2 Mechanical Properties of fluids

Syllabus

2.1 Introduction

Q.1 What is fluid?

Ans: Fluid is a substance that can flow.

Liquids and gases can flow. Therefore, they are fluids.

Note:

Fluid statics. *The branch of physics that deals with the study of fluids at rest is called fluid statics or hydrostatics.Its study includes hydrostatic pressure, Pascal's law, Archemedes' principle, floatation of bodies and surface tension.*

Fluid dynamics: *The branch of physics that deals with the study of fluids in motion is called fluid dynamics or hydrodynamics. Its study includes equation of countinuity, Bernoulli's theorem, Toricelli's theorem, viscosity, etc.*

2.2 Fluid

Note:

Difference between liquid and gas :

A liquid is incompressible and has a definite volume and a free surface of its own. A gas is compressible and it expands to occupuy all the space available to it.

Note:

Thrust at a point in fluid column:

i. *The total normal force exerted by a liquid on the surface is called thrust.*
ii. *If force acting on equal areas are equal then thrust is said to be uniform.*
iii. *Unit: S.1. unit is N and C.G.S. unit is dyne.*
iv. *Dimensions: $[M^1L^1T^{-2}]$*

⋆Q.2 What is an incompressible fluid?

Ans: An incompressible fluid is a fluid whose density does not change with change in pressure i.e., the density remains constant.

Q.3 State the properties of an ideal fluid.

Ans: An ideal fluid has the following properties:

i. It is incompressible: its density is constant.
ii. Its flow is irrotational: its flow is smooth, there are no turbulences in the flow.
iii. It is nonviscous: there is no internal friction in the flow, i.e., the fluid has no viscosity.
iv. Its flow is steady: its velocity at each point is constant in time.

Q.4 State the properties of fluid

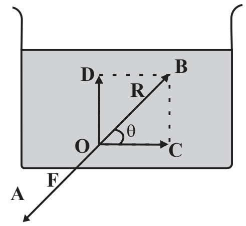
Ans: Properties of Fluids:

i. They do not oppose deformation, they get permanently deformed.
ii. They have ability to flow.
iii. They have ability to take the shape of the container.

2.2.1 Fluids at Rest

Note:

Consider a liquid contained in a vessel in the equilibrium state of rest. Suppose the liquid exerts a force F on the bottom surface in an inclined direction OA. The surface exerts an *equal reaction R to water along OB.*



The reaction R along OB has two rectangular components:

i. Tangential component, OC = R$\cos\theta$

ii. Normal component, OD = R$\sin\theta$

Since a liquid cannot resist any tangential force, so the liquid near O should begin to flow along OC. But the liquid is at rest, the force along OC must be zero.

$\therefore$ $R\cos\theta = 0$

As $R \neq 0$, *so* $\cos\theta = 0$ *or* $\theta = 90°$

Hence a liquid always exerts force perpendicular to the surface of the container at every point.

Q.5 Define pressure. State the S.I. and C.G.S. units and dimensions.

Ans:

i. Pressure at a point in liquid column:

a. The thrust exerted by a liquid at rest per unit area normal to the surface in contact with the liquid is called pressure.

$$P = \frac{F}{A}$$

b. The pressure at a point in a fluid is independent of direction, thus pressure is a scalar quantity.

ii. Units and dimentions:

a. Unit: Nm^{-2} or Pascal in S.I. system and dyne / cm^2 in C.G.S. system.

b. Dimensions: $[M^1L^{-1}T^{-2}]$

Type - I

Numerical Based on Thrust and pressure

1) The two thigh bones (femurs), each of cross-sectinal area 10 cm² support the upper part of human body of mass 40 kg. Estimate the average pressure sustained by the femurs. Take g = 10 ms⁻²

Data: $A = 20\ cm^2 = 20\times10^{-4}\ m^2$, $m = 40$ kg

To find: P_{av}

Formula: $P_{av} = \frac{F}{A} = \frac{mg}{A}$

Solution:

$$P_{av} = \frac{mg}{A} = \frac{\overset{2}{\cancel{40}}\times10}{\underset{1}{\cancel{20}}\times10^{-4}} = 2\times10^5\ Pa$$

Ans :	The average pressure sustained by the fermurs is 2×10^5 pa

2) How large the force does the air in the room exert on the inside window pan that is 20cm×60cm ? Atomospheric pressure is nearly 100 kPa.

Data: $P = 100\ kPa = 10^5 Pa$

$A = 20cm \times 60cm = 1200cm^2$

$= 1200 \times 10^{-4} m^2$

To find: F

Formula: $P = \frac{F}{A}$

$\therefore$ $F = P \times A$

Solution:

$F = P \times A = 10^5 \times 1200 \times 10^{-4}$

$= 1200\times10\ N = 12\ kN$

Ans :	Force exerted on window is 12 kN

★3) A student of mass 50 kg is standing on both feet. Estimate the pressure exerted by the student on the Earth. Assume reasonable value to any other quantity you need. Justify your assumption. You may use g = 10 ms⁻². By what factor will it change if the student lies on back?

Data: $m = 50$ kg, $g = 10\ m/s^2$

To find: i. $P_{standing}$ ii. P_{lying}

Formula: $P = \frac{Force}{Area}$

Solution:

i. <u>When the student is standing,</u>

let area of one foot = 150 m^2

$\therefore$ Area of both the feet = $2 \times 15 \times 10^{-4}$

$= 3 \times 10^{-4} m^2$

$$P_{standing} = \frac{Force}{Area} = \frac{Mg}{A} = \frac{50\times10}{3\times10^{-2}}$$

$= 166.67 \times 10^2$

$= 16.67 \times 10^3$ Pa

$= 16.67$ kpa

ii. When student lying on back

Area of back of student is approximately 0.3 m^2 (approximately)

$$P_{lying} = \frac{Force}{Area} = \frac{Mg}{Al_{ying}} = \frac{50\times10}{0.3}$$

= 1666.67 Pa

= 1.667 kpa

Factor by which the pressure changes while lying is given as

$$\frac{P_{lying}}{P_{standing}} = \frac{1.667}{16.67} = 0.1$$

$P_{lying} = 0.1\ P_{standing}$

Thus, the factor by which the pressure changes while lying is given as,

$$P_{lying} = \frac{0.3\times10^3}{25\times10^3} P_{Standing}$$

∴ $P_{lying} = \frac{3}{250} P_{Standing}$

∴ $P_{lying} = 0.012 P_{Standing}$

Ans : i. The pressure exerted by the student on the Earth when standing is 16.67 kpa.

ii. The pressure changes by a factor of 0.1 when the student lies on his back.

Problem for Practice

1. How much pressure will a man of weight 60 kgf exert on the ground when (i) he is lying and (ii) he is standing on his feet? Given that the area of the body of the man is 0.6 m^2 and that of a foot is 60 cm^2.

Ans: 980 Nm^{-2}, 4.9 × $10^4 Nm^{-2}$

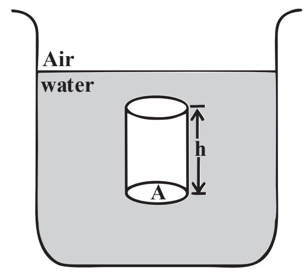
2. The force on a phonograph needle is 1.2N. The point has a circular cross-section whose radius is 0.1 mm. Find the pressure (in atm) it exerts on the records. Given 1atm = 1.013 × 10^5Pa. **Ans: 377 atm**

Q.6 Derive an expression for pressure exerted by liquid column.

Ans: Expression for pressure at a point due to liquid column:

i. Suppose a cylindrical vessel having area of cross-section 'A' contains a homogeneous liquid of density 'ρ' upto height 'h'

ii. Weight of liquid column exerts a downward thrust.



iii. Total normal force exerted at point A

= weight of liquid column

i.e. F = mg

mass of liquid =

density of liquid × volume of liquid

∴ m = ρV

where,

∴ F = Vρg

iv. Also, V = Ah

∴ F = Ahρg ....(1)

From definition,

From defination of pressure

$$pressure = \frac{F}{A}$$

∴ $P = \frac{\cancel{A}h\rho g}{\cancel{A}}$ ... (from 1)

$\boxed{P = h\rho g}$ ... (2)

Equation (2) represents the value of pressure due to liquid at any point inside the liquid.

upon:

a. height ofliquid column

b. density of the liquid and

c. acceleration due to gravity.

