e_0}{X_L}\right) \cos \omega t$$

But, $x_C = 1/\omega c$ & $x_L = \omega L$

$$= e_0 \left(\omega C - \frac{1}{\omega L}\right) \cos \omega t$$

v. When $\omega C = \frac{1}{\omega L}$ the current in the circuit is zero. This is called a parallel resonance.

vi. **Expression for resonant frequency (f_r):**

The frequency at which the resonance takes place and the minimum current flows through the circuit is called the resonating frequency.

At resonance,

$$X_L = X_C$$

$$\omega C = \frac{1}{\omega L}$$

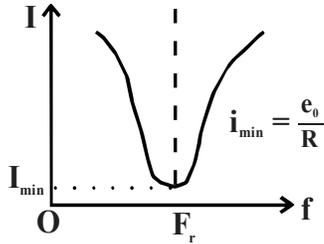
$$\omega^2 = \frac{1}{LC}$$

$$\therefore \omega = \frac{1}{\sqrt{LC}}$$

$$\omega = 2\pi f$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}$$

- vii. The variation of the current with frequency for a parallel resonance circuit is shown below.



- viii. This circuit called rejecter circuit because it rejects the current of the resonant frequency but allows the current of the other frequencies to pass through it.
- ix. This circuit use in wireless transmission or radio communication and filter circuits.

Q.31 State characteristics of series resonance circuit.

Ans:

- Resonance occurs when $X_L = X_C$
- Resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$
- Impedance is minimum and circuit is purely resistive.
- Current has a maximum value.
- When a number of frequencies are fed to it, it accepts only one frequency(f_r) and rejects the other frequencies. The current is maximum for this frequency. Hence it is called acceptor circuit.

Q.32 State the characteristics of parallel resonance

Ans:

- Resonance occurs when $X_L = X_C$.
- Resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$
- Impedance is maximum.
- Current is minimum.
- When alternating current of different frequencies are sent through parallel resonant circuit, it offers a very high impedance to the current of the resonant frequency (f_r) and rejects it. However, it allows the current of the other frequencies to pass through it, hence called a rejecter circuit.

Type - VI

Numerical based on Resonance

Formula used

$$f = \frac{1}{2\pi\sqrt{LC}}$$

- ★ 1) Calculate the value of capacity in picofarad, which will make 101.4 micro henry inductance to oscillate with frequency of one megahertz.

Data: $L = 101.4\mu\text{H} = 101.4 \times 10^{-6}\text{H}$;
 $f = 1\text{MHz} = 10^6\text{Hz}$

To find: Capacitance (C)

Formula: $f = \frac{1}{2\pi\sqrt{LC}}$

Solution: $f^2 = \frac{1}{4\pi^2 LC}$

$\therefore C = \frac{1}{4\pi^2 L f^2}$

$$= \frac{1}{4 \times 3.142 \times 3.142 \times 101.4 \times 10^{-6} \times 10^{12}}$$

$$= \frac{10^6}{4 \times (3.142)^2 \times 101.4} \times 10^{-12}$$

$$= 249.8 \times 10^{-12} \text{ F}$$

$$= 249.8 \text{ pF}$$

Ans: The capacitance is 249.8 pF.

- ★ 2) A 100 mH inductor, a 25μF capacitor and a 15 Ω resistor are connected in series to a 120 V, 50Hz AC source. Calculate

- impedance of the circuit at resonance
- current at resonance
- Resonant frequency

Data: $L = 100\text{mH} = 100 \times 10^{-3} \text{ H} = 10^{-1}\text{H}$
 $C = 25 \mu\text{F} = 25 \times 10^{-6}\text{F}$, $R = 15\Omega$
 $e_{\text{rms}} = 120\text{V}$, $f = 50 \text{ Hz}$

To find: i. (Z) ii. i_{rms}
iii. Resonant frequency (f_r)

Formula: i. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

ii. $i_{\text{rms}} = \frac{e_{\text{rms}}}{Z}$ iii. $f = \frac{1}{2\pi\sqrt{LC}}$

