15 Structure of Atoms and Nuclei

Syllabus

15.1 Introduction

Note :

i. *Greek philosophers Leucippus (-370 BC) and Democritus (460 – 370 BC) were the first scientists to propose, in the 5th century BC,that matter is made of indivisible parts called atoms.*

ii. *Dalton (1766-1844) gave his atomic theory in early nineteenth century. According to his theory (i) matter is made up of indestructible particles, (ii) atoms of a given element are identical and (iii) atoms can combine with other atoms to form new substances.*

iii. *That atoms were indestructible was shown to be wrong by the experiments of J. J. Thomson (1856-1940) who discovered electrons in 1887.*

iv. *He then proceeded to give his atomic model which had some deficiencies and was later improved upon by Ernest Rutherford (1871- 1937) and Niels Bohr (1885-1962).*

v. *an atom contains a tiny nucleus whose size (radius) is about 100000 times smaller than the size of an atom.*

vi. *The nucleus contains all the positive charge of the atom and also 99.9% of its mass.*

15.2 Thomson's Atomic Model

Q.1 Describe Thomson's model of an atom. Why was this model discarded later on?

Ans :

i. In 1898, J.J Thomson proposed that an atom is a sphere of positively charged matter with electrons embedded in it.

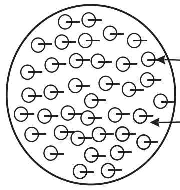
ii. The positive charge is uniformly distributed over the entire atom.

iii. The arrangement of electrons inside the continuous positive charge is similar to that of the seeds in a watermelon or the plums in a pudding. That is way Thomson's atomic model is also known as **plum pudding model**.

iv. The electrons are arranged in such a manner that their mutual repulsions are balanced by their attractions with the positively charged matter. Thus the atom as a whole is stable and neutral.

v. Thomson's model was able to explain with some success the processes like chemical reaction and radioactive disintegration.

vi. To explain the observed spectra of elements, Thomson assumed that slight perturbations of atoms cause vibrations of the electrons about their equilibrium positions.



vii. These vibrating electrons emit electromagnetic radiations of their own frequency of oscillations.

viii. **Failure of Thomson's model** Thomson model remained popular till about 1911 and was discarded later on due to the following drawbacks :

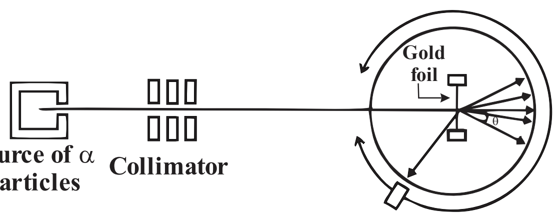
a. It could not explain the origin of several spectral series in the case of hydrogen and other atoms

b. It failed to explain the large angle scattering of α -particles in Rutherford's experiment.

15.3 Geiger - Marsden Experiment

Q.2 Explain Geiger-Marsden Experiment.

Ans:



Arrangement :

i. The experimental set up used by Geiger and Marsden is shown in fig

ii. A narrow beam of α - particles from radioactive source was incident on a gold foil with the help of lead bricks.

iii. When these α - particle passes through gold foil few of them get scattered.

iv. The scattered detector fixed on rotating stand.

v. Detector used had zinc sulphide screen and microscope.

vi. The whole setup is enclosed in evacuated chamber.

vii. The deviation of α-particles (θ) from its original direction is called scattering angle.

Observations :

i. Most of the α-particles passed undeviated.

ii. Only few (0.14%) scattered by more than 1^0.

iii. Very few (1 in 8000) were deflected by more than 90^0.

iv. Some particles even bounced back in 180^0.

Importance :

On the basis of these observations, Rutherford launched his atomic model.

15.4 Rutherford's Atomic Model

Q.3 Explain the Rutherford's model of an atom?

Ans: Postulates :

i. The atom has tiny positively charged core called nucleus.

ii. The total positive charge and entire mass (99.9%) of atom is confined in nucleus.

iii. The nucleus is surrounded by negatively charged electrons. The orbiting electrons round the nucleus in circular orbits similar to planets revolving round the sun.

iv. As an atom is electrically neutral, the positive charge on nucleus is equal to the total negative charge of all the orbiting electrons

v. As the size of nucleus is 10^{-15} m, about 100000 times smaller than the size of atom. Thus atom mostly consists of empty space.

Merit :

It explain the existence of nucleus inside an atoms and motion of electrons around the nucleus.

Demerit :

i. It could not explain stability of atomic model.

ii. It could not explain spectral lines of H-atom.

Q.4 Explain the drawbacks of Rutherford model of atom

Ans: Drawbacks of Rutherford model of atom:

i. **Stability of atomic model:**

a. The circular motion of electrons is accelerated motion, which radiates energy according to classical theory.

b. The energy of accelerated electron continuously decrease and follow inward spiral path of decreasing radius and finally fall into the nucleus. i.e. no stable atom could exist.

ii. Spectral lines of H-atom :

a. The frequency of electromagnetic wave emitted by the revolving electron is equal to the frequency of revolutions.

b. If electron spiral inward, their angular velocity and hence frequency; would increase continuously. Thus, they would emit energy with continuously increasing frequency. Hence, emit continuous spectra.

c. Experimentally atom has very stable structure and emit line spectra

15.5 Atomic Spectra

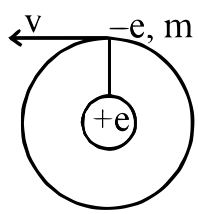
15.6 Bohr's Atomic Model

Q.5 State and explain the postulates of the Bohr's theory of the hydrogen atom.

Ans: Niels Bohr proposed his atomic model in 1913. based on following postulates

i. Postulate - 1: For circular orbits :

a. In hydrogen atom, an electron revolves in circular orbits round the nucleus.



b. The centripetal force for the circular motion is provided by the electrostatic force of attraction between the positively charged nucleus and the negatively charged electron.

$$\begin{pmatrix}\text{Centripetal}\\ \text{force}\end{pmatrix} = \begin{pmatrix}\text{Coulomb's force}\\ \text{of attraction}\end{pmatrix}$$

$$\therefore \quad \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \quad \text{... (1)}$$

where,

m = mass of an electron

v = velocity of an electron

r = radius of circular orbit of an electron

e = magnitude of electronic charge

ε_0 = permittivity of free space.

ii. Postulate - 2: For selected orbits:

a. An electron revolves without radiating energy only in those circular orbits in which its angular momentum is equal to the integral multiples of $\frac{h}{2\pi}$.

b. Angular momentum = $I\omega$ = mvr

Where,

c. According to postulate

Angular momentum = $n\frac{h}{2\pi}$

$\therefore$ mvr = $n\frac{h}{2\pi}$

where,

h - Planck's constant.

I - M.I. of electron

ω - angular velocity

n = principal quantum no. = 1, 2, 3...

i.e. electrons revolve in only those orbits that satisfies the above equation.

iii. Postulate - 3: Non radiating orbit:

a. When electron jumps from orbit of higher energy to an orbit of lower energy, it radiates energy in the form of quanta or photons.

Energy of emitted photon = h ν

b. The energy of emitted photon is equal to the difference between energies of two orbit in which transition is taking place.

Energy radiated = $E_m - E_n$

where,

E_m - energy of electron in m[th] orbit.

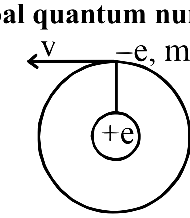
E_n - energy of electron in n[th] orbit.

$\therefore$ $h\nu = E_n - E_p$

i.e. an electron radiate energy when it jumps from higher orbit to lower orbit.

Q.6 On the basis of the Bohr's theory, derive an expression for the radius of the Bohr orbit. Show that the radius of a Bohr orbit is directly proportional to the square of the principal quantum number.

Ans :



i. Consider an electron revolving around the nucleus of a hydrogen atom in the n^{th} Bohr orbit.

ii. Let,

m = mass of an electron

v = velocity of an electron

r = radius of circular orbit of an electron

iii. According to the Bohr's first postulate

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

$$\therefore \quad v^2 = \frac{Ze^2}{4\pi\varepsilon_0 mr} \quad \text{... (1)}$$

iv. According to the 2^{nd} postulate

$$mvr = \frac{nh}{2\pi},$$

$$\therefore \quad v = \frac{nh}{2\pi mr}$$

$$\therefore \quad v^2 = \frac{n^2h^2}{4\pi^2m^2r^2} \quad \text{... (2)}$$

v. From equations (1) and (2)

$$\frac{n^2h^2}{4\pi^2m^2r^2} = \frac{Ze^2}{4\pi\varepsilon_0 mr}$$

$$\therefore \quad \frac{n^2h^2}{\pi mr} = \frac{Ze^2}{\varepsilon_0}$$

$$\therefore \quad r = \frac{\varepsilon_0 n^2h^2}{\pi mZe^2} \quad \text{....(3)}$$

This is the expression for the radius of the n^{th} Bohr orbit.

vi. Since, ε_0, h, m, e and π are constant ,

$\therefore \quad r \propto n^2$ ....(4)

Thus, the radius of a Bohr orbit is directly proportional to the square of the principal quantum number.

Type - I
Mumerical based on radius of Bohr orbit

Formulae used

1. $r_1 = \left(\frac{h^2\varepsilon_0}{\pi me^2}\right)$

2. $r_n \alpha n^2 \qquad r_n = 0.53\ n^2$

★ 1) Calculate the radius of the 3^{rd} orbit of the electron in hydrogen atom.

Data : n = 3,

To find : r_3

Formula : $r_n = 0.53n^2$

Solution : $r_n = 0.53n^2$

$= 9 \times 0.053$ nm = 0.477

Ans :	Radius of 3^{rd} orbit is 0.477 nm

2) Calculate the radius of the first orbit hence find radius of 2^{nd} Bohr orbit

Data : $m = 9 \times 10^{-31}$kg,

$e = 1.6 \times 10^{-19}$ C,

$h = 6.63 \times 10^{-34}$ Js,

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

To Find : i. r_1 ii. r_3

Formula: $r_1 = \left(\frac{h^2\varepsilon_0}{\pi me^2}\right)$

Solution :

i. For first orbit, n = 1

$$r_1 = \left(\frac{h^2\varepsilon_0}{\pi me^2}\right)$$

$$= \frac{8.85\times10^{-12}\times(6.63\times10^{-34})^2}{3.142\times9\times10^{-31}\times(1.6\times10^{-19})^2}$$

$= 5.374 \times 10^{-11}$ m

$= 0.5374$ Å

ii. $r \alpha n^2$

$$\frac{r_2}{r_1} = \left(\frac{2}{1}\right)^2$$

$\therefore \quad r_2 = 4 \times r_1 = 4 \times 0.5374$

$\therefore \quad r_2 = 2.1496$ Å

Ans :	Radius of 1^{st} Bohr orbit is 0.5374 Å and 2^{nd} Bohr orbit is 2.1496 Å

Problem for Practice

1. The radius of the first Bohr orbit of the hydrogen atom is 0.53 Å. Calculate the radius of the tenth orbit.

Ans : 53A⁰

2. The diameter of the innermost orbit of hydrogen is 1.06 Å. What is the diameter of the

10th orbit?

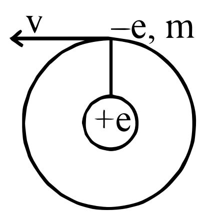
Ans : 106Å

3. Write down the expression for the radii of orbits of hydrogen atom. Calculate the radius of the smallest orbit

Ans : 0.53Å

Q.7 Show that angular speed of electron in n^{th} Bohr orbit equal to $\omega = \frac{\pi me^4}{2\varepsilon_0^2 h^3 n^3}$ or frequency of revolution, $f = \frac{me^4}{4\varepsilon_0^3 h^3 n^3}$

Ans:



i. Consider an electron circulating around the nucleus of a hydrogen atom in the n^{th} Bohr orbit.

ii. Let,

m = mass of an electron

v = velocity of an electron

r = radius of circular orbit of an electron

iii. According to the Bohr's first postulate

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$\therefore \quad mv^2r = \frac{e^2}{4\pi\varepsilon_0} \quad ....(1)$$

iv. According to the second postulate

$$mvr = \frac{nh}{2\pi}, \quad ....(2)$$

v. Dividing eq (1) by (2)

$$v = \frac{e^2}{2\varepsilon_0 hn} \quad ....(3)$$

vi. The radius of the n^{th} Bohr orbit.

$$r = \frac{\varepsilon_0 n^2 h^2}{\pi me^2} \quad ....(4)$$

vii. The angular velocity of electron is,

$$\omega = \frac{v}{r}$$

$$= \frac{e^2}{2\varepsilon_0 hn} \times \frac{\pi me^2}{\varepsilon_0 n^2 h^2}$$

$$\therefore \quad \omega = \frac{\pi me^4}{2\varepsilon_0^2 h^3 n^3} \quad ....(5)$$

viii. The frequency of revolution of a electron is,

$$f = \frac{\omega}{2\pi} \quad .... \text{ from eq}^n \ (5)$$

$$= \frac{\pi me^4}{2\varepsilon_0^2 h^3 n^3} \times \frac{1}{2\pi}$$

$$\therefore \quad f = \left(\frac{me^4}{4\varepsilon_0^2 h^3 n^3}\right) \quad ...(7)$$

Note :

i. Angular velocity is given by,

$$\omega = \frac{\pi me^4}{2\varepsilon_0^2 h^3 n^3}$$

Since, ε_0, h, m, e and π are constant

$$\therefore \quad \omega \propto \frac{1}{n^3}$$

i.e. Angular velocity of electron in n^{th} orbit is inversely proportional to cube of principal quantum number.

ii. Frequency of resolution is given by

$$\therefore \quad f = \left(\frac{me^4}{4\varepsilon_0^2 h^3 n^3}\right)$$

Since, ε_0, h, m, e and π are constant,

$$\therefore \quad f \propto \frac{1}{n^3}$$

i.e. frequency of revolution of electron in n^{th} orbit is inversely proportional to cube of principal quantum number.