Q.7 What is atmospheric pressure?

Ans:

i. The gaseous envelope surrounding the earth is called the atmosphere.

ii. The pressure exerted by the atmosphere is called atmospheric pressure.

iii. The force exerted by air column of air on a unit area of the earth's surface is equal to the atmospheric pressure.

iv. The atmospheric pressure at sea level is $1.013\times10^5 Nm^{-2}$ or Pa.

Q.8 Derive an expression for absolute pressure P at a depth d below the Surface of a liquid and at a height h above the liquid surface.

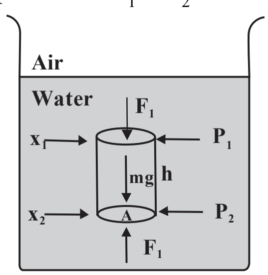
Ans:

i. Consider a tank filled with liquid of density 'ρ'. Suppose a cylinder having horizontal base area of cross-section 'A' and 'h' is imagined to be inside the tank.

Here,

$h = x_1 - x_2$

where x_1 and x_2 are the heights measured from a reference point and $x_1 > x_2$.



ii. Let P_1 and P_2 be the pressures of liquids at the points x_1 and x_2 respectively. The liquid cylinder is under the acation of following vertical forces:

a. Force due to the weight of the water column above the cylinder, acting vertically downwards on the top surface of the cylinder.

$F_1 = P_1A$ ....(1)

b. Force due to the water below the cylinder, acting vertically upwards on the lower surface of the cylinder.

$F_2 = P_2A$ ....(2)

c. Weight of the liquid cylinder acting vertically downwards.

$W = mg$ ....(3)

iii. But, $m = \rho V$

where,

m = mass of liquid, V = volume of the cylinder,

ρ = density of liquid

$\therefore$ $m = \rho Ah$ ....($\because V = Ah$)

$m = \rho A (x_1 - x_2)$ ....(4)

iv. As the water is in static equilibrium, the forces on the cylinder are balanced. Therefore, the net force on the cylinder is zero.

$F_1 + mg - F_2 = 0$

$\therefore$ $F_2 = F_1 + mg$ ....(5)

v. Substituting equations (1), (2), (3), and (4) in equation (5),

$P_2A = P_1A + \rho Ag(x_1 - x_2)$

$\therefore$ $P_2 = P_1 + \rho g (x_1 - x_2)$ ....(6)

vi. If the top of the cylinder is shifted to the surface of the liquid which is exposed to the atmosphere, then

$P_1 = P_0$ i.e., atmospheric pressure at the surface

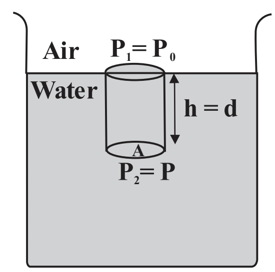
$x_1 = 0, x_2 = -h = -d$ (depth below the surface)

and $P_2 = P$

Substituting these values in equations (6),

$P = P_0 + d\rho g$ ....(7)

Equation (7) represents expression for absolute pressure P at a depth d below the surface of the liquid.



vii. Similarly, if the cylinder is above the liquid surface by a distance h, then,

$P_2 = P_0$ i.e., atmospheric pressure at the surface

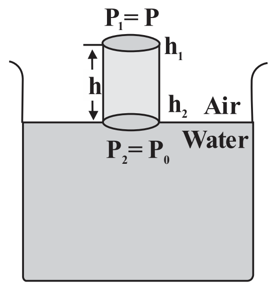
$P_1 = P, x_1 = h, x_2 = 0$ and $\rho = \rho_{air}$

Substituting these values in equation (6),

$P_0 = P + h\rho_{air} g$

$P = P_0 - h \rho_{air} g$ ....(8)

Equation (8) reprsents experssion for absolute pressure P at a height h above the surface of the liquid.



Q.9 Explain the term gauge pressur.

Ans:

i. The total pressure P at depth d below the surface of liquid at rest, which is open to atmosphere, is given as,
$P = P_a + d\rho g$ ....(1)

ii. This shows that the total pressre P is greater than atmospheric pressure by an amount equal to $d\rho g$.

iii. This excess of pressure at depth d in liquid is called gauge pressure.

∴ $P - P_a = d\rho g$ ....(2)

iv. Thus, gauge pressure at a point in a liquid is the difference between the absolute pressure and the atmospheric pressure.
Gauge pressure =
∴ Absolute pressure – Atmospheric pressure

Type - II
Numerical based on Pressure exerted by liquid column

Formula used:-

1. Pressure exerted by liquid column
$P = h\rho g$
2. Absolute pressure = Atmospheric pressure + Gauge pressure

Conversion used:-

1atm = 1.013×10^6 dyne cm^{-2}
= 1.013×10^5 Nm^{-2} (or Pa)
1bar = 10^6 dyne cm^{-2} = 10^5 Nm^{-2}
1 millibar (m bar) = 10^{-3}bar = 10^3dyne cm^{-2}
= 10^2 Nm^{-2}
1 torr = 1 mm Hg
1 atm = 101.3 kPa = 1.013 bar = 760 torr.

★1) Two different liquids of density ρ_1 and ρ_2 exert the same pressure ata a certain point. What will be the ratio of the heights of the respective liquid colums?

Data : For liquid 1
Density = ρ_1
height of liquid column = h_1
For liquid 2
Density = ρ_2
height of liquid column = h_2
Also $P_1 = P_2$

To Find : $\frac{h_1}{h_2}$

Formula: $P = h\rho g$

Solution:

i. Pressure exerted by liquid - 1
$P_1 = h_1\rho_1 g$ ... (1)

ii. Pressure exercted by liquid - 2
$P_2 = h_2\rho_2 g$

iii. According to given condition
$P_1 = P_2$

∴ $h_1\rho_2 \cancel{g} = h_2\rho_2 \cancel{g}$

$$\boxed{\frac{h_1}{h_2} = \frac{\rho_1}{\rho_2}}$$

Ans : The ratio of the height of the liquid column is $\frac{\rho_2}{\rho_1}$

★2) A swimmer is swimming in a swimming pool at 60 m below the surface of the water. Calculate the pressure on the swimmer due to water above.
(Density water = 1000 kg/m^3, g = 9.8 m/s^2)

Data: h = 60m, ρ = 1000kg/m^3, g = 9.8 m/s^2

To find: P

Formula: $P = h\rho g$

Solution: $P = h\rho g = 6 \times 1000 \times 9.8$
$= 5.88 \times 10^5$ N/m^2

Ans : The pressure on the swimmer due to water above is 5.88×10^5 N/m^2

★3) Find the pressure 200 m below the surface of the ocean if pressure on the free surface of liquid is one atmosphere. (Density of sea water = 1060 kg/m^3)

Data: ρ_{sw} = 1060 kg/m³, h = 200m,
P_0 = 1atm = 1.013 × 10^5 Pa
To find: P
Formula: $P = P_0 + h\rho g$
Solution:

$P = P_0 + h\rho g$
$P = 1.013\times10^5 + 200\times1060\times9.8$
$= 1.013\times10^5 + 20.776\times105$
$\therefore$ $P = 21.789 \times 10^5$ N/m²

Ans : Pressure at 200 m below the surface of ocean is 21.789×10^5 N / m²

4) **Express standard atmospheric pressure in i. Nm^{-2} ii. bars and iii. torr.**

Data: Standard atmospheric pressure
P = 1 atm
To Find : i. P (in N/m2)
ii. P (in bars)
iii. P (in torr)
Formula:
Solution:

i. P =1 atm
$= 1 \times 1.013 \times 10^5$
$= 1.013 \times 10^5$ N/me
ii. As 1 $Nm^{-2} = 10^{-5}$bar
$\therefore$ 1 atm = 1.013 bar = 1013 millibar.
iii. As 1 torr = 1mm of Hg
$\therefore$ 1 atm = 760 mm of Hg = 760 torr.