Solution: At resonance,

$X_L = X_C$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = R = 15 \Omega$

$i_{\text{rms}} = \frac{e_{\text{rms}}}{Z} = \frac{120}{15} = 8A$

$f = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2 \times 3.142 \times \sqrt{10^{-1} \times 25 \times 10^{-6}}}$

$= \frac{\sqrt{10}}{5 \times 2 \times 3.142 \times 10^{-3}}$

$= \frac{3.162}{3.142} \times 10^2$

$= 1.006 \times 10^2 \text{ Hz} = 100.6 \text{ Hz}$

Ans : i. Impedance of the circuit at resonance is 15Ω
 ii. The current at resonance is $8A$.
 iii. The resonant frequency is 100.6Hz .

13.9 Sharpness of Resonance:Q - factor

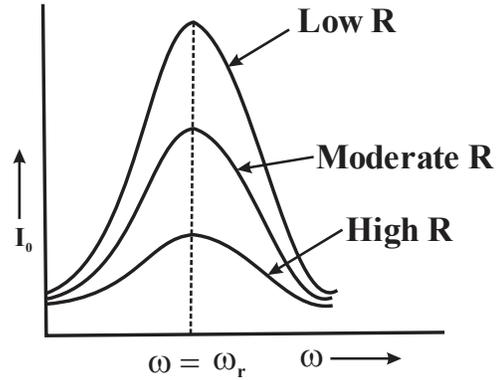
Q.33 What do you mean by sharpness of resonance in a series resonant circuit? (Q factor)

Ans:

i. Sharpness of resonance : Q-Factor. figure shows the variation of current amplitude I_0 in a series LCR-circuit with angular frequency ω , for three different values of R. The current amplitude has a peak at the

resonant frequency $\omega_r = \frac{1}{\sqrt{LC}}$ and falls to

zero in either direction. The resonant frequency is independent of R, but the sharpness of peak depends on R. The peak is higher for smaller values of R. Thus the resonance is sharp for small R and a flat one for large R. The sharpness of resonance is measured by a coefficient called the quality or Q-factor of the circuit.



ii. The Q-factor of a series resonant circuit is defined as the ratio of the resonant frequency to the difference in two frequencies taken on both sides of the resonant frequency such that at each frequency, the current amplitude becomes

$\frac{1}{\sqrt{2}}$ times the value at resonant frequency.

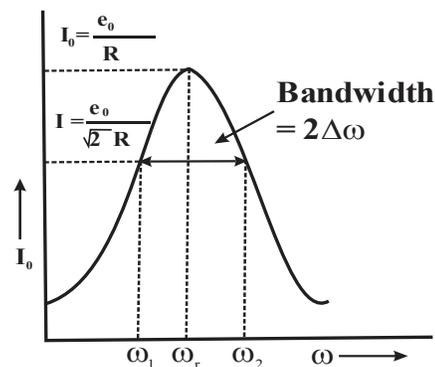
iii. Mathematically, the Q-factor can be expressed as

$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{2\Delta\omega} = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$

Where ω_1 and ω_2 are the frequencies at

which the current falls to $\frac{1}{\sqrt{2}}$ times its resonant value, as shown in figure

$\omega_1 = \omega_r - \Delta\omega ; \omega_2 = \omega_r + \Delta\omega$



iv. The frequency range $\omega_2 - \omega_1 = 2\Delta\omega$ is called bandwidth. The larger the value Q-factor, the smaller is the value of $2\Delta\omega$ or the bandwidth and sharper is the peak in the current. Q-factor is a dimensionless quantity.

13.10 Choke Coil

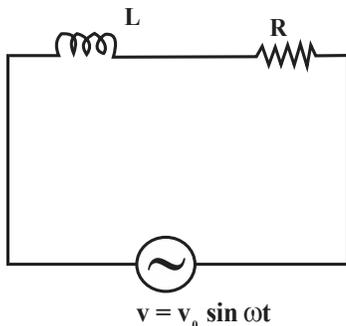
Q.34 What is a choke coil? Explain its action in a.c. circuits.