For period of revolution (T):

$$T = \frac{2\pi}{\omega}$$

Substituting value of ω

$$\therefore \quad T = \left(\frac{4\varepsilon_0^2 h^3 n^3}{me^4}\right) \quad ...(8)$$

Since, ε_0, h, m, e and π are constant,

$$\therefore \quad T \propto n^3$$

i.e. period of revolution of electron in n^{th} orbit is proportional to cube of principal quantum number.

iv. **For linear velocity (v):**

According to 2^{nd} postulate

$$v = \frac{nh}{2\pi mr}$$

Substituting r in above equation

$$v = \frac{e^2}{2\varepsilon_0 hn}$$

Since, ε_0, h, e are constant ,

$\therefore$ $v \propto \frac{1}{n}$

i.e. Linear velocity of electron in n^{th} orbit is inversely proportional to principal quantum number.

v. **For linear momentum (p):**

p = mv

Substitititing value of v

$$p = \left(\frac{me^2}{2\varepsilon_0 hn}\right) \quad ....(6)$$

Since, ε_0, h, e are constant ,

$\therefore$ $p \propto \frac{1}{n}$

i.e. Linear momentum of electron in n^{th} orbit is inversely proportional to principal quantum number.

vi. **For centripetal acceleration (a_c):**

$$a_c = \frac{v^2}{r}$$

Substituting value of v and r.

$$\therefore \quad a_c = \left(\frac{\pi me^6}{4\varepsilon_0^3 h^4 n^4}\right) \quad ...(9)$$

Since, ε_0, h, m, e and π are constant

$\therefore$ $a_c \propto \frac{1}{n^4}$

i.e. cen tripetal acceleration of electron in n^{th} orbit is inversely proportional to fourth power of principal quantum number.

Q.8 Derive an expression for the total energy of the electron in n^{th} Bohr orbit. Show that the energy of an electron in a Bohr orbit is inversely proportional to the square of the principal quantum number of that orbit.

OR

Derive the expression for the energy of electron in the atom.

Ans:

i. Consider an electron orbiting around the nucleus of an atom.

Let,

m = mass of an electron

v = velocity of an electron

r = radius of circular orbit of an electron

ii. The electron possesses kinetic energy due to its motion, and potential energy due to an electric field of the nucleus.

iii. **Kinetic energy :** According to Bohr's first postulate,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

$$\therefore \quad \frac{1}{2}mv^2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$$

$$\therefore \quad \text{K.E. of the electron} = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

iv. **Potential energy :** Potential at a point in the orbit due to the nucleus of charge + e is ,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{e}{r}$$

Therefore, the potential energy of the electron having charge – e is given by

P.E. = V × (– Ze)

$$= \frac{1}{4\pi\varepsilon_0}\frac{e\times(-Ze)}{r}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$$

v. **Total energy :**

Total energy of the electron is given by

E = K.E. + P.E.

$$= \frac{Ze^2}{8\pi\varepsilon_0 r} - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$= -\frac{Ze^2}{8\pi\varepsilon_0 r}$$

But, $r = \frac{\varepsilon_0 n^2 h^2}{\pi me^2 Z}$

$$E = -\frac{e^2 Z}{8\pi\varepsilon_0} \times \frac{\pi me^2 Z}{\varepsilon_0 n^2 h^2}$$

$$E = -\frac{me^4 Z^2}{8\varepsilon_0^2 n^2 h^2} = -13.6\frac{Z^2}{n^2}eV$$

This expression gives the energy of the

electron in the n^{th} Bohr orbit.

vi. Since, m, e, ε_0, h are constant,

$\therefore \quad E_n \propto \frac{Z^2}{n^2}$

Thus, the energy of an electron in the Bohr's orbit is inversely proportional to the square of the principal quantum number of that orbit.

Note:

i. **Energy in n^{th} orbit:**

$$E_n = -\frac{me^4 Z^2}{8\varepsilon_0^2 n^2 h^2}$$

As $m = 9 \times 10^{-31}$ kg,

$e = 1.6 \times 10^{-19}$ C,

$h = 6.63 \times 10^{-34}$ Js

$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2

i.e. $E_n = \frac{-13.6}{n^2}$ ...... in eV

ii. **For different orbits :**

a. For 1^{st} orbit i.e. n = 1

$\therefore \quad E_1 = -13.6$ eV

b. For 2^{nd} orbit i.e. n = 2

$\therefore \quad E_2 = \frac{-13.6}{4} = -3.4$ eV

c. For 3^{th} orbit i.e. n = 3

$\therefore \quad E_3 = \frac{-13.6}{9} = -1.5$ eV

iii. **Relation between P.E, K.E & T. E**

P. E. = – 2 K.E = 2 T. E

iv. **Binding energy (B.E):**

The binding energy of electron is the minimum energy required to make it free from the nucleus. B. E. of electron is,

B.E = – Total energy

$$= -\left(-\frac{me^4}{8\varepsilon_0^2 n^2 h^2}\right)$$

Since, m, e, ε_0, h are constant,

$\therefore \quad B.E \propto \frac{1}{n^2}$

Thus, binding energy of an electron in the Bohr's orbit is inversely proportional to the square of the principal quantum number of that orbit.

Binding energy is maximum for innermost orbit and minimum for outermost orbit.

v. **Ionisation:**

If external energy of 13.6 eV or greater is given to H-atom, electron become free. This process is called ionization

vi. **Ionisation energy (Ei):**

The minimum energy needed to ionise an atom or an electron in the ground state of H-atom is called as ionisation energy.

vii. **Ionisation potential of atom (V_i):**

The potential difference through which an electron should be accelerated to acquire ionisation energy is called ionisation potential.

$$V_i = \frac{Ei}{e}$$

$$= \frac{13.6}{n^2} \text{ ...in volts}$$

★ **Q.9 Define :**

i. excitation energy

ii. binding energy and

iii. ionization energy of an electron in an atom.

Ans :

i. The energy required to take an electron from the ground state to an excited state is called the excitation energy of the electron in that state.

ii. Binding energy of an electron is the minimum energy required to make it free from the nucleus.

iii. The ionization energy of an atom is the minimum amount of energy required to be given to an electron in the ground state of that atom to set the electron free.

Q.10 Explain the origin of the spectral lines on the basis of the Bohr's theory. Derive an expression for the wavelengths of the spectral lines in the hydrogen atom.

Ans: Origin of spectral lines :

i. In a stable hydrogen atom, an electron resides in innermost (n=1), is called ground state.

ii. The ground state, energy of an electron is –

13.6 eV.

iii. If the atom is subjected to external energy less than 13.6eV, the electron jumps to higher orbit. It is then said to be in an excited state.

iv. In the excited state the atom is unstable for 10^{-8} sec and jumps back to ground state in single step or intermediate state level.

v. During each such transition, difference between energies of two states is radiated in the form of photon of particular wavelength.

vi. Let, the electron jump from the m^{th} higher orbit to the n^{th} lower orbit.

Energy of an electron in respective orbits is,

$$E_n = \frac{-me^4}{8\varepsilon_0^2 m^2 h^2} \text{ and}$$

$$E_n = \frac{-me^4}{8\varepsilon_0^2 n^2 h^2}$$

vii. According to the Bohr's third postulate,

$$E_m - E_n = h\nu \quad ....(1)$$

where,

E_n – energy of an electron in the n^{th} orbit

E_p – energy of an electron in the p^{th} orbit.

$$h\nu = -\frac{me^4 Z^2}{8\varepsilon_0^2 n^2 h^2} + \frac{me^4 Z^2}{8\varepsilon_0^2 p^2 h^2}$$

$$= \frac{me^4 Z^2}{8\varepsilon_0^2 h^2}\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

But $\nu = \frac{c}{\lambda}$

Where, c – velocity of light

λ – wavelength of the radiation

$$\therefore \frac{hc}{\lambda} = \frac{me^4 Z^2}{8\varepsilon_0^2 h^2}\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = \frac{me^4 Z^2}{8\varepsilon_0^2 ch^3}\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

Where,

$$\frac{me^4}{8\varepsilon_0^2 ch^3} = R = \text{Rydberg's constant.}$$

$$R = 1.093 \times 10^7 / m$$

vii. Wavelength of radiation is, $\frac{1}{\lambda} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$

This is called Bohr's formula.

Remark :

Wave number ($\bar{\nu}$):

The number wave in unit distance is,

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

Q.11 Draw the energy level diagram for the H-atom. Mention different series in it. Hence explain the different series in the hydrogen atom.

Ans:

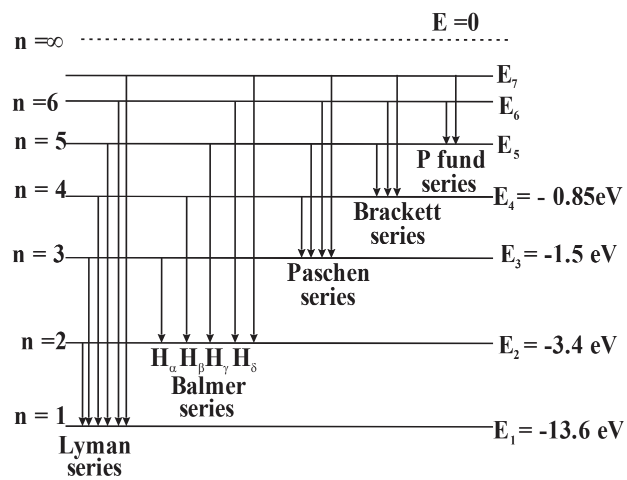
i. The energy level diagram shows the different series of lines observed in a spectrum of the hydrogen atom.

ii. The horizontal lines represent the different energy levels for different orbit.

iii The orbit numbers are given at the left and the corresponding energies in eV are given at the right.

iv The vertical arrow lines denote various electronic transitions .

v. Each transition corresponds to a definite characteristic wavelength. The transitions of the electrons in excited hydrogen gas gives different series of lines, as shown in the figure.



Series of spectral lines:

A. Lyman series :

i. This series arises due to transitions of the electrons from different outer orbit to the first Bohr orbit (n = 1).

ii. The wavelength for Lyman series is ,

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{m^2}\right)$$

where, m = 2, 3, 4, ...

iii. This series lines in the ultraviolet region.

B. Balmer series :

i. This series arises due to transitions of the electrons from different outer orbit to the second Bohr orbit (p = 2).

ii. The wavelength for Lyman series is ,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{m^2}\right),$$

where, m = 3, 4, 5, ...

iii. This series lines in the visible region.

C. Paschen series :

i. This series arises due to transitions of the electrons from different outer orbit to the third Bohr orbit (n = 3).

ii. The wavelength for Lyman series is ,

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{m^2}\right),$$

where, m = 4, 5, 6, ...

iii. This series lines in the infrared region.

D. Brackett series :

i. This series arises due to transitions of the electrons from different outer orbit to the fourth Bohr orbit (n = 4).

ii. The wavelength for Lyman series is ,

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{m^2}\right),$$

where, m = 5, 6, 7, ...

iii. This series lines in the near-infrared region.

E. Pfund series :

i. This series arises due to transitions of the electrons from different outer orbit to the fifth Bohr orbit (n = 5).

ii. The wavelength for Lyman series is ,

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{m^2}\right),$$

where, m = 6, 7, 8, ...

iii. This series lines in the far-infrared egion.

Note :

i. The smallest wavelength emitted in a series is called series limit

ii. The series limit for particular series is found by taking n = ∞ in Bohr's relation.

iii. Wavelength for different radiation

1	Gamma rays	6×10^{-19} m to 1×10^{-11}m
2	X-rays	1×10^{-11} m to 3×10^{-8}m
3	Ultraviolet	6×10^{-10} m to 4×10^{-7}m
4	Visible light	4×10^{-7} m to 8×10^{-7}m
5	Infra red	8×10^{-7} m to 3×10^{-5}m

★ **Q.12 Show that the frequency of the first line in Lyman series is equal to the difference between the limiting frequencies of Lyman and Balmer series.**

Ans:

i. $$\gamma = R_H Z^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

For first line in Lyman series,
n = 1, m = 2

$$\therefore \quad \bar{\gamma}_1 = R_H Z^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\bar{\gamma}_1 = \frac{3}{4} R_H Z^2 \qquad \text{... (1)}$$

ii. For limiting line of Lyman series ,
n = 1 , m = ∞

$$\therefore \quad (\gamma_\infty)_L = R_H Z^2\left(\frac{1}{1^2} - \frac{1}{\infty}\right)$$

$$= R_H Z^2 \qquad \text{... (2)}$$

iii. For limiting frequency of Balmer series
n = 1 , m = ∞

$$\therefore \quad (\gamma_\infty)_B = R_H Z^2\left(\frac{1}{2^2} - \frac{1}{\infty}\right)$$

$$\therefore \quad (v_\infty)_B = \frac{R_H Z^2}{4} \qquad \text{...(3)}$$

iv. Difference between two limiting frequencies,

$$(\gamma_1)_L - (\gamma_\infty)_B = R_H Z^2 - \frac{R_H Z^2}{4}$$

$$= \frac{3}{4} R_H Z^2$$

$$= \gamma_1$$

Type - II
Numerical based on speed of electron in Bhor orbit

Formulae Used

1. $r = \frac{\varepsilon_o h^2 n^2}{\pi m e^2 z}$ $\therefore$ $r \alpha n^2$

2. $v = \frac{e^2 z}{2\varepsilon_o hn}$ $\therefore$ $v \alpha \frac{1}{n}$

3. $\omega = \frac{v}{r} = \frac{\pi m e^4 z^2}{2\varepsilon_o^2 n^3 n^3}$ $\therefore$ $\omega \alpha \frac{1}{n^3}$

4. $f = \frac{\omega}{2\pi} = \frac{m e^4 z^2}{4\varepsilon_o^2 h^3 n^3}$ $\therefore$ $f \alpha \frac{1}{n^3}$

5. $T = \frac{1}{f} = \frac{4\varepsilon_o^2 h^3 n^3}{m e^4 z^2}$ $\therefore$ $T \alpha n^3$

6. $a = \frac{v^2}{r} = \frac{\pi m e^6 z^3}{4\varepsilon_o^3 h^3 n^3}$ $\therefore$ $a \alpha \frac{1}{n^3}$

1) Calculate the linear velocity of an electron in the first and third Bohr orbits of the hydrogen atom.