Ans :

Problem For Practice

1. What is the pressure on a swimmer 10 m below the surface of a lake?
Ans: 86.6cm

2. The density of the atmosphere at sea level is 1.29 kg m^{-3}. Assume that it does not change with altitude. Then how high would the atmosphere extend?
Take g = 9.81 ms^{-2}.
Ans: 8 km

3. At a depth of 1000 m in an ocean (a) What is the absolute pressure? (b) What is the gauge pressure? (c) Find the force acting on the window of area 20 cm × 20 cm of a submarine at this depth, the interior of which is maintained at sea-level atmospheric pressure. (The density of sea water is 1.03×10^3 kg m^{-3}, g = 10 ms^{-2})
Ans:104 atm, 103 atm, 4.12×10⁵N.

Q.10 Explain the term hydrostatic paradox?

Ans:

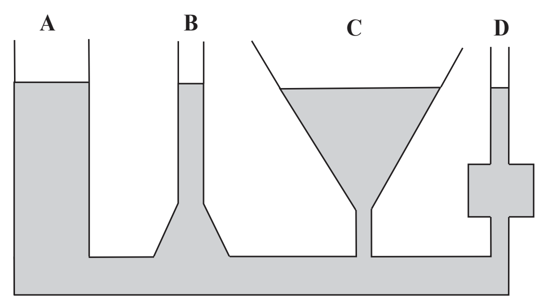
i. Consider the inter connected vessels as shown in figure. When a liquid is poured in any one of the vessels, it is noticed that the, level of liquids in all the vessels is the same. This obervation is somewhat puzzling. It was called 'hydrostatics paradox.

ii. One can feel that the pressure of the base of the vessel C would be more than the at the base of the vessel B and the liquid from vessel C would rise into the vessel B. However, it is never observed.

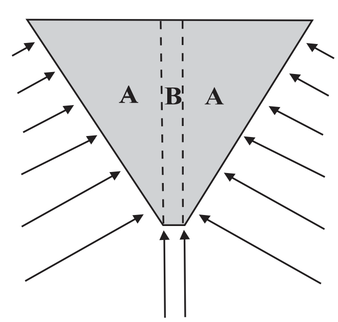
iii. $P = h\rho g$ tells that the pressure at a point depends only on the height of the liquid column above it. It does not depend on the shape of the vessel.

iv. In this case, height of the liquid column is the same for all the vessels. Therefore, the pressure of liquid column in each vessel is the same and the system is in equilibrium.

v. That means the liquid in vessel C does not rise in to vessel B.



vi. Consider the forces exerted by the walls of the vessel on the liquid. These forces are perpendicular to walls of the vessel at each point as shown in figure.



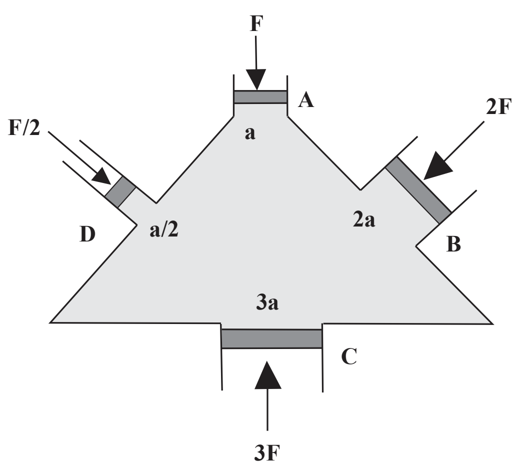
Each of these forces can be resolved into their vertical and horizontal components. The vertical components which acts in the upward direction supports the weight of the liquid above the wall. Thus, only the weight in cylindrical column of the liquid in section B is not balanced and contributes towards the pressure at the base. This face resolves the hydrostatic paradox.

Q.11 State Pascal's law of fluid pressure. Describe the experimental proof for the same.

Ans: Statement: *The pressre applied at any point of an enclosed fluid at rest is transmitted equally and undiminished to every point of the fluid and also on the walls of the container, provided the effect of gravity is neglected.*

Explanation:

i. Consider a vessel with four arms A, B, C and D having different cross-sectional areas a, 2a, 3a and a/2 respectively filled with incompressible fluid and fitted with frictionless, water tight pistons



ii. Initially, the enclosed water is at rest. Now Suppose that the piston A is pushed down with a force F so that the pressure exerted by it on the fluid as,

$$P_A = \frac{F}{a}$$

iii. It is found that the pistons B, C and D can be prevented from moving backwards only if forces 2F, 3F and F/2 are exerted on them respectively.

iv. The pressure on the pistons B, C and D are,

piston B : $P_B = \frac{2F}{2a} = \frac{F}{a} = P_A$

piston C : $P_C = \frac{3F}{3a} = \frac{F}{a} = P_A$

piston D : $P_D = \frac{F/2}{a/2} = \frac{F}{a} = P_A$

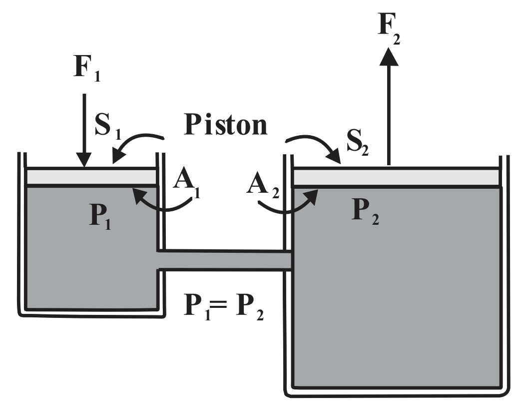
i.e., $P_A = P_B = P_C = P_D$

v. This shows that the applied pressure on A is transmitted undiminished on pistons B, C and D as required by Pascal's law.

Q.12 Explain the concept of a hydraulic lift.

Ans: Hydraulic lift:

i. Hydraulic lift is used to lift a heavy object using small force.



ii. A tank containing a fluid is fitted with two pistons S_1 and S_2. S_1 has a smaller area of cross section A_1 while S_2 has a much larger area of cross section A_2 (i.e., $A_2 >> A_1$).

iii. Piston S_1 is used to exert a force F_1 directly on the liquid. The pressure $P_1 = \frac{F_1}{A_1}$ is transmitted throughout the liquid undiminished to the larger piston S_2.

iv. This results in an upward force on the larger piston S_2, given as,

$F_2 = P_2 A_2$

But, according to Pascal's law $P_2 = P_1 = \frac{F_1}{A_1}$

$\therefore \quad F_2 = F_1 \times \frac{A_2}{A_1}$

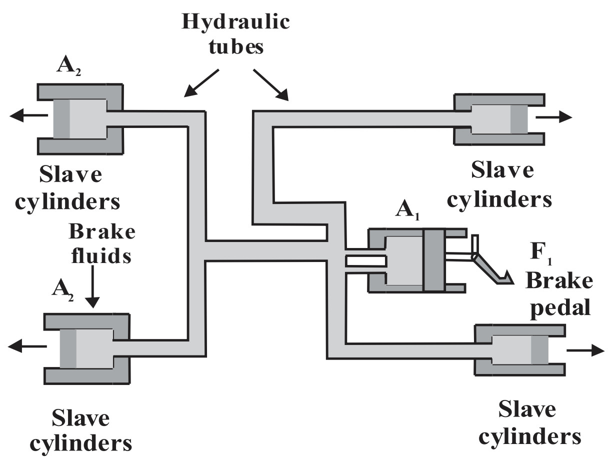
Since, A_2 is bogger than A_1 So F_2 is much larger than F_1

v. This shows that a small force applied on the smaller piston will be appearing as a very large force on the larger piston. As a result of it, a heavy load placed on the larger piston can be easily lifted upwards or moved down.