Ans: **Choke Coil.** A choke coil is simply an inductor with large inductance which is used to reduce current in a.c. circuits without much loss of energy.

Principle. The working of a choke is based on the fact that when a.c. flows through an inductor, current lags behind the emf by a phase angle of $\pi/2$ rad.

Construction. It is made of thick insulated copper wire wound closely in a large number of turns over a soft-iron laminated core. Choke coil offers a large reactance $X_L = 2\pi fL$ to the flow of a.c. and hence current is reduced. Laminated core reduces losses due to eddy currents.

Working. As shown in figure, a choke is put in series across an electrical appliance of resistance R and is connected to an a.c. source. This forms an LR-circuit.



Average power dissipated per cycle in the circuit is

$$P_{av} = V_{eff} I_{eff} \cos \phi = V_{eff} I_{eff} \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

Inductance L of the choke coil is very large so that $R \ll \omega L$. Then

$$\text{Power factor, } \cos \phi \approx \frac{R}{\omega L} \approx 0$$

i.e., Average power dissipated by the coil is very small. As $Z = \sqrt{R^2 + \omega^2 L^2}$ is large, so current is reduced without appreciable wastage of power.

MULTIPLE CHOICE QUESTIONS

Entrance Corner (Set 4)

- In an LCR-circuit, capacitance is changed from C to $2C$. For the resonant frequency to remain un-changed, the inductance should be changed from L to
 - $4L$
 - $2L$
 - $L/2$
 - $L/4$
- The self inductance of the motor of an electric fan is $10H$. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of
 - $4\mu F$
 - $8\mu F$
 - $1\mu F$
 - $2\mu F$
- In a series LCR circuit $R = 200\Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30° . On taking out the inductor from the circuit the current leads the voltage by 30° . The power dissipated in the LCR circuit is
 - 242 W
 - 305 W
 - 210 W
 - 0 W
- An inductor L and a capacitor C are connected in the circuit as shown in the figure. The frequency of the power supply is equal to the resonant frequency of the circuit. Which ammeter will read zero amperes?

$E = E_0 \sin \omega t$

 - A_1
 - A_2
 - A_3
 - None of these
- What is the value of inductance L for which the current is maximum in a series LCR-circuit with $C = 100\mu F$ and $\omega = 1000$ s⁻¹?
 - 100 mH
 - 1 mH
 - 10 mH
 - cannot be calculated unless R is known.

Try Yourself

6. The power delivered by the source circuit is maximum when
 a. $\omega L = \omega C$ b. $\omega L = 1/\omega C$
 c. $\omega L = \omega C^2$ d. $\omega L = \sqrt{\omega C}$
7. The square root of the product of inductance and capacitance has the dimension of
 a. length b. mass
 c. time d. no dimension
8. For a series, LCR-circuit, the power loss at resonance is
 a. $\frac{V^2}{\omega L - 1/\omega C}$ b. $\frac{V^2}{\omega L + 1/\omega C}$
 c. $I^2 \omega C$ d. $I^2 R$
9. The average emf during the positive half cycle of an AC supply of peak value E_0 is
 a. $\frac{E_0}{\pi}$ b. $\frac{E_0}{\sqrt{2}\pi}$
 c. $\frac{E_0}{2\pi}$ d. $\frac{2E_0}{\pi}$

AnswerKey

Set - 1 (MCQ)

1	c	2	d	3	d	4	d	5	a
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Try Yourself

6	c	7	b	8	b	9	d	10	c
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Set - 2 (MCQ)

1	c	2	d	3	c	4	c	5	a
6	b	7	a	8	a				

Try Yourself

9	b	10	c	11	c	12	d	13	d
14	b	15	a						

Set - 3 (MCQ)

1	b	2	c	3	b	4	d	5	a
6	c	7	a	8	b	9	c	10	a

Try Yourself

11	a	12	c	13	c	14	b	15	d
16	d	17	b	18	b	19	c	20	a
21	a	22	c						

Set - 4 (MCQ)

1	c	2	c	3	a	4	c	5	a
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Try Yourself

6	b	7	c	8	d	9	d
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