Data : $r = 0.53 \times 10^{-10}$ m,
$m = 9.1 \times 10^{-31}$ kg,
$h = 6.63 \times 10^{-34}$ Js

To Find : i. v_1 ii. v_3

Formula : i. For Hz atom = $V = \frac{e^2 z}{2\varepsilon_0 nh}$

ii. $v \alpha \frac{1}{n}$

Solution :

i. $V = \frac{e^2 z}{2\varepsilon_0 nh}$

$$= \frac{(1.6\times10^{-19})^2}{2\times8.85\times10^{-12}\times1\times6.63\times10^{-34}}$$

$$= \frac{(1.6)^2\times10^{-38+12+34}}{2\times6.63\times8.85}$$

$$= \frac{(1.6)^2\times10^{8}}{2\times6.63\times8.85}$$

$$= \frac{(1.6)^2\times100\times10^{6}}{2\times6.63\times8.85}$$

= Atlog

log N		log N	
log 1.6	0.2041	log 2	0.3010
log 1.6	+0.2041	log 6.63	+0.8215
log 100	2.0000	log 8.85	0.9469
	2.4082		2.0694

$\times 10^6$

$$= \text{At}\log\begin{bmatrix} 2.4082 \\ -2.0694 \\ \hline 0.3388 \end{bmatrix}\times10^6$$

$= 2.182 \times 10^6$ m/s

ii. $v \alpha \frac{1}{n}$

$$\frac{v_3}{v_1} = \frac{n_1}{n_3} = \frac{1}{3}$$

$\therefore$ $v_3 = \frac{v_1}{3} = \frac{2.182\times10^6}{3}$

$= 0.7274 \times 10^6$ m/s

$v_3 = 7.274 \times 10^5$ m/s

Ans : The linear velocity of e^- in 1st orbit is 2.182×10^6 m/s
The linear velocity of e^- in 2nd orbit is 7.274×10^5 m/s

2) Find frequency of electron in 1st Bohr orbit of Hz atom.

Data : For hydrogen atom,
n = 1

To Find : f

Formulae : $f = \frac{me^4}{4\varepsilon_o^2 h^3 n^3}$

Solution: $f = \frac{me^4}{4\varepsilon_o^2 h^3 n^3}$

$$= \frac{9.1\times10^{-3}\times(1.6\times10^{-19})^4}{4\times(8.85\times10^{-12})^2\times(6.63\times10^{-34})^3\times(1)^3}$$

$$= \frac{9.1\times(1.6)^4}{4\times(8.85)^2\times(6.63)^3}\times10^{-31-76+24+102}$$

$$= \frac{9.1\times(1.6)^4}{4\times(8.85)^2\times(6.63)^3}\times10^{19}$$

$$= \frac{910000 \times (1.6)^4}{4 \times (8.85)^2 \times (6.63)^3} \times 10^{14}$$

= Atlog

log N	
log 9100	5.9590
4 log 1.6	+0.8164
	5.7754

log N	
log 4	0.6021
2 log 8.85	+1.8938
3 log 6.63	2.4645
	4.9604

$\times 10^{14}$

$$= \text{Atlog}\begin{bmatrix} 5.7754 \\ -4.9604 \\ \hline 0.8150 \end{bmatrix} \times 10^{14}$$

$f = 6.531 \times 10^{14}$ Hz

Ans : Frequency of electron in 1^{st} orbit of Hz atom is 6.531×10^{14} Hz

3) If period of revolution of electron in Bohr's 1st orbit is 1.52 × 10–16 a find period of revolution of electron in second Bohr orbit

Data : For 1^{st} orbit

$n_1 = 1$ and $T_1 = 1.52 \times 10^{-16}$s

For 2^{nd} orbit

$n_2 = 2$

To Find : T_2

Formula : $T \propto n^3$

Solution : $T \propto n^3$

$$\frac{T_2}{T_1} = \left(\frac{n_2}{n_1}\right)^3 = \left(\frac{2}{1}\right)^3 = \frac{8}{1}$$

$T_2 = 8 \times T_1$

$= 8 \times 1.52 \times 10^{-16}$ s

$T_2 = 12.16 \times 10^{-16}$ sec.

Ans : Time period of electron in second orbit is 12.16×10^{-16} second

Problem for Practice

1. Determine the speed of the electron in n = 3 orbit of He^+ ion.

Ans : 1.46×10^6 ms^{-1}

2. Find the speed of the electron in the second orbit in a hydrogen atom.

(**Given :** $h = 6.63 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$C, $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2)

Ans : 1.091×10^6 m/s

3. Show that hte speed of an electron in the innermost orbit of H-atom is 1/137 times the speed of light in vacuum.

Ans : $\frac{1}{137}$c

4. Calculate the frequency of revolution of an electron in the first orbit of Hydrogen atom if the radius of the first orbit is 0.5 A° and the velocity of electron in the first orbit is 2.24×10^6 m/s.

Ans : 7.1×10^{15} Hz

Type - III

Numerical based on angular momentum

Formulae used

1. According to Bohr's 2nd orbit

$$mvr = \frac{nh}{2\pi}$$

$$\therefore \quad L = \frac{nh}{2\pi}$$

1) Find the change in the angular momentum of the electron when it jumps from the fourth orbit to the first orbit in hydrogen atom.

Data : For 1^{st} orbit, n = 1

For 4^{th} orbit n = 4

To Find : ΔL

Formula : $L = \frac{nh}{2\pi}$

Solution :

i. For 1st orbit $L_1 = \frac{nh}{2\pi}$

ii. For 2nd orbit $L_4 = \frac{4h}{2\pi}$

iii. Change in angular momentom

$\Delta L = L_4 - L_1$

$$= \frac{4h}{2\pi} - \frac{h}{2\pi} = \frac{3h}{2\pi}$$

$$\Delta L = \frac{3 \times 6.63 \times 10^{-34}}{2 \times 3.14}$$

$$= \text{Atlog}\left[\begin{array}{cc|cc} \multicolumn{2}{c|}{\log N} & \multicolumn{2}{c}{\log D} \\ \log 3 & 0.4771 & \log 2 & 0.3010 \\ \log 6.63 & +0.8215 & \log 3.14 & +0.4969 \\ & 1.2986 & & 0.7979 \end{array}\right] \times 10^{-34}$$

$$= \text{Atlog}\left[\begin{array}{r} 1.2986 \\ -\ 0.7979 \\ \hline 0.5007 \end{array}\right] \times 10^{-34}$$

$= 3.167 \times 10^{-34}$ kg m²/s

Ans : The change in angular momentum is 3.167×10^{-34} kg m^2/s

Problem for Practice

1. What is the angular momentum of an electron in the third orbit of an atom?
   Ans : 3.15×10^{-34} Js

2. Calculate the angular momentum of the electron in the second orbit in the hydrogen atom. (**Given :** h= 6.63×10^{-34} Js)
   Ans : 2.111×10^{-34} kgm²/S

Type - IV
Numerical based energy of electron in Bohr orbit

Formulae used

1. T.E of electron in Bohr orbit

$$E = \frac{-me^4 z^2}{8\varepsilon_o^2 h^2 n^2} = 13.6\frac{z^2}{n^2} eV$$

2. K.E = – (T.E)
3. P.E = 2 (T.E)
4. Ionising energy = $E_\infty - E_n$

1) Determine the energies of the first two excited states of the electron in hydrogen atom. What are the excitation energies of the electrons in these orbits?

Data : For the excited level n = 2,
for second excited level n = 3
$E_1 = -13.6$ eV

To Find : i. E_2 ii. E_3
iii. Excitation energy of 1st excited level
iv. Excitation energy of 2nd excited level

Formula: i. Energy of nth level,

$$E_n = \frac{13.6}{n^2} eV$$

ii. Excitation energy = $E_n - E_1$

Solution :

i. $$E_n = \frac{13.6}{n^2} eV$$

$$E_2 = -13.6 \times \frac{1}{2^2} = -3.4 eV \text{ and}$$

$$E_3 = -13.6 \times \frac{1}{3^2} = -1.51 eV$$

ii. Excitation energy of 1st excited level
$E_2 - E_1$
$= -3.4 - (-13.6) = 10.2$ eV
Excitation energy of 2nd excited level
$= E_3 - E_1$
$= -1.51 - (-13.6) = 12.09$ eV

Ans : Excitation energies for 2nd and 3rd orbit are 10.2 eV and 12.09 eV respectively

2) The energy of an excited hydrogen atom is –3.4 eV. Find the angular momentum of the electron.

Data : $E_n = -3.4$ eV

To Find : L

Formula : i. $E_n = \frac{-13.6}{n^2}$ eV

ii. $L = \frac{nh}{2\pi}$

Solution :

i. $$En = -\frac{13.6}{n^2}$$

$\therefore$ $$3.4 = -\frac{13.6}{n^2} \quad n^2 = \frac{13.6}{3.4}$$

$$= n^2 = 4 \qquad n = 2$$

ii. $$\text{Angular momentum} = \frac{nh}{2\pi}$$

$$= \frac{2 \times 6.63 \times 10^{-34}}{2 \times 3.142} \text{ joule-second}$$

$\therefore$ Angular momentum
$= 2.11 \times 10^{-34}$ joule -second

Ans : Angular momentum of electron is 2.11×10^{-34} Js

3) Energy of electron in 2^{nd} Bohr orbit is –3.4 eV. Find the energy of electron in 3^{rd} Bohr orbit and 1^{st} Bohr orbit.

Data : $E_2 = -3.4$ eV.

To Find : i. E_3 ii. E_1

Formulae : $E = \frac{-13.6}{n^2} e$

$\therefore \quad E \propto \frac{1}{n^2}$

Solution :

i. $E_n \propto \frac{1}{n^2}$

$\therefore \quad \frac{E_3}{E_2} = \frac{2^2}{3^2} = \frac{4}{9}$

$\therefore \quad E_3 = (4/9) \times E_2 = (4/9) \times (-3.4 \text{ eV})$

$E_3 = -1.51$ eV

ii. $E \propto \frac{1}{n^2}$

$\frac{E_1}{E_2} = \frac{(2)^2}{(1)^2}$

$\therefore \quad E_1 = -(4/1) \times E_2 = 4 \times (-3.4\text{eV})$

$E_1 = -13.6$ eV

Ans :	Energy of electron in 1st orbit is 13.6 eV and energy of electron in 2nd orbit is - 1.51 ev

4) Energy of electron in 2^{nd} Bohr orbit is –3.4 eV. Calculate its K.E. and P.E. in the 3^{rd} orbit.

Data: $E_2 = -3.4$ eV

To Find : i. $(K.E)_3$ ii. $(P.E)_3$

Formula : i. $E \propto \frac{1}{n^2}$ ii. $K.E = -(T.E)$

iii. $P.E = 2\,(T.E)$

Solution :

i. $E \propto \frac{1}{n^2}$

$\frac{E_3}{E_2} = \frac{(2)^2}{(3)^2}$

$E_3 = \frac{4}{9} \times E_2 = \frac{4}{9} \times (-3.4)$

$E_3 = -1.51$ eV

ii. $K.E = -(T.E) = -(-1.51)$

$= +1.51$ eV

iii. $P.E = 2(T.E) = 2 \times (-1.51)$

$= -3.02$ eV

Ans :	K.E of electron in 3rd orbit is 1.51 eV P.E of electron in 3rd orbit is − 3.02 eV

Problem for Practice

1. The energy of an electron in the n^{th} orbit is given by $E_n = -13.6/n^2$ eV. Calculate the energy required to excite an electron from ground state to the second excited state.

Ans : 12.09 eV

2. Calculate (i) the radius of the second orbit of hydrogen atom, and (ii) total energy of electron moving in the second orbit .

Ans : 2.12Å, –3.4eV

3. The ground state energy of hydrogen atom is –13.6 eV. (i) What is the kinetic energy of an electron in the 2^{nd} excited state ? (ii) What is the potential energy of an electron in the 3^{rd} excited state?

Ans : –0.85ev (i) 1.51 eV (ii) –1.70 eV

4. Find the value of energy of the electron in eV in the third Bohr orbit of H_2 atom.
Given : $R = 1.097 \times 10^7 m^{-1}$; $h = 6.63 \times 10^{-34}$Js and $c = 3 \times 10^8$ m/s.

Ans : –1.51 eV

Type - V
Numerical based on spectral line

Formulae used

1. $\frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$

2. $R = \frac{me^4}{8\varepsilon_0^2 h^3 c} = 1.097 \times 10^7 \text{ m}^{-1}$

3. $E = h\upsilon = \frac{hc}{\lambda}$

1) An electron in hydrogen atom stays in its second orbit for 10^{-8} s. How many revolutions will it make around the nucleus in that time?