Q.13 Explain the use of Pascal's law in hydraulic brakes

Ans: Hydraulic brakes:

i. Hydraulic brakes are based on the principle of Pascal's law.



ii. It consists of a master cylinder of area of cross section A_1. One end of the cylinder is attached to a frictionless airtight piston P. The piston is connected to a brake pedal.

iii. The other end of the master cylinder is filled with brake fluid (oil) and connected through hydraulic tubes to slave cylinders of area of cros section A_2 such that A_2 is much greater than A_1.

iv. When the brake pedal is applied by a small force F_1, the piston of master cylinder is pushed in the forward direction. The pressure generated is given as,

$$P = \frac{F_1}{A_1} \qquad ....(1)$$

v. As a result, the piston in the slave cylinder also moves in forward direction so as to maintain the volume of the oil constant. The slave piston pushes the friction pads against the rotating disc, which is connected to the wheel causing the moving vehicle to stop.

vi. The force F_2 on slave cylinder is then,

$F_2 = PA_2$

$$\therefore \quad F_2 = F_1 \frac{A_2}{A_1} \qquad ...(2)$$

Since A_2 is greater than A_1 much greater F_2 is also much greater than F_1.

vii. This shows that a small force applied on the brake pedal gets converted into a large force which slows down or stops a moving vehicle.

Type - III

Numerical based on Pascal's law

Formlae used

1. The pressure applied at any point of an enclosed mass of fluid is transmitted equally in all directions.

2. $P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$

★1) A hydraulic brake system of a car of mass 1000 kg having speed of 50 km/h, has a cylindrical piston of radius of 0.5 cm. The slave cylinder has a radius of 2.5 cm. If a constant force of 100N is applied on the brake, what distance the car will travel before coming to stop?

Data: $m = 1000 kg, F_1 = 100 N,$

$r_1 = 0.5\text{ cm} = 0.5 \times 10^{-2}\text{ m}$

$r_2 = 2.5\text{ cm} = 2.5 \times 50^{-2}\text{m}$

$v = 0,$

$u = 50\text{ km/h} = 50 \times \frac{5}{18}\text{ m/s}$

$= \frac{5}{18} = \frac{125}{9}\text{ m/s}$

To find: s

Formulae: i. $\frac{F_2}{A_2} = \frac{F_1}{A_1}$ ii. $a = \frac{F}{m}$

iii. $v^2 = u^2 - 2as$

Solution:

i. $\frac{F_2}{A_2} = \frac{F_1}{A_1}$

$$\frac{F_2}{\pi r_2^2} = \frac{F_1}{\pi r_1^2} = \frac{F_2}{\pi(2.5 \times 10^{-2})^2} = \frac{100}{\pi(0.5 \times 10^{-2})^2}$$

$\therefore \quad F_2 = \dfrac{100\times(2.5\times10^{-2})^2}{(0.5\times10^{-2})^2} = 2500\text{ N}$

ii. Acceleration of the car,

$$a = \frac{F_2}{m}$$

$\therefore \quad a = \dfrac{2500}{1000} = 2.5\text{m/s}^2$

iii. $v^2 = u^2 - 2as$

$$0^2 = \left(\frac{125}{9}\right)^2 - 2(2.5)s$$

$$5s = \left(\frac{125}{9}\right)^2$$

$\therefore \quad s = \left(\dfrac{125}{9}\right)^2 \times \dfrac{1}{5}$

$$= \frac{3125}{81} = 38.58\text{m}$$

Ans : The car stops after travelling 38.58m.

★2) In a hydraulic lift, the input piston had surface area 30 cm² and the output piston has surface area of 1500 cm². If a force of 25N is applied to the input piston, calculate weight on output piston.

Data: $A_1 = 30\text{ cm}^2$, $A_2 = 1500\text{ cm}^2$, $F_1 = 25\text{ N}$

To find: F_2

Formula: $\dfrac{F_2}{A_2} = \dfrac{F_1}{A_1}$

Solution:: $F_2 = \dfrac{F_1}{A_1} \times A_2 = \dfrac{25\times1500}{30}$

$\therefore \quad F_2 = 1250\text{ N}$

Ans : Weight on the output piston is 1250N.

Problem for Practice

1. In a car lift compressed air exerts a force F_1 on a small piston having a radius of 5cm. This pressure is transmitted to a second piston of radius 15 cm. If the mass of the car to be lifted is 1350 kg, what is F_1? What is the pressure necessary to accomplish this task? Take $g = 9.81\text{ ms}^{-2}$.

Ans: 1.9×10^5 Pa

2. The neck and bottom of a bottle are 2cm and 10cm in diameter respectively. If the cork is pressed with a force of 1.2 kg f in the neck of the bottle, calculate the force exerted on the bottom of the bottle.

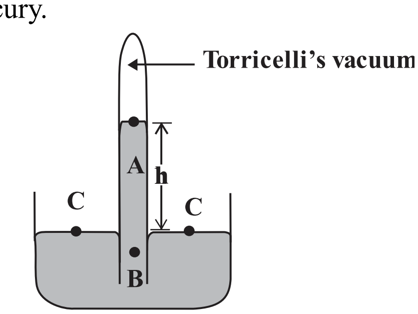
Ans : 30 kg f

Q.14 What is barometer. Write the note on mercury barometer.

Ans:

i. An instrument that measures atmospheric pressure is called a barometer.

ii. The barometer is in the form of a glass tube completely filled with mercury and placed upside down in a small dish containing marcury.



iii. A glass tube of about 1 meter length and a diameter of about 1 cm is filled with mercury up to its brim. It is then quickly inverted into a small dish containing mercury. The level of mercury in the glass tube lowers as some mercury spills in the dish. A gap is created between the surface of mercury in the glss tube and the closed end of the glass tube. The gap does not contain any air and it is called Torricelli's vacuum. It do contain some mercury vapors.

iv. Thus, the pressure at the upper end of the mercury column inside the tube is zero, i.e. pressure at point such as A is p_A = zero.

v. Let us consider a point C on the mercury surface in the dish and another point B inside the tube at the same horizontal level as that of the point C.

vi. The pressure at C is equal to the atmospheric pressure p_0 because it is open to atmosphere. As points B and C are at the same horizontal level, the pressure at B is also equal to the atmospheric pressure p_0, i.e. $p_B = p_0$.

vii. Suppose the point B is at a depth h below the point A and ρ is the density of mercury then,
$p_B = p_A + h\rho g$
$p_A = 0$ (there is vacuum above point A) and $p_B = p_0$, therefore, $p_0 = h\rho g$, where h is the length of mercury column in the mercury barometer.

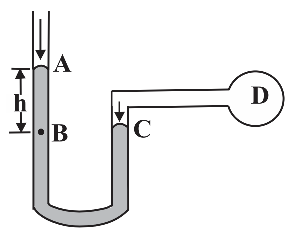
Q.15 Write a note on open tube manometer.

Ans:

i. A manometer consists of a U – shaped tube partly filled with a low density liquid such as water or kerosene.

ii. This helps in having a larger level difference between the level of liquid in the two arms of the manometer.

iii. One arm of the manometer is open to the atmosphere and the other is connected to the container D of which the pressure p is to be measured.



iv. The pressure at point A is atmospheric pressure p_0 because this arm is open to atmosphere.

v. To find the pressure at point C, which is exposed to the pressure of the gas in the container, we consider a point B in the open arm of the manometer at the same level as point C.

vi. The pressure at the points B and C is the same, i.e.,
$p_C = p_B$ ....(1)

vii. The pressure at point B is,
$p_B = p_0 + h\rho g$ ....(2)
where, ρ is the density of the liquid in the manometer, h is the height of the liquid column above point B, and g is the acceleration due to gravity.

viii. According to Pascal's principle, pressure at C is the same as at D, i.e., inside the chamber. Therefore, the pressure p in the container is,
$p = p_C$

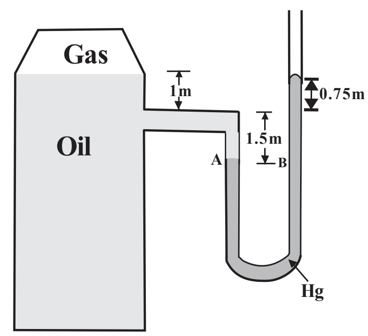
ix. Using equation (1) and (2) we can write,
$p = p_0 + h\rho g$

x. As the manometer measures the gauge pressure of the gas in the container D, we can write the gauge pressure in the container D as
$p - p_0 = h\rho g$

Type - IV

Numerical based on open tube manometer.