Solution :

The electron in 2nd orbit will transit to 1st orbit
Energy difference between two levels
$\Delta E = h\nu$

$$\nu = \frac{\Delta E}{h}$$

$= 2.46 \times 10^{15}$ Hz

Thus in one sewcond electron completes 2.46×10^{15} revolutions

In 10^{-8} s, total revolutions performed

$= 2.46 \times 10^{15} \times 10^{-8}$

$= 2.46 \times 10^{7}$

Ans : Total number of revolutions made by electron is 2.46×10^7

★ **2) Calculate the wavelengths of the first three lines in Paschen series of hydrogen atom.**

Data : $R = 1.097 \times 10^7\ m^{-1}$
For Paschen series, n = 3

To Find : Wavelength of 1st three lines of paschen series $(\lambda_1, \lambda_2, \lambda_3)$

Formula : For Paschen series,

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{m^2}\right)$$

Solution :

i. For 1st line of Paschen series
From formula ; m = 4

$$\therefore \quad \frac{1}{\lambda_1} = 1.097 \times 10^7\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= 1.097 \times 10^7\left(\frac{7}{9 \times 16}\right) \quad = \quad \lambda_1 = \frac{9 \times 16}{1.097 \times 10^7}$$

$\therefore \quad \lambda_1 = 1.875 \times 10^{-6}$ m = 18750 Å

ii. For 2nd line of Paschen series
From formula, m = 5

$$\frac{1}{\lambda_2} = 1.097 \times 10^7\left(\frac{1}{3^2} - \frac{1}{5^2}\right)$$

$$\frac{1}{\lambda_2} = 1.097 \times 10^7\left(\frac{16}{9 \times 25}\right)$$

$$\lambda_2 = \frac{9 \times 25}{1.097 \times 10^7}$$

$\lambda_2 = 1.282 \times 10^{-6}$ m = 12820 Å

iii. For 3rd line of Paschen series , m = 6

$$\therefore \quad \frac{1}{\lambda_3} = 1.097 \times 10^7\left(\frac{1}{3^2} - \frac{1}{6^2}\right)$$

$$\frac{1}{\lambda_3} = 1.097 \times 10^7\left(\frac{27}{9 \times 36}\right)$$

$$\lambda_3 = \frac{9 \times 36}{1.097 \times 10^7}$$

$\lambda_3 = 1.094 \times 10^{-6}$ m = 10940Å

Ans : Wavelength of first three lines of Paschen series are 1875 Å, 1282 Å and 1094 Å, respectively.

3) Calculate the shorlest and longest wavelength of lyman series Given R = 1.097 ×10⁷ m⁻¹

Data : For lyman series, n = 1
$R = 1.097 \times 10^7\ m^{-1}$

To Find : i. λ_s ii. λ_L

Formula : $\frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$

Solution :

i. For shortlest wavelength of lyman series
$n = 1,\ m = \infty$

$$\frac{1}{\lambda_s} = R\left[\frac{1}{(1)^2} - \frac{1}{(\infty)^2}\right]$$

$$\frac{1}{\lambda_s} = R$$

$$\lambda_s = \frac{1}{R} = \frac{1}{1.097 \times 10^7}$$

$= 0.9116 \times 10^{-7}$

$= 911.6 \times 10^{-10}$m

$\lambda_s = 911.6$ A°

ii. For longest wavelength of lyman series
n = 1, m = 2

$$\frac{1}{\lambda_L} = R\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right]$$

$$\frac{1}{\lambda_L} = R\left[1 - \frac{1}{4}\right]$$

$$\frac{1}{\lambda_L} = \frac{3R}{4}$$

$$\lambda_L = \frac{4}{3R} = \frac{4}{3}\times\left(\frac{1}{R}\right) = \frac{4}{3}\times 911.6A^\circ$$
$$= 1215.4\ A^\circ$$

Ans : Longest wavelength of lyman series is 1215.4 A° and shortest wavelength is 911.6A°

4) The H_β line of Balmer series of hydrogen atom has a wavelength 4860 A° Find wavelength and the wave number of H_α line

Data : For Balmer series

$\lambda_\beta = 4860A^\circ$

To Find : λ_α , $\bar{\upsilon}_\alpha$

Formulae : $\frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$

$\bar{\upsilon} = \frac{1}{\lambda}$

Solution :

i. For H_β line of Balmer series
n = 2, m = 4

$$\frac{1}{\lambda_\beta} = R\left[\frac{1}{(2)^2} - \frac{1}{(4)^2}\right]$$

$$\frac{1}{\lambda_\beta} = R\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$\frac{1}{\lambda_\beta} = R\left[\frac{4-1}{16}\right]$$

$$\frac{1}{\lambda_\beta} = R\times\frac{3}{16}$$

$$\lambda_\beta = \frac{16}{3R} \quad \text{... (1)}$$

ii. For H_α line of Balmer series
n = 2, m = 3

$$\frac{1}{\lambda_\alpha} = R\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right]$$

$$\frac{1}{\lambda_\alpha} = R\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\lambda_\alpha = \left[\frac{5}{36}\right]$$

$$\lambda_\alpha = \frac{36}{5R} \quad \text{...(2)}$$

∴ Dividing equation (2) by (1)

$$\frac{\lambda_\alpha}{\lambda_\beta} = \frac{\overset{9}{\cancel{36}}}{5\cancel{R}}\times\frac{3\cancel{R}}{\underset{4}{\cancel{16}}}$$

$$\lambda_\alpha = \frac{27}{20}\times\lambda_\beta$$

$$= \frac{27}{20}\times 4860$$

$$\lambda_\alpha = 6561A^\circ$$

iv. $\bar{\upsilon} = \frac{1}{\lambda_\alpha} = \frac{1}{6.561\times10^{-7}}$

$= 0.1524\times10^7$

$= 1.524\times10^6\ m^{-1}$

Ans : Wavelength of H_α line is 6561A° and wave number of H_α line is $1.524 \times 10^6\ m^{-1}$

Problem for Practice

1. Taking the wavelength of H_α line is 6563 A°, calculate the wavelength of the first line of the Lyman series.
   Ans :1215.4 A°
2. The wavelength of the longest line of the Balmer series is 6560 A. U. Calculate the wavelength of the first line of (i) Lyman series (ii) Paschen series.
   Ans :1214.8 A°; 18742.8 A°
3. The wavelength of H_α line of the Balmer series of H_2 is 6563 A°. Find (1) the wavelength of the H_y line of the series and (2) shortest wavelength of Brackett series.
   Ans : 4340.6 A°; 14584.4 A°
4. The shortest wavelength line in the Lyman series is 912 A°. Find the shortest wavelength line in (1) Balmer series (2) Paschen series.
   Ans : 3648 A°; 8208 A°
5. The energy of the electron in the ground state of Hydrogen is - 13.6 eV. Find the Rydberg constant and the wavelength of the first line of

Paschen series, $e = 1.6 \times 10^{-19}$ C ; h=6.63×10^{-34}Js.

Ans : 1.094 ×10^{-7}m^{-1}; 18.75 ×10^{-7}m

6. If the first member of the Lyman series is at 121.5 nanometres, calculate the wavelengths of the first members of Brackett and Paschen series.

Ans : 4050 nm, 1874.6 nm

7. The ionisation potential of H_2 atom when the electron is in the ground state is 13.6 eV. Find R. Given h= 6.63 × 10^{-34}Js; e= 1.6 × 10^{-19} C and c= 3 × 10^{8} m/s.

Ans : 1.094 × 10^{7}m^{4}

15.7 Atomic Nucleus

Note:

i. *The discovery of neutrons by chadwick, led Heisenberg to propose proton - neutron hypothesis in 1932.*

ii. *According to this hypothesis, protons and neutrons are the main building blocks of the nuclei of all atoms. According to this hypothesis a nucleus of mass number A and atomic . According to this hypothesis a nucleus of mass number A and atomic number Z contains Z protons and (A–Z) neutrons.*

iii. *The protons give positive charge to the nucleus, while protons and neutrons together give it mass. To neutralise the positive charge of the nucleus, i.e, to make the atom electrically neutral, the number of extra-nuclear electrons is Z.*

iv. ***Proton :** It is fundamental particle which may be called the nucleus of hydrogen. It has a positive charge of 1.6 ×10^{-19} C. It has a rest mass of 1.6726 ×10^{-27} kg which is about 1836 times the rest mass of an electron. A proton has an intrinsic (spin) angular momentum equal to 1/2. It also possesses a magnetic moment much smaller than that of an electron.*

v. ***Neutron :** It is a chargeless fundamental particle having mass slightly greater than that of a proton. Its rest mass is 1.6749 × 10^{-27} kg. It has intrinsic angular momentum equal to that of a proton. Inspite of being neutral, a neutron also possesses a small magnetic moment.*

Neutrons and protons are identical particles in the sense that their masses are nearly the same and the force, called nuclear force, does not distinguish them. So the neutrons and protons have common name, the nucleons. However, as the proton is positively charged and the neutron is electrically neutral, so the electromagnetic force can distinguish the two types of particles.

Q.13 Explain the following terms.

Ans :

i. **Nucleons :**

Protons and neutrons which are present in the nuclei of atoms are collectively known as nucleons

ii. **Atomic number :**

The number of protons in the nucleus is called the atomic number of the element. It is denoted by Z

iii. **Mass number :**

The total number of protons and neutrons present in a nucleus is called the mass number of the element.It is denoted by A.

Hence for a neutral atom, we have the following relations :

Number of protons in an atom = Z

Number of electrons in an atom = Z

Number of nucleons in an atom = A

Number of neutrons in an atom = N = A – Z

Q.14 Define Isotopes, isotones and isobars ?

Ans:

i. Isotopes :

The nuclei having same number of protons but different number of neutrons are called isotopes

e.g. deuterium $^{2}_{1}H$ is isotopes of $^{1}_{1}H$

ii. Isobars :

The nuclei having same mass number (A) but having different atomic number (Z) are called as isobars.

e.g. ${}^{3}_{1}H$ and ${}^{3}_{2}H$ are isobars.

iii. Isotones :

The nuclei of elements having same number of neutrons but different atomic numbers are called as isotones.

e.g. ${}^{198}_{80}Hg$ ${}^{197}_{79}Au$ are the isotones.

Q.15 Define the terms atomic unit and electron volt express atomic mass unit in terms of MeV.

Ans : Atomic mass unit : The mass of the carbon - 12 atom is 1.992678×10^{-26} kg, which is very small. Therefore, it is useful to choose a convenient unit for expressing the mass of atoms. This unit is defined by taking mass of carbon-12 atom equal to 12 atomic mass units.

i. One atomic mass unit is defined as $\frac{1}{2}^{th}$ of the actual mass of carbon - 12 atom.

ii. Atomic mass unit is denoted by amu or just by u. Thus

$$1 \text{ amu} = \frac{1}{12} \times \text{Mass of carbon - 12 atom}$$

$$= \frac{1}{12} \times 1.992678 \times 10^{-26} \text{ kg}$$

1 amu = 1.660565×10^{-27} kg

iii. Mass of an electron,

$m_e = 0.00055$ amu $= 9.11 \times 10^{-31}$ kg

Mass of a proton

$m_p = 1.0073$ amu $= 1.6726 \times 10^{-27}$ kg

Mass of a neutron

$m_n = = 1.0086$ amu $= 1.6749 \times 10^{-27}$ kg

Mass of a hydrogen atom,

$m_H = m_p + m_e = 1.0078$ amu

v. The atomic masses can be measured accurately by using an instrument called mass spectrometer.

vi. **Relation between amu and MeV : Energy equivalent of amu.** The Einstein's mass-energy equivalence relation is

$E = mc^2$

This relation shows that the energy content of an object is equal to its mass times the square of the speed of light. To determine the energy equivalent of one atomic mass unit, we take

$m = 1 \text{ amu} = 1.66 \times 10^{-27}$ kg

$c = 2.998 \times 10^8 \text{ ms}^{-1}$

Then $E = 1.66 \times 10^{-27} \times (2.998 \times 10^8)^2$ J

$$= \frac{1.66 \times 10^{-27} \times (2.998 \times 10^8)^2}{1.602 \times 10^{-19}} \text{eV}$$

$= 931$ MeV

1 amu = 931 MeV

Q.16 Describe the size of Nucleus?

Ans: Size of Nucleus:

i. The positively charged, heavy and high density centre which holds 99.9 mass of atom is called as Nucleus.

ii. The size of atom is of the order of 10^{-10} m, and the size of nucleus is about 10^{-15} m.

iii. Size of nucleus can be measured by using fast moving electrons in scattering experiment.

iv. The radius of nucleus is given by,

$R = R_0 A^{1/3}$,

where,

R_0 - linear constant $= 1.2 \times 10^{-15}$ m.

A - mass number

i.e. volume of nucleus is proportional to A.

v. Also, density of nucleus (app.2.3×10^{17} kg/m^3) is constant and independent of mass number A

e.g. Radius of carbon nuclei is

$R_c = 1.2 \times 10^{-15} \text{ m} \times (12)^{1/3}$

$= 2.7473 \times 10^{-15}$ m

Q.17 Prove that the nuclear density is same for all nuclei. Give an estimate of nuclear density.

Ans : Nuclear density:

The density of nuclear matter is the ratio of the mass of a nucleus to its volume. As the volume of a nucleus is directly proportional to its mass number A, so the density of nuclear matter is independent of the size of the nucleus. Thus the nuclear matter behaves like a liquid of constant density. Different nuclei are like drops of this liquid, of different sizes but of same density,

Let A be the mass number and R be the radius of a nucleus. If m is the average mass of a nucleon, then

Mass of necleus = mA

Volume of nucleus $= \frac{4}{3}\pi R^3$

$$= \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

Nuclear density,

$$\rho_{nu} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Clearly, nuclear density is independent of mass number A or the size of the nucleus.

Taking $m = 1.67 \times 10^{-27}$ kg

and $R_0 = 1.2 \times 10^{-15}$ m, we get

$$\rho_{nu} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.142 \times (1.2 \times 10^{-15})^3}$$

$= 2.30 \times 10^{17}$ kg m^{-3}

Thus the nuclear mass density is of the order 10^{17} kg m^{-3}. This density is very large as compared to the density of ordinary matter, say water, for which $\rho = 1.0 \times 10^3$ kg m^{-3}.