1) What is the absolute and gauge pressure of the gas above the liquid surface in the tank shown in figure? Density of oil = 820 kg m^{-3}, density of mercurry = 13.6×10^{3} kgm^{-3}. Given 1 atmospheric pressure = 1.01 × 10^{5} Pa.



Solution:

i. Let p is the pressure of the gas in the tank.
Pressure at point A
$P_A = P + h_{oil}\rho_{oil}\, g$
$= p + (1+1.5) \times 820 \times 9.8$
$= P + 20090$

ii. Pressure at point B
$P_B = P_0 + h_m \rho_m\, g$
$= P_0 + (1.50 + 0.75) \times 13.6 \times 10^3 \times 9.8$
$= P_0 + 299880$

iii. $P_A = P_B$
$P + 20090 = P_0 + 299880$
$P - P_0 = 299880 - 20090 = 279790$
Gauge pressure = 2.8 × 10^{5} pa

iv. Absolute pressure =
Gauge pressure + Atmospheric pressure
= 2.8 × 10^{5} + 1.01 ×10^{5} = 3.81 × 10^{5} Pa.

Ans : Gauge pressure of gas is 2.8×105 pa and
Absolute pressure of gas is 3.81×10^{5} Pa

Problem for Practice

1. A vertical U-tube of uniform inner cross-section contains mercury in both of its arms. A glycerine (density 1.3 g cm^{-3}) column of length 10 cm is introduced into one of the arms. Oil of density 0.8 g cm^{-3} is poured in the other arm until the upper surfaces of the oil and glycerine are in the same horizontal level. Find the length of the oil column.

Ans : 9.6cm

2.4 Surface Tension

Q.16 Explain the term Intermolecular force of attraction.

Ans: Intermolecular force of attraction :

i. Between any two molecules of a liquid, there exists a force of attraction. This force is called the intermolecular force.

ii. The intermolecular force is a short range force, i.e., it is effective over a very short distance, (about 10^{-9}m). Beyond this distance, the force is negligible.

iii. Intermolecular forces of attraction are of two types :

a. Cohesive force b. Adhesive force.

Q.17. Define:

i. Cohesion ii. Cohesive force

iii. Adhesion iv. Adhesive force

v. Range of molecular attraction

vi. Sphere of influence.

Ans:

i. Cohesion :

Attraction between two molecules of the same substance is called cohesion.

ii. Cohesive force:

a. The force of attraction between two molecules of the same substance is called the cohesive force.

b. It is strongest in solids and weakest in gases.

c. Example : Force of attraction between water molecule, two oxygen molecules etc.

iii. Adhesion:

Attraction between two molecules of the different substances is called adhesion.

iv. Adhesive force:

The force of attraction molecules of the different substances is called adhesive force.

Example : Force of attraction between water molecules and steel glass.

v. Range of molecular attraction :

The maximum distance between two molecules up to which the intermolecular forces are effective is called the range of molecular attraction.

vi. Sphere of influence:

An imaginary sphere drawn round a molecule as a centre, with a radius equal to the range of molecular attraction, is called the sphere of influence of that molecule.

Q.18 Define surface film and free surface of liquid.

Ans: Surface film:

A layer of surface of a liquid whose thickness is equal to the range of intermolecular force is called surface fim.

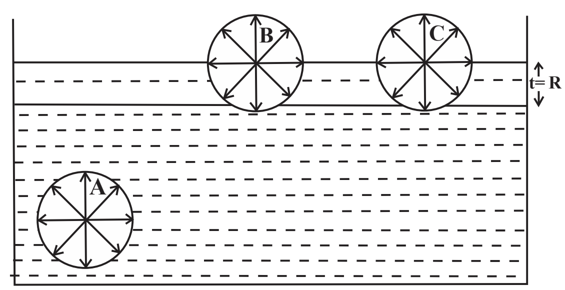
Free suface of liquid:

Surface of liquid which does not experience any shear stress is called free surface of liquid.

Example : The interface between liquid water and air above.

Q.19 Explain the phenomenon of surface tension on the basis of molecular theory.

Ans:



i. The molecule inside the liquid is attracted equally in all directions, so that the resultant force acting on it is zero.

ii. Consider three molecules A, B, C.

Molecule A : molecule lies well within the liquid surface and If we draw sphere of influence then it lies within the liquid.

It is equally attracted in all direction with cohesive force, hence resultant force on molecule A is zero.

Molecule B : Molecule 'B' lies within the surface film and if we draw sphere of influence then larger part of sphere lies inside the liquid and small part will lie on the surface (in air) Net cohesive force is stronger than adhesive force; hence molecule 'B' is pulled downwords.

Molecule C : molecule 'C' lies on the surface of water and if we draw sphere of influence then half part will lie in the air and half part in the liquid. Resultant cohesive force is stronger than adhesive force. Hence molecule 'C' experiences downword pull.

iii. Hence, when any molecule is brought to the surface from inside the liquid, the work is done against the resultant force pulling the molecule in the downward direction. This work gives potential energy to the liquid surface.

iv. Again, the liquid surface must have minimum potential energy. Therefore, minimum number of molecules should remain on the surface.

v. Therefore, the surface develops a tendency to contract to minimise its area, and to behave like a stretched elastic membrane.

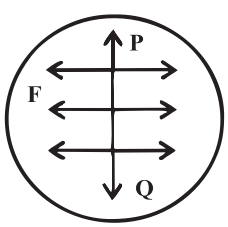
Q.20 Define surface energy and explain the term surface tension.

Ans: Surface energy :

The potential energy per unit increase in area of the liquid surface is called the surface energy.

Surface tension :

i. Surface tension of a liquid is defined as the force per unit length, acting at right angles to an imaginary line drawn in the free surface of a liquid.



ii. Let, L - length of the line

F - force acting on it,

The surface tension (T) of the liquid is given by,

$$T = \frac{F}{L}$$

iii. **Units and dimensions:**

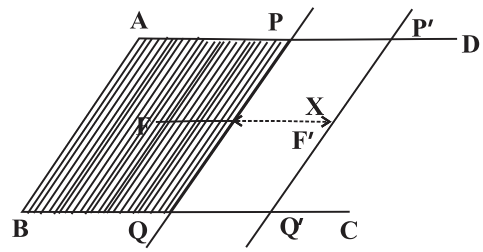
a. **SI unit :** N/m

b. **CGS unit :** dyne/cm

c. **Dimensions :** $[M^1L^0T^{-2}]$

Q.21 Derive the relation between surface tension and surface energy per unit area.

Ans:



i. ABCD is an open rectangular frame of wire on which a wire PQ can slide without friction.

ii. Dip the frame into soap solution. A soap film APQB is formed. Due to surface tension, a force F acts on the wire PQ of length L tending to pull it towards AB.

F = 2 TL

(The factor 2 indicates that the soap film has two surfaces, both of which are in contact with the wire)

iii. Let, the wire PQ be pulled outward through a small distance dx to be in a position P'Q'.

Work done in the process,

dw = applied force × displacement

dw = F dx,

dw = 2 TL dx

Now, the change in surface area of the film is,

dA = 2L dx

iv. dw = T × (2 L dx)

dw = T × d A

$$T = \frac{dw}{dA}$$

v. This potential energy of the surface per unit area is surface energy.

∴ Surface energy per unit area = Surface tension

★Q.22 How much amount of work is done in forming a soap bubble of radius r?

Ans:

i. Work done against force due to surface tension,

$W = T(dA)$ ...(1)

ii. In case of bubble of radius r, there are two surfaces in contact with air. Therefore the total surface area of bubble is given as,

$dA = 2 \times (4\pi r^2) = 8\pi r^2$

iii. Substituting dA in eq (1)

$W = T(dA) = T(8\pi r^2) = 8\pi r^2 T$

★Q.23 Why is the surface tension of paints and lubricating oils kept low?

Ans: Surface tension of lubricating oils and paints is kept low in order to help them spread over a large area.

Type - V
Numerical based on Surface tension

Formulae Used

Surface tension $(T) = \frac{F}{L}$

1) Calculate the force required to take away a flat circular plate of radius 0.01 m from the surface of water. The surface tension of water is 0.075 N/m

Data: r = 0.01m, T = 0.075 N/m.

To find: F

Formula: $F = T \times \ell$

Solution:

$F = T \times \ell$

$F = T \times 2\pi r$

$\therefore$ $F = 0.075\,(2 \times 3.142 \times 0.01)$

$= 0.150 \times 3.142 \times 0.01$

$\therefore$ $F = 0.004713$ N

Ans : Force required to take away flat circular plate is 0.004713 N

2) A needle 5 cm long can just rest on the surface of water without wetting. What is its weight? Surface tension of water = 0.07 N/m.

Data: $l = 5$ cm $= 5 \times 10^{-2}$ m

T = 0.07 N/m

To find: W

Formula: Weight of needle = Force due to surface tension

$\therefore$ $W = T \times L$

$\therefore$ $W = T \times 2l$

Solution:

$W = T \times 2l = 0.07 \times 2 \times 5 \times 10^{-2} = 0.007$ N

Ans : Weight of needle is 0.007 N

3) A light square wire frame each side of which is 10 cm long hangs vertically in water with one side just touching the water surface. Find the additional force necessary to pull the frame clear of water. (T = 0.074 N/m).

Data: $(l) = 10$ cm $= 10 \times 10^{-2}$ m $= 0.1$m

T = 0.074 N/m

To find: F

Formula: $T = \frac{F}{L}$

$\therefore$ $F = T \times L \Rightarrow F = T \times 2l$

Solution:

$F = T \times 2l = 0.0742 \times 10^{-1} = 0.0148$ N

Ans : Force required to pull square frame is 0.0148 N

4) A beaker of radius 10 cm is filled with water. Calculate the force of surface tension on any diametrical line on its surface. Surface tension of water is 0.075 N/m.

Data: $l = 2 \times 10 = 20$ cm $= 0.2$m

T = 0.075 N/m

To find: F

Formula: $F = T \times l$

Solution:

$F = T \times l$

$F = 0.075 \times 0.2 = 0.015$

$F = 1.5 \times 10^{-2}$ N

Ans : The force of surface tension on any diametrical line on its surface is 1.5×10^{-2} N

Problem for Practice

1. A thin and light ring of a material of radius 3 cm is rested flat on a liquid surface. When slowly raised, it is found that the pull required is 0.03 N more before the film breaks than after. Find the surface tension of the liquid.

Ans: 0.08 N/m

2. A thin wire is bent in the form of a rectangle of length 4 cm and breadth 3 cm. What force due to the surface tension do the sides experience when a soap film is formed in the frame? S.T. of soap solution = 0.030 N/m.

Ans: 8.4×10^{-3} N

3. A wire ring of radius 6 cm is rested flat on the surface of a liquid and then slowly raised. The pull required is 0.06 N more before the film breaks than it is after. What is the surface tension of the liquid? Neglect the thickness of the ring.

Ans: 0.0796 N/m

4. A glass plate 8 cm long is 2 mm thick. It is suspended with the long side horizontal and just touching the water surface in a trough full of water. Calculate the downward force on the plate due to surface tension. T = 0.072 N/m.

Ans: 11.8×10^{-3} N

Type - VI
Numerical Based on work done in blowing soap bubble

Formula used

1. $$T = \frac{dW}{dA}$$

$\therefore$ $dW = T \times dA$

2. As soap bubble have two

The surface area of bubble = $2 \times 4\pi r^2$

★ 1) Calculate the work done in blowing a soap bubble to a radius of 1 cm. The surface tension of soap solution is 2.5×10^{-2} N/m.

Data: $T = 2.5 \times 10^{-2}$ N/m,

r = 1cm = 0.01m

To find: W

Formula: Work done = $T \times dA$

Solution:

i Initial surface area of bubble $(A_1) = 0$

ii A soap bubble has two surfaces, outer surface and inner surface.

$\therefore$ Final surface area of soap bubble is,

$A_2 = 2\times (4\pi r^2) = 8\pi r^2$

iii. change in area = $dA = A_2 - A_1 = 8\pi r^2$

$= 8 \times 3.142 \times (0.01)^2 = 25.13\times10^{-4}$ m^2

Work done = $T \times dA$

iv. $W = 2.5\times10^{-2} \times 25.13 \times 10^{-4}$

$= 6.275\times10^{-5}$J

Ans :	Work done in blowing a soap bubble to a radius of 1 cm is 6.275×10^{-5}J

★ 2) How much work is required to form a bubble of 2 cm radius from the soap solution having surface tension 0.07 N/m.

Data: $r = 2\text{cm} = 2 \times 10^{-2}$ m, T = 0.07 N/m

To find: W

Formula: Work done = $T \times dA$

Solution:

i. Initial surface area of bubble $(A_1) = 0$

ii. A soap bubble has two surfaces, outer surface and inner surface.

$\therefore$ Final surface area of soap bubble is,

$A_2 = 2 \times (4\pi r^2) = 8\pi r^2$

iii. Change in area = $dA = A_2 - A_1 = 8\pi r^2$

$= 8 \times 3.142 \times (2\times10^{-2})^2$

$= 25.136 \times 4 \times 10^{-4}$ m^2

iv. Work done = $T \times dA$

$\therefore$ $W = 0.07 \times 25.136 \times 4 \times 10^{-4}$

$= 0.703 \times 10^{-3}$J

Ans :	Work done required to form bubble is 0.703×10^{-3}J

3) Calculate the work done in increasing the radius of soap bubble in air from 1 cm to 2 cm. The surface tension of soap solution is 30 dyne/cm.

Data : r_1 = 1cm, r_2 = 2cm,

T = 30 dyne/cm,

To find: W

Formula: Work done = T.dA

Solution : A soap bubble has two surface, outer surface and inner surface

i. Initial surface area of bubble

$A_1 = 2 \times 4\pi r_1^2 = 8\pi r_1^2$

ii. Final surface area of bubble

$A_2 = 2 \times 4\pi r_2^2 = 8\pi r_2^2$

iii. Change in surface area

dA = A2 – A1

$= 8\pi r_2^2 - 8\pi r_1^2$

$= 8\pi (r_2^2 - r_1^2)$

v. Work done = T.dA

$= 8\pi T(r_2^2 - r_1^2)$

$= 8 \times 3.14 \times 30\ (2^2 - 1)$

$= 2260.8 = 2261$ erg

Ans : Work done in blowing bubble from 1cm to 2cm is 2261 erg.

★ **4) A rectangular wire frame of size 2 cm × 2 cm, is dipped in a soap solution and taken out. A soap film is formed, if the size of the film is changed to 3 cm × 3 cm, calculated the work done in the process. The surface tension of soap film is 3×10^{-2} N/m.**

Data: $A_1 = 2cm \times 2cm = 4\times10^{-4} m^2$,

$A^1 = 3cm \times 3cm = 9\times10^{-4} m^2$,

$T^2 = 3\times10^{-2}$ N/m

To find: W

Formula: W = T dA

Solution:

i. $dA = 2(A_2 - A_1) = 2\ (9 - 4\ ) \times 10$

$= 10 \times 10^{-4} = 10^{-3} m^2$

ii. $W = T \times dA = 3 \times 10^{-2} \times 10^{-3} = 3 \times 10^{-5}$ J

Ans : The work done in increasing film is 3×10^{-5} J.

Problem for Practice

1. A soap bubble of radius 12 cm is blown. Surface tension of soap solution is 30 dyne/cm. Calculate the work donein blowing the soap bubble.

   Ans: 1.085×10^{-2}J

2. Two soap bubbles have radii in the ratio 4 : 3. What is the ratio of work done to blow these bubbles?

   Ans: 16:9

3. Calculate the work done in blowing a soap bubble from a radius of 2 cm to 3 cm. The surface tension of the soap solution is 30 dyne cm^{-1}.

   Ans: 3770.4 erg.