Type - VI

Numerical based on Nuclear size

Formulae used

1. $E = mc^2$

2. 1 amu = $\frac{1}{12}$ × Mass of carbon - 12 atom

3 $R = R_0 A^{1/3}$,
where,
$R_0 = 1.2 \times 10^{-15}$ m.

4. $\rho = \frac{3m}{4\pi R_0^3}$

1) Calculate the radius and density of ^{70}Ge nucleus given its mass to be approiximately 69.924 u.

Data : m(Ge) = 69.924 u
$= 69.924 \times 1.66 \times 10^{-27}$ kg
We know that, $R_0 = 1.2 \times 10^{-15}$ m

To Find: i. Radius of ^{70}Ge nucleus
ii. Density of ^{70}Ge nucleus

Formula : i. Radius of any nuclei $R = R_0 A^{1/3}$
ii. Density of nucleus $\rho = \frac{3m}{4\pi R_0^3}$

Solution :

i. $R = R_0 A^{1/3}$
$R_{Ge} = 1.2 \times 10^{-15} \times 70^{1/3}$
$= 1.2 \times 10^{-15} \times 4.121$
$= 4.9452 \times 10^{-15}$ m

ii. $\rho = \frac{3m}{4\pi R_0^3}$

$$\rho = 3 \times 69.924 \times 1.66 \times \frac{10^{-27}}{4\pi (4.9452 \times 10^{-15})^3}$$

$= 2.292 \times 10^{17}$ kg/m^3

Ans : i. The radius of the Ge nuclei is 4.952×10^{-15}m
ii. The density of the Ge nuclei is 2.292×10^{17} kg/m^3

2) Express 16 mg mass into equivalent energy in eV.

Solution : Here m = 16 mg = 16×10^{-6} kg,
$c = 3 \times 10^8$ ms^{-1}
Equivalent energy,
$E = mc^2 = 16 \times 10^{-6} \times (3 \times 10^8)^2$ J

$$= \frac{16 \times 10^{-6} \times (3 \times 10^8)^2}{1.6 \times 10^{-19}} \text{eV}$$

$= 9 \times 10^{30}$ eV

Problem for Practice

1. Calculate the radius of Ge^{70}. Given $R_0 = 1.1$ fm. **Ans:2.9×10^{17} kg m^{-3}**

2. Calculate the density of hydrogen nucleus in SI units Given $R_0 = 1.1$ fermi and $m_p = 1.007825$ amu
Ans :2.98×10^{17} kgm^{-3}

15.8 Nuclear Binding Energy

Q.18 Explain the mass- energy relations?

Ans:

i. According to the Einstein, mass and energy are inter-convertible.

ii. Energy for equivalent mass is given by,
$E = mc^2$
where,

E = energy in joule
m = mass of matter in kg.
c = velocity of light in vacuum This equation gives the mass - energy relation.

iii. If a single electron is completely destroyed the energy released can be obtained as,

$E_e = m_e c^2$

$\therefore\ E_e = 9.1 \times 10^{-31} \times (3\times 10^8)$

$= 81.9 \times 10^{-15}$ J

$= \dfrac{81.9\times10^{-15}}{1-6\times10^{-19}}$

$= 0.511 \times 10^6$eV

$\therefore\ E_e = 0.511$ MeV

Remark:

a. Energy released by proton is ,
$E_p = 941.1$ MeV

b. Energy released by neutron is ,
$E_n = 942.2$ MeV

Q.19 What is mass defect of a nucleus ? Express it mathematically.

Ans :

Mass defect : It is found that the mass of a stable nucleus is always less than the sum of the masses of its constituent protons and neutrons in their free state.

The difference between the rest mass of nucleus and the sum of the rest masses of its constituent nucleous is called its **mass defect.** Consider the nucleus $^{A}_{Z}X$. It has Z protons and (A–Z) neutrons. Therefore, its mass defect will be

$\Delta m = Zm_p + (A - Z)\, m_n - m$

where m_p, m_n and m are the rest masses of a proton neutron and the nucleus $^{A}_{Z}X$ respectively.

Q. 20 Explain the term nuclear binding energy ?

Ans :

i. The amount of energy required to separated all the nucleon from the nucleus is called as binding energy of nucleus or nuclear binding energy.

ii. B.E. of nucleus is about 10^6 larger than that of atom.

ii. The B.E. of nucleus is given by,

$\therefore$ B.E. $= \Delta m \times c^2$....... in joule.

$= [Zm_p + (A- Z)\, m_n] - M \times c^2$

iii. B.E. per nucleon (E) is given by,

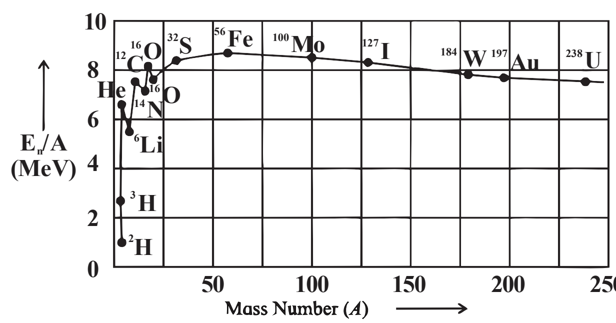
$E = \left(\dfrac{\text{B.E.of nucleus}}{A}\right)$

$= \left(\dfrac{Zm_p + (A - Z)m_n - M}{A}\right)c^2$

where,
A - Atomic mass number.

Q.21 Draw a binding energy curve to show the variation of binding energy per nucleon with mass number. What inferences can be drawn from B.E. curve?

Ans :



i. The binding energy of hydrogen nucleus having a single proton is zero.

ii. Deuterium nucleus has the minimum value of binding energy per nucleon EB/A and is therefore, the least stable nucleus.

iii. The value of E_B/A increases with increase in atomic number and reaches a plateau for A between 50 to 80. Thus, the nuclei of these elements are the most stable.

iv. The peak occurs around A = 56 corresponding to iron, which is thus one of the most stable nuclei.

v. The value of E_B/A decreases gradually for values of A greater than 80, making the nuclei of those elements slightly less stable.

vi. The value of binding energy per nucleon (EB/A) goes on decreading till A ~ 238 which is the mass number of the heaviest naturally occuring element which is uranium.

Note :

Packing fraction . *The packing fraction of a nucleus is its mass defect per nucleon. Thus*
If P.F. is positive (as in case of nuclei with mass number lesss than 20 and above 200) then the nucleus is unstable. If P.E. is negative (as in case of nuclei with mass number between 20 and 200), then it indicates that some mass has been converted into energy which binds the nucleons together and so the nucleus is stable. Thus the P.F. is directly related to the availabiliuty of nuclear energy and the stability of the nucleus.

Type - VII

Numerical based on B.E Nucleus

Formulae used

1. Mass defect, $\Delta m = [Zm_p + (A - Z)\, m_n - m_N]$
2. B.E = $(\Delta m)\, c^2$
3. B.E / nucleon = $= \frac{B.E}{A}$
4. Packing fraction = $= \frac{\Delta m}{A}$

1) **Calculate the hindinf energy of $^{7}_{3}Li$, the masses of hydrogen and lithium atoms being 1.007825 u and 7.016 u respectively.**

Data : m (Li) = 7.016 u
M(H) [or m(p)] = 1.007825 u
We know that m(n) = 1.00866 u

To Find : Binding energy of $^{7}_{3}Li$ (E_B)

Formula : $E_B = \Delta Mc^2$

Solution : $\Delta M = (Zm_p + Nm_n - M)$
$= (3m_H + 4m_n - M_{Li})$
From formula
$E_B = (3m_H + 4m_n - M_{Li})c^2$
We know that
$1u = 931.5\ \frac{MeV}{c^2}$
$E_B = (3\times1.007825 + 4\times1.00866 - 7.016)\times931.5$
$\therefore$ $E_B = 39.23$ MeV

Ans : The binding energy of Lithium nuclei is 39.23 MeV

2) **Determine the binding energy per nucleon of the americium isotop $^{244}_{95}Am$, given the mass of $^{244}_{95}Am$ to 244.06428 u.**

Data : $m_{Am} = 2.44.06428$ u: $A_{Am} = 244$
N(p) = 95
N(n) = 244 – 95 = 149

To find : Binding energy per nucleon $(E_{B/A})$

Formula: $EB = [\Delta M]c^2$

Solution : $\Delta M = Zm_p\ Nm_n - M$
$= (95\ m_p + 149m_n - m_{Am})$
From formula
$E_B = [95\times m_p + 149\times m_n - m_{Am}]\ c2$
We know that $1\ u = 931.5\ \frac{MeV}{c^2}$
$E_B = [95\times938.272 + 149\times939.565 - 244.06429\times931.5]$
$= 8.913\times10^4 + 1.401\times10^5 - 2.274\times10^5$
$= 1.83\times10^3$ MeV
From formula (ii)
$E_B = \frac{1.83\times10^3}{244} = 7.5MeV$

Ans : The binding energy per nucleon is 7.5 MeV

3) **Calculate the energy released in the following reactions, given the masses to be $^{223}_{88}Ra$: 223.0185 u, $^{209}_{82}Pb$: 208.9811, $^{14}_{6}C$: 14.00324, $^{236}_{92}U$: 236.0456, $^{140}_{56}Ba$: 139.9106, $^{94}_{36}Kr$: 93.9341, $^{11}_{6}C$: 11.01143, $^{11}_{5}B$: 11.0093. Ignore neutrino energy.**

i. $^{223}_{88}Ra \rightarrow\ ^{209}_{82}Pb + ^{14}_{6}C$
ii. $^{236}_{92}U \rightarrow\ ^{140}_{56}Ba + ^{94}_{36}Kr + 2n$
iii. $^{11}_{6}C \rightarrow\ ^{11}_{5}B + e^{+} + \text{neutrino}$

Solution : Using $Q = \Delta Mc^2$ in each case,
i. $Q = [\ M_{Ra} - M_{pb} - Mc\ ]\ c^2$

$= [223.0185 - 208.9811 - 14.00324]\ c^2$

We know that, $1\ u = 931.5 \dfrac{MeV}{c^2}$

$= [\ 0.03416\ ] \times 931.5$ MeV

$= 31.8200$ MeV

ii. $Q = [M_U - M_{Ba} - M_{Kr} - 2\times m_n]\ c^2$

$= [236.0456 - 139.9106 - 93.9341] - (2\times 1.00866)]$

$= [0.18358] \times 931.5$ MeV

$= 171$ MeV

iii. $Q = [M_c - M_B - M_{e+}\ ]\ c^2$

$=[11.01143 - 11.0093 - 0.00055]c^2$

$= [0.00158]\times 931.5$ MeV

$= 1.472$ MeV

Ans : i. Energy released in the reaction is 31.8200 MeV
ii. Energy released in the reaction is 171 MeV
iii. Energy released in the reaction is 1.472 MeV

Problem for Practice

1. In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotops and their masses are $^{24}_{12}Mg$ (23.98504 u) $^{25}_{12}Mg$ (24.98584 u) and $^{26}_{12}Mg$ (25.98259 n). The natural abundance of $^{24}_{12}Mg$ is 78.99 by mass. Calculate the abundances of other two isotops.

Ans: 8.73%, 12.28%

2. Calculate the binding energy of an α - particle in MeV. Given : m_p (mass of proton) = 1.007825 amu, m_n (mass of neutron = 1.008665 amu, mass of He nucleus = 4.002800 amu, 1 amu = 931 MeV

Ans :28.097 MeV

3. Calculate the binding energy per nucleon of $^{40}_{20}$ Ca nucleus. Given : m $^{40}_{20}$ Ca = 30.962589 amu, m_n (mass of a neutron) = 1.008665 amu, m_p (mass of a proton) = 1.007825 amu

Ans :8.547 MeV

4. Calculate the binding energy per nucleon of (B.E/nucleon) in the nuclei of $^{31}_{15}P$. Given: $\left[^{31}_{15}P\right] = 30.97376$ amu, $m\left[^{1}_{0}n\right] = 1.00865$ amu, $m\left[^{1}_{1}H\right] = 1.00782$ amu

Ans : 8.47 MeV

5. Calculate the binding energy per nucleon of $^{35}_{17}Cl$ nucleus. Given that mass of $^{35}_{17}Cl$ = 34.980000 u, mass of proton = 1.007825 u, mass of neutron = 1.008665 u and 1 atomic mass unit (1u) = 931 MeV.

Ans : 8.22 MeV

6. Calculate the binding energy per nucleon of $^{56}_{26}Fe$ nucleus. Given that mass of $^{56}_{26}Fe$ = 55.934939 amu, mass of a neutron = 1.008665 amu, mass of a proton = 1.007825 amu

Ans : 8.79 MeV

6. Calculate binding energy per nucleon of $^{209}_{83}Bi$ Given that: m $^{209}_{83}Bi$ = 208.980388 amu, m (neutron) = 1.008665 amu, m(proton) = 1.007825 amu

Ans : 7.85 MeV

15.9 Radioactive Decays

Q.22 Define radioactive decay. Why does it happen?

Ans : The nuclear phenomenon in which an unstable nucleus undergoes a decay is called radioactive decay.

Reason : Nuclei having a particular ratio of mass number to atomic number are stable in nature. Hence, they can remain unchanged for very long time. Other nuclei, not being stable undergo changes in their structure by emission of some particles.

Q.23 What is parent nucleus and daughter nucleus?

Ans: Unstable nuclei change or decay to other nuclei through radioactivity. The decaying nucleus is called the parent nucleus while the nucleus produced after the decay is called the daughter nucleus.

Q.24 What is Q-value?

Ans :

i. The total mass of the products after radioactive decay is less than the mass of the parent atom.

ii. The excess mass appears as the kinetic energy of the products.

ii. The difference in the energy equivalent of the mass of the parent atom and that of the sum of masses of the products is called the Q-value, Q, of the decay

iv. It is equal to the kinetic energy of the products. In general, $Q = [M_{parent} - M_{products}]c^2$

Q.25 What are alpha, bneta and gamma decays?