4. A soap bubble of radius $1/\sqrt{\pi}$ cm is expanded to radius $2/\sqrt{\pi}$ cm. Calculate the work done. Surface tension of soap solution = 30 dyne cm^{-1}. **Ans:720 erg**

Type - VII

Numerical based on droplets

Formula used

W = T × dA

1) A drop of mercury of radius 0.2 cm is broken into 8 deplets of the same size. Find the work done if the surface tension of mercury is 435.5 dyne/cm

Data: $R = 0.2cm = 2\times10^{-3}$ m, n = 8,

T = 435.5 dyne/cm = 0.4355N/m

To find: W

Formula: W = TdA

Solution:

Step1: Volume of n droplets = Volume of single drop

$$n \times \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R^3$$

$\therefore$ $nr^3 = R^3$

$$r = \frac{R}{(n)1/3} = \frac{2\times10^{-3}}{(8)1/3} = \frac{2\times10^{-3}}{2} = 10^{-3}m$$

Step2: To Find change in area

$dA = n \times 4\pi r^2 - 4\pi R^2$

$dA = 4\pi\ (nr^2 - R^2)$

$= 4 \times 3.14\ [8 \times (10^{-3})^2 -)2\times10^{-3})^2]$

$= 12.56\ [8\times10^{-6} - 4\times10^{-6}]$

$= 12.56 \times 4 \times 10^{-6}$

$= 50.24 \times 10^{-6} m^2$

Step3: To Find work done

$W = T \times dA$

$= 0.4355 \times 50.24 \times 10^{-6}$

$= 21.88 \times 10^{-6}$ J

$W = 2.188\times10^{-5}$ J

Ans : Work done to form 8 dropletes is 2.188×10^{-5}J

2) Twenty seven droplets of water, each of radius 0.1 mm coalesce into a single drop. Find the change in surface energy. Surface tension of water is 0.072 N/m.

Data: $r = 0.1mm = 10^{-4}$m, n = 27,

T = 0.072 N/m

To find: dE

Solution:

Step1: To find radius of big drop

Volume of single drop = volume of n droplets

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi R^3$$

$\therefore$ $R = (n)^{1/3}\, r$

$R = (27)^{1/3} \times 10^{-4}$

$R = 3\times10^{-4}$ m

Step2 : To find change in surface area

$dA = n \times 4\pi r^2 - 4\pi R^2$

$= 4\pi\,(nr^2 - R^2)$

$= 4\times3.14\,[\,27\times(10^{-4})^2 - (\,3\times10^{-4})^2]$

$= 12.56\,[27-9]\times10^{-8}$

$= 12.5\times18\times10^{-8}$

$dA = 226.08\times10^{-8}$ m^2

Step3 : To find change in surface energy

$dfe = dw = T \times dA$

$= 0.072\times226.08\times10^{-8}$

$= 16.28\times10^{-8}$

$= 1.628\times10^{-7}$

Ans : Change in t he surface energy is 1.628×10^{-7} J.

Probelm for Practice

1. Hight droplets of mercury each of radius 1mm coalesces into a single drop. Find change in the surface energy. (T = 0.465J/m^2)

   Ans: 23.37×10^{-6} J

2. A drop of mercury 2 mm in diameter breaks into a million small spherical droplets, all of same size. Calculate the workdone. (Surface tension of mercury = 460 × 10^{-3} N/m)

   Ans: 5.7856×10^{-4} J

3. A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Take surface tension of the liquid as T.

   Ans: 2 π D^2 T

4. A liquid drop of diameter 4 mm breaks into 1000 droplets of equal size. Calculate the resultant change in surface energy, the surface tension of the liquid is 0.07 Nm^{-1}.

   Ans: 3168 × 10^{-8} J.

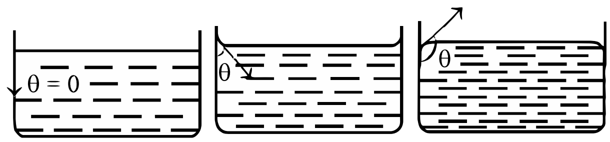
Angle of contact

Q.24 What is angle of contact ? State its characteristics.

Ans: **Angle of contact :**

When a liquid is in contact with a solid, the angle between the surface of the solid and the tangent drawn to the surface of the liquid at the point of contact, measured on the side of the liquid, is called the angle of contact of that liquid with that solid.

Characteristics of angle of contact :



(a) Water (b) Kerosene (c) Mercury

Angle of contact

i. The angle of contact is constant for a given liquid - solid pair.

ii. For a liquid which completely wets the solid, the angle of contact is equal to zero.In this case the tangent to the liquid surface is almost along the surface of the solid.

Example:1 water in contact with glass. (fig.a)

iii. For a liquid which partially wets the solid, the angle of contact is acute.

Example: kerosene in contact with glass. (fig. b)

iv. For a liquid which does not wet the solid, the angle of contact is obtuse.

Example: mercury in contact with glass. (fig.c)

v. For a given liquid - solid pair, even a small contamination causes a large change in the value of angle of contact.

vi. The angle of contact depends on.

   a. The relative magnitudes of the adhesive forces between the molecules of the liquid and the container.

   b. The cohesive forces among the liquid molecules.

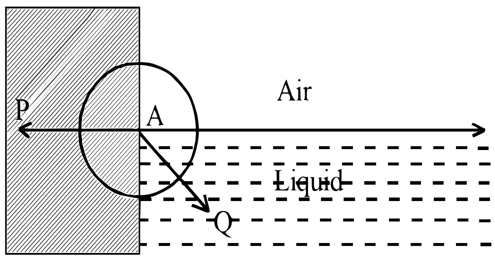
Q.25 Explain why the free surface of some liquid in contact with a solid is not horizontal.

OR

Explain the phenomenon of angle of contact on the basis of intermolecular force

Ans: **Consider** a liquid molecule A, situated in the liquid surface and in contact with the solid.

Its sphere of influence is partly in solid, partly in liquid and partly in air.



Liquid in contact with solid

The molecule is acted upon by the forces of attraction, as below:

i. The resultant adhesive force exerted by the molecules of the solid.
It is represented by $\overrightarrow{AP}$

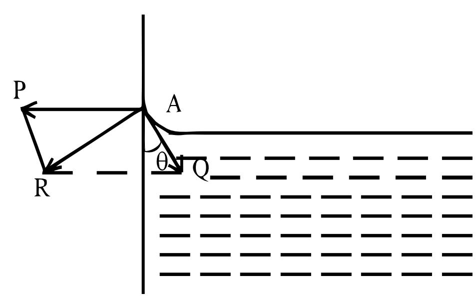
ii. The resultant cohesive force exerted by the molecules of the liquid.
It is represented by $\overrightarrow{AQ}$

iii. The resultant adhesive force exerted by the molecules of the air.
It is very small and negligible force.

iv. Gravitational force equal to the weight of the molecule.
It is also very small and negligible force.

v. Let, $\overrightarrow{AR}$ be the resultant of $\overrightarrow{AP}$ and $\overrightarrow{AQ}$. In the state of equilibrium, the surface of the liquid will be perpendicular to the $\overrightarrow{AR}$.

i. Liquid which partially wets the solid :



In this case,

$\vec{AP} > \vec{AQ}$.

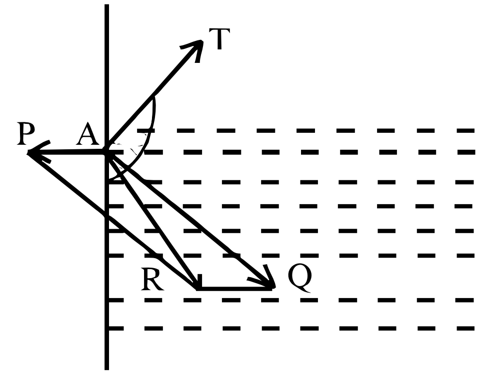
∴ $\vec{AR}$ lies on the side of the solid. In the state of equilibrium, the surface of the liquid will be perpendicular to the $\overrightarrow{AR}$.

∴ The liquid surface at A becomes concave, forming an acute angle of contact.

Example:

For ether - glass pair $\theta = 16^0$.

ii. Liquid which does not wet the solid :



In this case,

$\vec{AP} < \vec{AQ}$.

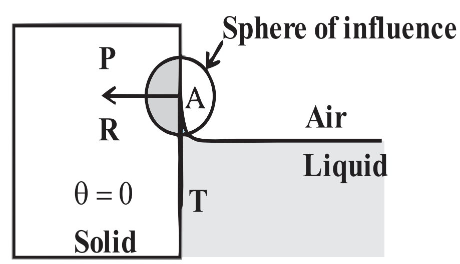
∴ $\vec{AR}$ lies inside the liquid. In the state of equilibrium, the surface of the liquid will be perpendicular to the $\overrightarrow{AR}$.