OR

State types of radioactive decay.

Ans: Radioactive decay is classified into three types:

i. **α - decay :** It is a type of decay in which helium nucleus ${}^{4}_{2}$He is emitted.

ii. **β-decay :** In this decay, electrons or positrons are emitted.

iii. **γ -decay :** In this decay, high energy photons are emitted.

Q.26 Explain alpha (α) decay.

Ans: α - decay :

i. In α decay, the parent nucleus loses heliumem nucleus (i.e. two protons and two neutrons)

ii. The decay can be expressed as

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$$

iii. All nuclei with A > 210 undergo α decay.

iv. Example :

$$\underset{\text{(Bismuth)}}{{}^{212}_{83}\text{Bi}} \xrightarrow{\rightarrow} \underset{\text{(Thallium)}}{{}^{208}_{81}\text{T}l} + \alpha$$

v. Q - value of α decay is

$Q = [m_X - m_Y - m_{He}]c^2$

where m_X, m_Y and m_{He} are atomic masses of the parent atom, the daughter atom and the helium atom respectively.

Q.27 Explain why nuclei with A > 210 undergo α - decay

Ans :

i. Nuclei with A>210 have a large number of protons.

ii. Hence, the electrostatic repulsion between them is very large and the attractive nuclear forces between the nucleons are not able to cope with it.

iii. This makes the nucleus unstable and it tries to reduce the number of its protons by ejecting them in the form of alpha particles.

Q.28 Explain beta(β) and beta plus decay.

Ans: B - decay :

i. In β decay, the nucleus emits an electron produced by converting a neutron in the nucleus into a proton.

i.e., $n \rightarrow p + e^- + \text{antineutrino}$

ii. During beta decay, the number of nucleons i.e., the mass number of the nucleus remains unchanged.

iii. The decay can be written as,

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \text{antineutrino}$$

iv. Example :

$$\underset{\text{(Cobalt)}}{{}^{60}_{27}\text{Co}} \xrightarrow{\rightarrow} \underset{\text{(Nickel)}}{{}^{60}_{28}\text{Ni}} + e^- + \text{antineutrino}$$

Q – value of β - decay is,

$Q = [m_X - m_Y - m_e]c^2$

Here, mass of the antineutrino is ignored as it is negligible compared to the masses of the nuclei.

Beta - plus decay :

i. In beta plus decay, a proton gets converted to a neutron by emitting a positron and a neutrino.

i.e., $p \rightarrow n + e^+ + \text{neutrino}$

ii. During the decay the mass number remains unchanged but Z decreases by one and N increases by one.

iii. The decay can be written as

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^- + \text{neutrino}$$

iv. Example :

$$\underset{\text{(Sodium)}}{{}^{22}_{11}\text{Na}} \xrightarrow{\rightarrow} \underset{\text{(Neon)}}{{}^{22}_{11}\text{Na}} + e^+ + \text{neutrino}$$

v. In this case, mass of a neutron is higher than the mass of a proton. extra energy needed to produce a neutron can be obtained from the rest of the nucleus.

Q.29 Explain gamma decay.

Ans :

i. In gamma (γ) decay, gamma rays (high energy protons) are emitted by the parent nucleus.

ii. The daughter nucleus is same as the parent nucleus as no other particle is emitted, but it has less energy as some energy goes out in the form of the emitted gamma ray.

iii. It can be written as,

$$X_{excited} \xrightarrow{\rightarrow} X_{relaxed} + \gamma$$

Where, $X_{excited}$ is excited nucleus, $X_{relaxed}$ is same nucleus at lower energy states and γ is released gamma ray photon.

iv. Energy release (Q value) in -γ decay is difference in energy levels through which nucleus is transiting.

v. Gamma decays usually occur after parent nucleus has undergone α or β decay.

Type - VIII

Numerical based on α, β and λ decay

1) Calculate the binding energy of an alpha particle given its mass to be 4.00151 u

Data : $m_\alpha = 4.00151u$

$$m_p = 1.00728u = \frac{938.281 MeV}{c^2}$$

$$m_n = 1.00866u = \frac{939.567 MeV}{c^2}$$

We know that $1\ u = 931.5\frac{MeV}{c^2}$

To find : Binding energy of an alpha particle

Formula : $E_B = (\Delta M)c^2$

Solution : $\Delta M = Zm_p\ Nm_n - M$
$= (2m_p + 2m_n - m_\alpha)$

From formula

$E_B = [2m_p + 2m_n - m_\alpha)]$ c2
$= [(2 \times 938.281) + (2 \times 939.567) - (4.00151 \times 931.5)]$
$= 28.289$ MeV

Ans :	Binding energy of Helium atom is 28.289 MeV

2) Calculate the energy released in the alpha decay of ^{238}Pu to ^{234}U, the masses involved being m_{Pu} = 238.04955 u, m_U = 234.04095 u and m_{He} = 4.002603u.

Data : $m_{Pu} = 238.04955$ u
$m_U = 234.04095$ u,
$m_{He} = 4.002603$ u

Reaction : $^{238}Pu \rightarrow {}^{234}U + {}^4He$

To Find : Energy released in alpha decay (Q value)

Formula : $Q = [m_{parent} - m_{product}]\ c^2$

Solution : From formula

$Q = [m_{pu} - m_U - m_{He}]\ c^2$

$Q = [(238.04955 - 234.04095 - 4.002603)]c^2$

We know that

$$1u = 931.5\frac{MeV}{c^2}$$

$= 0.005997 \times 931.5$ MeV
$= 5.5862$ MeV

Ans :	Energy released in the alpha decay is 5.5862 MeV

3) Calculate the maximum kinetic energy of the beta particle (positron) emitted in the decay of $^{22}_{11}Na$, given the mass of $^{22}_{11}Na$ = 21.994437 u, $^{22}_{10}Ne$ = 21.991385 u and me = 0.00055 u

Data : $^{22}_{11}Na = 21.994437$ u,
$^{22}_{10}Na = 21.991385$u and
me = 0.00055 u

Reaction : $^{22}_{11}Na \rightarrow {}^{22}_{10}Na + e^+ +$ neutrino

To Find : Maximum energy kinetic energy of beta particle (Q value)

Formula : $Q = [m_{parent} - m_{product}]\ c^2$

Solution : Considering neutrino having zero energy,

$Q = [m_{Na} - m_{Ne} - m_e]\ c^2$
$= [21.994437 - 21.991385 - 0.00055]\ c^2$

We know that

$$1u = 931.5\frac{MeV}{c^2}$$

$= 0.002502 \times 931.5$
$= 2.3306$ Mev

Ans :	Maximum kinetic energy of the beta particle is 2.3306 MeV

4) **Calculate the energy released in the nuclear reaction $^7_3\text{Li} + p \rightarrow 2\alpha$ given mass of ^7_3Li atom and of helium atom to be 7.016 u ans 4.0026 u respectively.**

Data : $m_{Li} = 7.016 \text{ u} = 7.016 \times 931.5 \frac{\text{MeV}}{c^2}$

$m_p = 938.272 \frac{\text{MeV}}{c^2}$

$m_\alpha = 4.0026\text{u} = 4.0026 \times 931.5 \frac{\text{MeV}}{c^2}$

Reaction : $^7_3\text{Li} + p \rightarrow 2\alpha$

To Find : Emergy released in the nuclear reaction (Q value)

Formula : $Q = [m_{parent} - m_{product}]\ c^2$

Solution : From formula

$Q = [m_{Li} - m_p - 2m_\alpha]\ c^2$

$= [7.016 \times 931.5 + 938.272 - 2 \times 4.0026 \times 931.5]$

$= 6.535 \times 10^3 + 938.272 - 7.456 \times 10^3$

$= 17.272$ MeV

Ans :	The energy released in the nuclear reaction is 17.272 MeV

Problem for Practice

1.
2.
3.
4.
5.

15.10 Law of Radioactive Decay

Q.30 State the law of radioactive disintegration and hence show that radioactive decay is exponential in nature.

Ans: Law of radioactive disintegration :

The number of nuclei undergoing the decay per unit time is proportional to the number of unchanged nuclei present at that instant.

Explanation:

i. Let, N_0 - number of nuclei present at $t = 0$.

N - number of nuclei present at time t.

dN - number of nuclei that disintegrated in dt.

According to the law of radioactive decay,

$-\frac{dN}{dt} \propto N,$

$\therefore \frac{dN}{dt} = -\lambda\ N$ .....(1)

where,

λ is a constant called decay constant or disintegration constant.

The negative sign indicates that the number of atoms decreases as the time increases.

From equation (1)

$\frac{dN}{N} = -\lambda\ dt$

ii. Integrating both sides and separating the variables,

$\int \frac{dN}{N} = \int -\lambda\ dt$

$\therefore \log_e N = -\lambda t + c$ ...(2)

where,

c is constant of integration.

At t = 0, $N = N_0$

$\therefore \log N_0 = 0 + c$

$\therefore c = \log_e N_0$

Substituting in equation (2),

$\therefore \log_e N = -\lambda t + \log_e N_0$

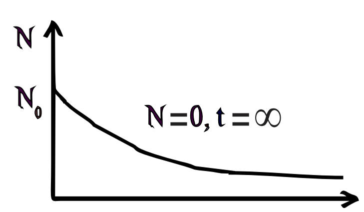
$\log_e N - \log_e N_0 = -\lambda t$

$\therefore \log_e \left(\frac{N}{N_0}\right) = -\lambda\ t$

$\therefore \frac{N}{N_0} = e^{-\lambda t}$

$\therefore N = N_0 e^{-\lambda t}$ ...... (2)

This equation shows that the radioactive substance decays exponentially with time t.



For any radioactive substance, all the atoms will disintegrate only after infinite time.

Q.31 Define half life. Derive an expression for the half life.

Ans: Half life period (T):

Half life period of a radioactive element is defined as the time required for the atoms of that element to decrease the number of atoms to half of the original number.

Explanation:

i. Let, N_0- number of atoms present at time t = 0.

N be number.of atoms present after time t

ii. We have,

$N = N_0 e^{-\lambda t}$

According to definition,

when $t = T\frac{1}{2}$ then $N = \frac{N_0}{2}$

$\therefore \frac{N_0}{2} = N_0 e^{-\lambda T_{\frac{1}{2}}}$

$\therefore \frac{1}{2} = e^{-\lambda T_{\frac{1}{2}}}$

$\therefore e^{-\lambda T_{\frac{1}{2}}} = 2$

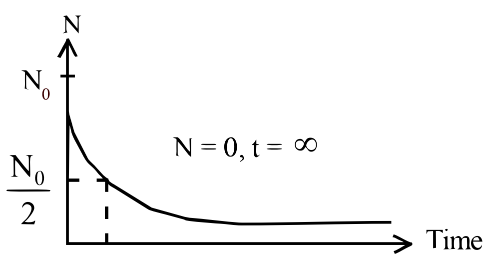
$\therefore \lambda T_{\frac{1}{2}} = \log_e 2$

$\therefore \lambda T_{\frac{1}{2}} = 2.303 \times \log_{10} 2$

$\therefore \lambda T_{\frac{1}{2}} = 2.303 \times 0.3010 = 0.693$

$\therefore T_{\frac{1}{2}} = \frac{0.693}{\lambda}$

i.e. half life of a radioactive substance depends on the decay constant.



Note :

i. Half life is different for different substances, e.g. half life of radium is 1620 years and that of radon is 4 seconds.

ii. Rate of disintegration is also called activity:

iii. Units of Radioactivity:

1Bq = 1d.p.s.

1 rd = 10^6 d.p.s

1 ci = 3.7×10^{10} d.p.s

★ Q.32 What is activity of radioactive materials? Give its units.

Ans :

i. The rate of decay, i.e., the number of decays per unit time $-\frac{dN(t)}{dt}$, is called as activity A (t).

ii. It is given as, A (t)

$= -\frac{dN(t)}{dt} = \lambda N(t) = \lambda N_0 e^{-\lambda t}$

At t = 0, the activity is given by $A_0 = \lambda N_0$

$A(t) = A_0 e^{-\lambda t}$

iii. a. S.I.unit : Becquerel (Bq)

1 Bq = one decay per second.

b. CGS unit is curie (Ci)

1 Ci = 3.7×10^{10} Bq

Q.33 Derive relation between average life and half life of a radioactive material.

Ans :

i. The number of nuclei decaying between time t and t + dt is given by $\lambda N_0 e^{-\lambda t} dt$

ii. The life time of these nuclei is t. Thus, the average lifetime τ of a nucleus is

$$\tau = \frac{1}{N_0}\int_0^\infty t\lambda N_0 e^{-\lambda t} dt = \lambda \int_0^\infty t e^{-\lambda t} dt = \frac{1}{\lambda}$$

Now $T_{\frac{1}{2}} = \frac{0.693}{\lambda}$

$$\boxed{\tau = \frac{T_{\frac{1}{2}}}{0.693}}$$

Type - IX

Numerical based on α,β and λ decay

Formulae used

1. $N = N_0 e^{-\lambda t}$

2. $A(t) = A_0 e^{-\lambda t}$

3. $T_{1/2} = \frac{0.693}{\lambda}$

4. $\tau = \frac{1}{\lambda}$

5. $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

6. $\frac{dN(t)}{dt} = \lambda N(t)$

★ 1) The activity of a radioactive sample decreased from 350 s^{-1} to 175 s^{-1} in one hour. Determine the half life of the species

Data : $A_0 = 350\ s^{-1}$, $A(t) = 175\ s^{-1}$

$T = 1\ h = 3600\ s$

To find : $(T_{1/2})$

Formula : i. $A(t) = A_0 e^{-\lambda t}$ ii. $T_{1/2} = \frac{0.693}{\lambda}$

Solution :

i. $A(t) = A_0 e^{-\lambda t}$

$175 = 350\, e^{-3600\lambda}$

Taking natural log of both sides, we get

$ln = 175 = -3600\ ln\ 350$

$\therefore \quad -3600\ \lambda = ln \frac{175}{350}$

$\therefore \quad \lambda = \frac{ln\left(\frac{350}{175}\right)}{3600} = \frac{ln 2}{3600} = \frac{2.303 \log 2}{3.6\times10^3}$

$= \frac{2.303 \times 0.3010 \times 10^{-3}}{3.6}$

$= \frac{2.303 \times 3.010 \times 10^{-4}}{3.6}$

= Atlog [ ] $\times 10^{14}$

log N		log D	
log 2.303	0.3623	log 3.6	0.5563
log 3.010	+0.4786		
	0.8409		

=Atlog $\left[\begin{array}{r} 0.8409 \\ -0.5563 \\ \hline 0.2846 \end{array}\right] \times 10^{-4}$

$= 1.926 \times 10^{-4}$ Sec

ii. $T_{1/2} = \frac{0.693}{\lambda}$

$T_{1/2} = \frac{0.693}{1.925\times10^{-4}} = \frac{6.93}{1.925}\times10^3$

= Atlog [ ] $\times 10^3$

log 6.93	0.8407
log 1.925	−0.2844
	0.5563

$= 3.599 \times 10^3$ s $= 3.6 \times 10^3$ s

Ans : The half life of the species is $3.6 \times 10^3 s$

★ 2) A source contains two species of phosphorous nuclie $^{32}_{15}P$ ($T_{1/2}$=14.3d)and $^{33}_{15}P$ ($T_{1/2}$=25.3d). At time t = 0, 90% of the decays are from $^{32}_{15}P$. How much time has to elapse for only 15% of the decays to be from $^{32}_{15}P$

Data : <u>**Species - 1**</u> : $^{32}_{15}P$

$T_{1/2}$ = 14.3 days

<u>**Species - 2**</u> : $^{33}_{15}P$

$(T_{1/2})_2$ = 25.3 days

At time t = 0, 90% of the decay are form $^{32}_{15}P$

At time t, 15% of decay are form $^{32}_{15}P$

To find: t

Formula : $\left|\frac{dN(t)}{dt}\right| = |-\lambda N(t)| = \lambda N(t)$

$T_{1/2} = \frac{0.693}{\lambda}$

Solution :

i. At t = 0, 90% of decay from for $^{32}_{15}P$ (species -1) and 10% for $^{33}_{15}P$ (species-2)

$\therefore \quad \frac{(N_0)_1\lambda_1}{(N_0)_2\lambda_2} = \frac{90\%}{10\%} = \frac{9}{1} = 9$ ... (1)

ii. At t = t, 15% of decay is form $^{32}_{15}P$ and 85% from $^{33}_{15}P$.

$$\frac{(N_0)_1\lambda_1 e^{-\lambda_1 t}}{(N_0)_2\lambda_2 e^{-\lambda_2 t}} = \frac{15\%}{85\%} = \frac{3}{17} \quad ...(2)$$

Dividing (1) by (2)

$$\frac{(N_0)_1\lambda_1}{(N_0)_2\lambda_2} \times \frac{(N_0)_2\lambda_2 e^{-\lambda_2 t}}{(N_0)_1\lambda_1 e^{-\lambda_1 t}} = \frac{9\times17}{3}$$

$$e^{(\lambda_1-\lambda_2)} = 51$$

Taking log on b.t.s

$$(\lambda_1-\lambda_2)t = \log_e 51$$

$$\therefore \left[\frac{0.693}{(T_{1/2})_1} - \frac{0.693}{(T_{1/2})_2}\right] t = 2.303\log_{10} 51$$

$$\therefore 0.693\left[\frac{1}{14.3} - \frac{1}{25.3}\right] t = 2.303\times1.7076$$

0.693 [0.0699 – 0.0395] t = 3.933

[0.693 × 0.0304] ^{t} = 3.933

$$t = \frac{3.933}{0.694\times0.0304}$$

$$= \frac{3.933}{0.02107} = 186.6 \text{ days}$$

Ans : Time taken for only 15% of decays to be from $^{33}_{15}P$is 186.6 days

★ **3) The half-life of a nuclear species ^{N}X is 3.2 days, Calculate its (i) decay constant, (ii) average life (iii) the activity of its sample of mass 1.5 mg.**

Data : $T_{1/2}$ = 3.2 days
= 3.2 × 24 × 60 × 60s
= 2.765 ×10^5 s,
m = 1.5 mg = 1.5 × 10^{-3} g

To Find : i. λ ii. τ iii. A (t)

Formula : i. $T_{1/2} = \frac{0.693}{\lambda}$ ii. $\tau = \frac{1}{\lambda}$

iii. $A(t) = \lambda N(t)$

Solution :

i. $T_{1/2} = \frac{0.693}{\lambda}$

$$2.765\times10^5 = \frac{0.693}{\lambda}$$

$$\therefore \lambda = \frac{0.693}{2.765\times10^5} = 2.506\times10^{-6}\ s^{-1}$$

ii. $\tau = \frac{1}{\lambda}$

$$\tau = \frac{1}{2.506\times10^{-6}} = 0.4\times10^6 = 4\times10^5\ s$$

$$\tau = \frac{4\times10^5}{8.64\times10^4}\text{days} = 4.63 \text{ days}$$

iii. As number of nuclei in given sample
$N(t) = N_A$ × no. of moles
Assuming 'Y' as atomic mass of nuclear species 'X'

$$N(t) = 6.022\times10^{23}\times\frac{1.5\times10^{-3}}{Y} = \frac{9.033\times10^{20}}{Y}$$

iv. $A(t) = \lambda N(t)$

$$= 2.5\times10^{-6}\times\frac{9.033\times10^{20}}{Y}$$

$$= \frac{2.258\times10^{15}}{Y}$$

$$= \frac{2.258\times10^{15}}{Y\times3.7\times10^{10}} = Ci = \frac{6.1\times10^4}{Y}Ci$$

Ans : i.The decay constant of reaction is $2.506\times10^{-6}\ s^{-1}$ ii. The mean life of the species is 4.63 days iii. The activity of its 1.5 mg sample is $\frac{6.1\times10^4}{Y}$Ci

4) The half-life of $^{90}_{38}Sr$ is 28 years. Determine the disintegration rate of its 56 mg sample

Data : $T_{1/2}$ $^{90}_{38}Sr$ = 28 years
= 28 ×365×24×60×60 sec
= 8.83 ×10^8 sec,
M = 5mg = 5×10^{-3} g

To Find : Disintegration rate of 5 mg sample

$$\left(\frac{dN(t)}{dt}\right)$$

Formula : i. $\lambda = \frac{0.693}{T_{1/2}}$ ii. $\left|\frac{dN(t)}{dt}\right| = \lambda N(t)$

Solution :

i $$\lambda = \frac{0.693}{T_{1/2}}$$

$$\lambda = \frac{0.693}{8.83\times10^{8}} = 7.847 \times10^{-10} \text{ sec}$$

ii. 90 gm of ${}^{90}_{38}Sr$ contains 6.023×10^{23} atoms

$\therefore$ in 5×10^{-3} gm of ${}^{90}_{38}Sr$, number of atoms are

$$N = \frac{5\times10^{-3}\times6.023\times10^{23}}{90}$$

$= 3.347 \times10^{10}$ atoms

iii. $$\left|\frac{dN(t)}{dt}\right| = \lambda N(t)$$

$= (7.847 \times10^{-10})\ (3.347\times10^{19})$

$= 26.27 \times109$

$= 2.627 \times10^{10}\ s^{-1}$

Ans : The disintegration rate of ${}^{90}_{38}Sr$ is $2.627\times10\ s^{-1}$

★ **5) What is the amount of ${}^{60}_{27}Co$ necessary to provide a radioactive source of strength 10.0 m Ci, its half life being 5.3 Years?**

Data: $$\frac{dN(t)}{dt} = 10 \text{ m Ci}$$

$= 10\times10^{-3} \times 3.7 \times10^{10}$ decay / s

$= 3.7 \times10^{8}$ decay / s

$T_{1/2} = 5.3$ years

$= 5.3\times3.1526\times10^{7}$ s

$= 1.67 \times 10^{8}$ s

To Find : M

Formula : i. $\lambda = \frac{0.693}{T_{1/2}}$ ii. $\left|\frac{dN(t)}{dt}\right| = \lambda N(t)$

Solution:

i $$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.67\times10^{8}} = 0.415 \times10^{-8} \text{ s}$$

ii. $$N(t) = \frac{\left|\frac{dN(t)}{dt}\right|}{\lambda}$$

$$N = \frac{3.7\times10^{8}}{0.415\times10^{-8}} = 8.917 \times 10^{16}$$

iii. Mass of 6.022×10^{23} atoms = 60 g

Mass of 8.915×10^{16} atoms = m

$$\therefore \quad m = \frac{60\times8.915\times10^{16}}{6.022\times10^{23}} = 88.84 \times10^{-7} \text{ g}$$

$= 8.884 \times10^{-6}$ g

Ans : The amount of ${}^{60}_{27}Co$ necessary to provide a radioactive source of strength 10.0 mCi is 8.884×10^{-6} g.

★ **6) Disintegration rate of a sample is 10^{10} per hour at 20 hrs from the start. reduces to 6.3×10^{9} per hour after 30 hours. Calculate its half life and the initial number of radioactive atoms in the sample.**

Data : $A(t_1) = 10^{10}$/hr for $t_1 = 20$ hrs

$A(t_2) = 6.3 \times10^{9}$/hr for $t_2 = 30$ hrs

To Find : i. $T_{1/2}$ ii. N_0

Formula : i. $A(t) = A_0 e^{-\lambda t}$ ii. $T_{1/2} = \frac{0.693}{\lambda}$

iii. $A(t_0) = \lambda N_0 e^{-\lambda t}$

Solution :

i. $$A(t_1) = A_0 e^{-\lambda t_1}$$

$$10^{10} = A_0 e^{-20\lambda} \quad ...(1)$$

$$A(t_2) = A_0 e^{-\lambda t_2}$$

$$6.3\times10^{9} = A_0 e^{-30\lambda} \quad ...(2)$$

Dividing equation (1) by (2)

$$\therefore \quad \frac{10^{10}}{6.3\times10^{9}} = e^{-20\lambda+30\lambda} = e^{10\lambda}$$

$1.587 = e^{10\lambda}$

$10\lambda = 2.303 \log 1.587$

$10\lambda = 2.303 \times 0.2007$

$\therefore$ $10\lambda = 0.4621$

$\therefore$ $\lambda = 0.04621\ hr^{-1}$

ii. $$T_{1/2} = \frac{0.693}{\lambda}$$

$$T_{1/2} = \frac{0.693}{0.04621} \approx 15 \text{hours}$$

iii. $A(t_1) = \lambda N_0 e^{-\lambda t_1}$

$10^{10} = 0.0462\ N_0 e^{-0.0462\times20}$

$10^{10} = 0.0462\ N_0\ e^{-0.924}$

$$N_0 = \frac{10^{10}}{0.0462} \times e^{0.924}$$

$N_0 = 2.1645 \times 10^{11} \times e^{0.924}$

$$\frac{N_0}{2.1645 \times 10^{11}} = e^{0.924}$$

Taking logarithm on both sides

$$\therefore \quad 2.303 \log\left(\frac{N_0}{2.1645 \times 10^{11}}\right) = 0.924$$

$$\therefore \quad \log\left(\frac{N_0}{2.1645 \times 10^{11}}\right) = \frac{0.924}{2.303} = 0.4012$$

Taking antilog on both sides

$$\frac{N_0}{2.1645 \times 10^{11}} = 2.516$$

$N_0 = 2.1645 \times 10^{11} \times 2.519 = 5.452 \times 10^{11}$

Ans : i. Half life of sample is 15 h
ii. Initial number of atoms is 5.452×10^{11}

7) The isotope 57_{Co} deays by electron capture to 57_{Fe} with a half-life of 272 d. The 57_{Fe} nucleus is produced in an excited state, and it almost instantaneously emits gamma rays

i. Find the mean lifetime and decay constant for 57_{Co}

ii. If the activity of a radiation source 57_{Co} is 2.0 μCi now, how many 57_{Co} nuclei does the source contain?

iii. What will be the activity after one year?

Data : $T_{1/2} = 272 \times 24 \times 60 \times 60$ sec $= 2.35 \times 10^7$ sec

$$A_0 = \left(\frac{dN(t)}{dt}\right) = 2.0\mu Ci$$

$= 2.0 \times 3.7 \times 10^{10} \times 10^{-6}$ decays/sec

$= 7.4 \times 10^4$/s

To Find : i. τ ii. λ iii. N0
iv. A(t) after one year

Formulae : i. $\tau = \frac{T_{1/2}}{0.693}$ ii. $\lambda = \frac{1}{\tau}$

iii. $A_0 = N_0 \lambda$ iv. $A(t) = A_0 e^{(-\lambda t)}$

Solution :

i. $$\tau = \frac{T_{1/2}}{0.693} = \frac{2.35 \times 10^7}{0.693} s = 3.391 \times 10^7 s$$

ii $$\lambda = \frac{1}{\tau} = \frac{1}{3.39 \times 10^7} = 2.945 \times 10^{-8} s^{-1}$$

Now for number of nuclei when activity is 2 μCi

iii. $A_0 = N_0 \lambda$

$$\therefore \quad N_0 = \frac{A_0}{\lambda}$$

$$= \frac{7.4 \times 10^4}{2.95 \times 10^{-8}} = 2.513 \times 10^{12} \text{ nuclei}$$

iv. $A(t) = A_0 e^{-\lambda t}$

After 1 year i.e 3.156×10^7 sec

$A(t) = 2e^{-(2.949 \times 10^{-8} \times 3.156 \times 10^7)}$

$= 2e^{-0.9307}$

$$A(t) = \frac{2}{e^{0.9307}} \quad ...(1)$$

let $x = e^{0.9307}$

Taking log on b.t.s

$2.303 \log x = 0.9307$

$$\log x = \frac{0.9307}{2.303}$$

$x = \text{Atlog}(0.4041)$

$x = 2.536$

$\therefore$ From (1)

$$A(t) = \frac{2}{2.536} = 0.7886 \ \mu Ci$$

Ans : i. Mean life time of ^{57} Co is $3.391\ 10^7$ s
ii. Decay constant of ^{57}Co is $2.945\ 10^{-8}$ s^{-1}
iii. Initial nuclei number is 2.513×10^{12}
iv. Activity after one year is 0.7886μCi

8) Before the year 1900 the activity per unit mass of atmospheric carbon due to the presence of ^{14}C averaged about 0.255 Bq per gram of carbon. (i) What fraction of carbon atoms were ^{14}C? (ii) An archaeological specimen containing 500 mg of carbon, shows 174 decays in one hour. What is the age of the specimen, assuming that its activity per unit mass of carbon when the specimen died was equal to the average value of the air? Half life of ^{14}C is 5730 years?

Solution :

Before 1900

activity i.e $\left|\frac{dN(t)}{dt}\right| = 0.255$ Bq = 0.255 s^{-1}

T1/2 = 5730 year
= 5730 ×365 ×24 ×60×60s
= 1.81 ×10^{11}s

Disintegration constant,

$$\lambda = \frac{0.693}{1.81\times10^{11}} = 3.83\times10^{-12}s^{-1}$$

Total number of radioactive carbon atoms in 1 g. Atmospheric carbon

$$N(t) = \frac{\left|\frac{dN(t)}{dt}\right|}{\lambda} = \frac{0.255}{3.83\times10^{-12}}$$
$$= 6.66\times10^{10}/g$$

Number of carbon atoms available is one gram carbon.

$N_c = \frac{N_A}{M}$, where N_A os a vogadro's constant and M is molecular mass of carbon.

$$= \frac{6.022\times10^{23}}{12} = 5.02\times10^{22}/g$$

Fraction of radioactive carbon in one gram of atmospheric carbon

$$F = \frac{N}{N_c} = \frac{6.66\times10^{10}}{5.02\times10^{22}} = 1.326\times10^{-12}$$

Now, $= \frac{1}{1.326\times10^{-12}} = 0.75\times10^{12}$

i.e. 1 atom is radioactive in every 7.5×10^{11} atoms

Considering standard conventions of expressing quantity with appropriate order of magnitude there were 4 radioactive atoms in every 3×10^{12} carbon atoms.

ii. As 174 decays per hour is shown for 500 mg 348 decays per hour will take place for 1g

$$= \frac{348}{60\times60} \text{ decays per sec per 1 g}$$
= 0.097 decays per sec per 1 g

For age of specimen,

$$t = \frac{2.303}{\lambda}\log\frac{A_0}{A(t)}$$

$$t = \frac{2.303}{3.83\times10^{-12}}\log\frac{0.255}{0.097}$$
$$= \frac{2.303}{3.83\times10^{-12}}\log\frac{255}{97}$$
$$= \frac{2.303}{3.83\times10^{-12}}(\log 255 - \log 97)$$
$$= \frac{2.303}{3.83\times10^{-12}}(2.4065-1.9868)$$
$$= 2.524\times10^{11} \text{ s}$$
$$= \frac{2.524\times10^{11}}{365\times24\times60\times60} \text{ years} = 8003 \text{ years}$$

Ans : i.4 radioactive atoms in every 3×10^{12} atoms were ^{14}C
ii. Age of specimen is 8003 years.

Problem for practice

1.

2.

3.

4.

15.11 Nuclear Energy

Q.34 Write a note on nuclear energy.

Ans :

i. Nuclear energy is non- conventional sources of energy.

ii. Nuclear energy is the energy released when nuclei undergo a **nuclear reaction**, i.e., when one nucleus or a pair of nuclei, due to their interaction, undergo a change in their structure resulting in new nuclei and generating energy in the process.

iii. Nuclear reactions are abundant source of energy. Energy generated in nuclear reaction is of order of few MeV as against chemical reaction producing energy of few eV.

iv. Nuclear energy generation is a very complex and expensive process. It can also be extremely harmful and hence requires extreme caution while generating nuclear energy.

Q.35 Explain what is nuclear fission giving an example. Write down the formula for energy generated is the process.

Ans:

i. The process of splitting a heavy nucleus into two lighter nuclei after bombardment with release of energy is called nuclear fission.

ii. Example : ${}^{236}U$ undergoes fission producing barium (Ba) and krypton (Kr). In the process, it release 2 neutrons (n) and energy.

i.e., ${}^{236}_{90}U \longrightarrow {}^{140}_{56}Ba + {}^{94}_{36}Kr + 2n + Q$

iii. Energy generated,

$Q = (\Delta M)c^2$

Where,

$\Delta M = M_{parent} - M_{product}$

$= \text{mass of } {}^{236}U - [\text{mass of } {}^{140}Ba + \text{mass of } {}^{94}Kr + (\text{mass of neutron})]$

iv. The energy produced in the fission is in the form of kinetic energy of the products, i.e., in the form of heat which can be collected and converted to other forms of energy as needed.

Q.36 Describe the principles of a nuclear reactor.

Ans: A nuclear reactor is an apparatus or a device in which nuclear fission is carried out in a controlled manner to produce energy in the form heat which is then converted to electricity.

Q.37 Explain chain reaction in an uranium reactor.

Ans:

i. In an uranium reactor, ${}^{235}_{92}U$ is used as the fuel. It is bombarded by slow neutrons to produce ${}^{236}_{92}U$ which undergoes fission i.e.,

a. ${}^{235}_{92}U + n \longrightarrow {}^{236}_{92}U$

b. 1. ${}^{236}_{92}U \to {}^{137}_{53}I + {}^{97}_{39}Y + 2n$

2. ${}^{236}_{92}U \to {}^{140}_{56}Ba + {}^{94}_{36}Kr + 2n$

3. ${}^{236}_{92}U \to {}^{133}_{51}Sb + {}^{99}_{41}Bb + 4n$

4. ${}^{236}_{92}U \to {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3n$

ii. Some decay reactions produce 2 neutrons while others produce 3 ot 4 neutrons. The average number of neutrons per is 2.7.

iii. These neutrons are in turn absorbed by other ${}^{235}_{92}U$ nuclei to produce ${}^{235}_{92}U$ which undergo fission and produce further 2.7 neutrons per fission.

iv. Though the reaction can be self sustaining to proceed, in a nuclear reactor it is allowed to occur in a controlled fashion.

v. The energy generated, which is in the form of heat, is carried away and converted to electricity by using turbines etc.

Q.38 What is a chain reaction? Explain with example.

Ans : The successive fissions due to large number of prompt (neutrons released during the fission process) neutrons gives the tremendous increase in magnitude of energy release in the reactions. This process is known as chain reaction.

Example : When ${}^{236}_{92}U$ undergoes fission on an average 2.7 neutrons are produced during process. Therse neutrons get absorbed by ${}^{235}_{92}U$ nuclei around producing more ${}^{236}_{92}U$ nuclei and causing further nuclear fissions.

This means reaction one started proceeds by itself like a chain, releasing huge amount of energy.

Q.39 What is the difference between a nuclear reactor and a nuclear reactor bomb?

Ans : Nuclear reactor works on the principle of controlled chain reaction, where in number of neutrons released per fission are kept limited by absorption of excess neutrons.

Nuclear bomb works on the principle of uncontrolled chain reaction, where in there is no control on number of neutrons released per fission. This has cascading effect, increasing amount of energy produced rapidly which leads to an explosion.

Q.40 Explain what is nuclear fusion giving an example. Write down fomula for energy generated in the process.

Ans :

i. The nuclear reaction in which two lighter nuclei are fused to form a heaavier nucleus is called nuclear fusion.

ii. If any two of the lighter nuclei come sufficiently close, within about one fm of each other, then they can undergo nuclear reaction anf form a heavier nucleus.

iii. For two bare nuclei to come into proximity overcoming repulsive force between their positive charge, requires high energy, Hence, fusion process requires very high temperature.

iv. The newly formed nucleus has smaller mass than the sum of masses of fused muclei.

v. Example : At centre of sun at temperature of 107 K four hydrogen nuclei (i.e protons) fuse to form a helium nucleus. The effective reaction can be written as

$$4p \rightarrow \alpha + 2e^{+} + \text{neutrinos} + 26.7\ \text{MeV}$$

vi. As mass defect is converted into energy, energy value, $Q = (\Delta M)c^2$

where $\Delta M = M_{parent} - M_{product}$

For given example, ignoring energy taken by neutrinos energy value,

$Q = [4\ m_p - (m_\alpha + 2m_e)]c^2$

Q.41 What does term bare nuclei mean?

Ans : Electrons when are given energies larger than the ionization potentials of atoms by heating a gas of atoms, atoms get stripped off their electrons. Such atoms with only nuclei are terms bare nuclei.

Q.42 Explain nuclear fusion in Sun

Ans :

i. The temperature at the centre of the Sun is about 107 K.

ii. The nuclear reactions taking place at the centre of the Sun are the fusion of four hydrogen nuclei, i.e., protons to form a helium nucleus.

iii. The fusion proceeds in several steps.

iv. The effective reaction can be written as

$$4p \rightarrow \alpha + 2e^{+} + \text{neutrinos} + 26.7\ \text{MeV}$$

v. These reactions have been going on inside the Sun since past 4.5 billion years and are expected to continue for similar time period in the future.

Q.43 Distinguish between nuclear fission and nuclear fusion.

Ans:

No	Nuclear fission	Nuclear fusion
i	It is the process in which a heavy nucleus splits up into two lighter nuclei of nearly equal masses.	It is the process in which two lighter nuclei combine together to form a heavy nucleus.
ii.	Nuclear fission may take place at ordinary temperature.	A very high temperature of the order of million of degree is required.
iii.	The sources of fissionable materials is limited.	The sources or fusion reaction i.e haydrogen is more plentiful (air and water)
iv.	The products of nuclear fission are in general radioactive and hence pose a radiation hazard.	The products of fusion are non-radioactive and pose no radiation hazard.

Type - X

Numerical based on Nuclear energy

1) Calculate the energy release in the reaction.

$${}^{236}_{92}\mathbf{U} \rightarrow {}^{137}_{53}\mathbf{I} \rightarrow {}^{97}_{39}\mathbf{Y} \rightarrow \mathbf{2n}.$$

The masses of ${}^{236}_{92}U, {}^{137}_{53}I$ and ${}^{97}_{39}Y$ are 236.04557, 136.91787 and 96.91827 respectively.

Data : $m_U = 236.04557$ u,
$m_I = 136.91787$ u
$m_Y = 96.91827$ u

Reaction: ${}^{236}_{92}U \rightarrow {}^{137}_{53}I \rightarrow {}^{97}_{39}Y \rightarrow 2n$

To Find : Energy released in the reaction (Q value)

Formula : $Q = [m_{parent} - m_{product}]c^2$

Solution : $Q = [m_{parent} - m_{product}]c^2$

$Q = [m_U - m_I - m_Y - 2mn]c^2$

$= [236.04557 - 136.91787 - 96.91827 - 2\times1.00856]c^2$

We know that

$1\ u = 931.5\ \frac{MeV}{c^2}$

$= 0.19213\times931.5$ MeV

$= 178.969095$ MeV

Ans :	Energy released in the equation is 178.969095 MeV

2) **Calculate the energy released in the fusion reaction taking place inside the Sun. $4p \rightarrow \alpha + 2e^+ + \text{neutrino}$ neglecting the energy given to the neutrinos. Mass of alpha particle being 4.001506 u.**

Data : $m_\alpha = 4.001506$ u

We know that, mp = 1.00728 u

me+ = 0.00055u

Reaction : $4p \rightarrow \alpha + 2e^+ + \text{neutrino}$

To Find : Energy release in the fusion reaction (Q value)

Formula : $Q = [m_{parent} - m_{product}]c^2$

Solution :

$Q = [m_{parent} - m_{product}]c^2$

$Q = [4\times m_p - m_\alpha - 2\times m_e]c^2$

$\ldots[\because M_{neutrino} = 0]$

$Q = [4\times1.00728 - 4.001506 - 2\times0.00055]c^2$

We know that

$1\ u = 931.5\ \frac{MeV}{c^2} = 0.026514\times931.5$

$= 24.698$ MeV

Ans :	Energy released in the fusion reaction is 24.698 MeV

3) **How much mass of ^{235}U is required to undergo fission each day to provide 3000 MW of thermal power? Average energy per fission is 202.79 MeV**

Data : Average energy per fission of ^{235}U

$= 202.79$ MeV

$= 202.79\times1.6\times10^{-19}\times10^6$ J

$= 3.245\times10^{-11}$ J

Also total power generation by thermal Power plant = 3000 MW per day

$= 3\times10^6\times24\times60\times60$ s

$= 2.592\times10^{14}$ J

To Find : Mass of U - 235

Formulae :

i. Total number atoms producing this energy

$$(N) = \frac{\text{Total energy}}{\text{Energy Produced by 1 atoms}}$$

ii. Mass(m) = no. of mass × molar mass

Solution :

i. $$(N) = \frac{\text{Total energy}}{\text{Energy Produced by 1 atoms}}$$

$$(N) = \frac{2.592\times10^{14}}{3.245\times10^{11}} = 7.99\times1024$$

ii. Mass(m) = no. of mass × molar mass

m =

$$m = \frac{N}{N_A}\times235$$

$$= \frac{7.99\times10^{24}}{6.022\times10^{23}}\times235$$

$= 1.326\times2350$

$= 3116.1$ g $= 3.1$ kg

Ans :	Mass of 235 U required is 3.1 kg

❑❑❑