The liquid surface at A becomes convex, forming an obtuse angle of contact .

Example:

For mercury - glass pair $\theta = 128^0$.

iii. Liquid which completely wet the solid surface.



For this case, the angle of contact us 0

In this case, the liquid molecule near the contact refion are so less in number that coehessive fore is negligible

∴ $\overrightarrow{AQ} = 0$

the resultant force is along the net adhesive force.

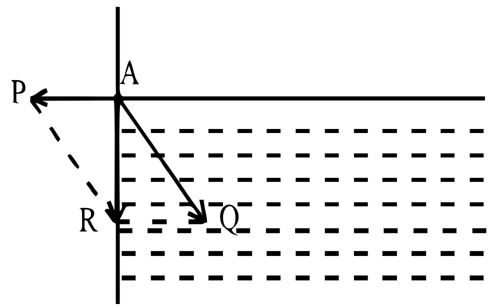
∴ $\overrightarrow{AP} = \overrightarrow{AR}$

In equilibrium state, surface of liquid is perpendicular to $\overrightarrow{AR}$ which is a long the wall

Angle of contat is o.

Example : highly pure water with clean glass.

iv. Liquid which just do not wets the surface :



For this case the angle of contact is 90^0 .

In this case, the resultant ($\overrightarrow{AR}$) is along the wall.

The liquid surface in contact at A is almost horizontal. The angle of contact is 90^0 .

Example: hypothetical case

Q.26 Discuss the various conditions for formation of drop on the surface.

Ans:

i. Consider a liquid drop at a equilibrium on the surface of a glass plate (solid).

ii. Three interfaces are formed:

a. solid-liquid b. solid-air and

c. liquid-air

iii. Let,

T_1 = surface tension for the solid-liquid interface.

T_2 = surface tension for the solid-air interface.

T_3 = surface tension for the liquid-air interface.

θ = angle of contact between solid liquid pair.

AS the force due to surface tension is tangential to the surface in contact,direction of T_1, T_2 and T_3 are shown in figure.

iv. T_3 can be resolved into two components:

$T_3 \cos\theta$ along horizontal and

$T_3 \sin\theta$ along vertical.

v. The drop is in equilibrium. Therefore, the horizontal components must balance each other.

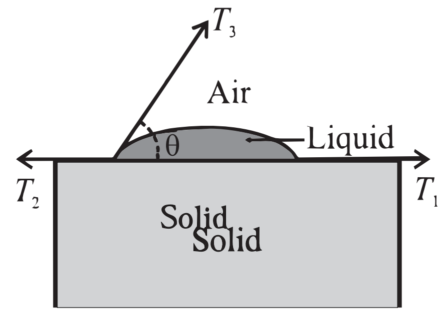
$T_3\cos\theta + T_1 = T_2$

$\therefore$ $T_3\cos\theta = T_2 - T_1$

$\therefore$ $\cos\theta = \dfrac{T_2 - T_1}{T_3}$ ....(1)

vi. If $T_2 > T_1$ and $(T2 - T1) < T3$.

$\cos\theta$ is positive. Hence, angle of contact is acute. Thus shape of meniscus is concave.



vii. If $T_2 < T_1$ and $(T_1 - T_2) < T_3$,

$\cos\theta$ is negative.

θ lies between 90° and 180° i.e., angle of contact is obtuse.

Thus, shape of meniscus is convex.

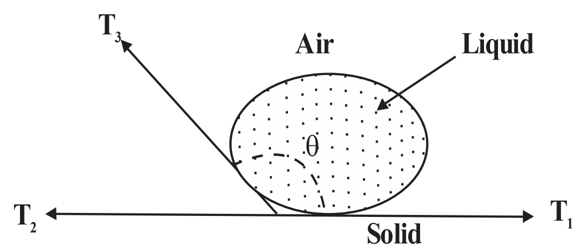


Figure (b)

vii. If $T_2 - T_1 = T_3$, $\cos\theta = 1$ and θ is nearly equal to zero.

ix. If $T_2 - T_1 > T_3$ or $T_2 > T_1 + T_3$, $\cos\theta > 1$ which is impossible. Hence liquid is spread over the solid surface and drop will not be formed.

Q.27 State the factor affecting angle of contact.

Ans: Factors affecting the angle of contact:

The value of the angle of contact depends on the following Factors

i. The nature of the liquid and the solid in contact.

ii. Impurity: Impurities present in the liquid change the angle of contact.

iii. Temperature of the liquid: Any increase in the temprature of a liquid decreases its angle of contact. For a given a solid-liquid surface, the angle of contact is constant at a given temperature.

Q.28 Explain effect of impurities on surface tension of liquid

Ans: Effect of impurities:

i. When soluble substance such as Common salt (i.e., sodium chloride) is dissolved in water, the surface tension of water increases.

ii. When a sparingly soluble substance such as phenol or a detergent is mixed with water, surface tension of water decreases. For example, a detergent powder is mixed with water to wash clothes. Due to this, the surface tension of water decreases and water makes good contact with the fabric and is able to remove tough stains.

iii. When insoluble imputiy is added into water, surface tension of water decreases. When impurity gets added to any liquid, the cohesive force of that liquid deceases which affects the angle of contact and hence the shape of the meniscus.

If mercury gathers dust then its surface tension is reduced. It does not form spherical droplets unless the dust is competely remoed.

Q.29 Explain effect of temperature on surface tension of liquid.

Ans: Effet of Temperatur:

i. In most liquids, as temprature increases surface tension decreases

ii. For example, it is suggested that new cotton fabric should be washed in cold water. In this case, water does not make good contact with the fabric due to its higher surface tension. Thoes fabric does not lose its colour because of this.

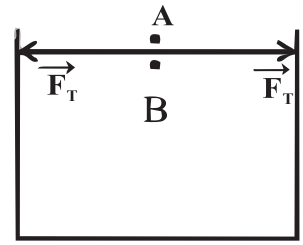
iii. Hot water is used to remove tough stains on fabric because of its lower surface tension. In the case of molten copper or molten cadmium, the sarface tension increases with increase in its tempreture.

iv. The surface of a liquid becomes zero at critical tempratur.

Q.30 Explain the nature of a pressure on two sides of a liquid surface. Also, state their causes.

Ans:

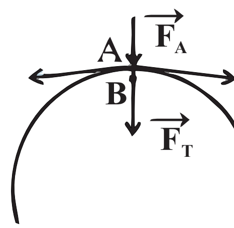
1) Plane liquid surface: Consider planar free surface of the liquid. In this case, the resultant force due to surface tension, $\vec{F}_T$ on the molecule at B is zero. The force $\vec{F}_A$ itself decides the pressure and the pressure at A and B is the same.



2) Convex liquid surface :

i. Surface of the liquid in the Fig.2.21 (b) is upper convex. (Convex, when seen from above).

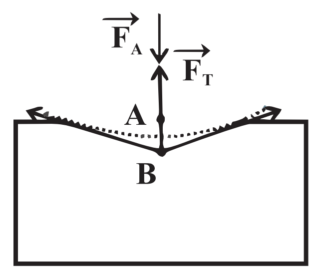
ii. In this case, the resultant force due to surface tension, $\vec{F}_T$ on the molecule at B is vertically downwards and adds up to the downward force $\vec{F}_A$.



iii. This develops greater pressure at point B, which is inside the liquid and on the concave side of the meniscus.

iv. Thus, the pressure on the concave side i.e., inside the liquid is greater than that on the convex side i.e., outside the liquid.

3) Concave liquid surface:



i. Surface of the liquid is upper concave (concave, when seen from above).

ii. In this case, the force due to surface tension $\vec{F}_T$, on the molecule at B is vertically upwards. The force $\vec{F}_A$ due to atmospheric pressure acts downwords.

iii. Forces $\vec{F}_A$ and $\vec{F}_T$ thus, act in opposite direction. Therefore, the net downward force responsible for the pressure at B is less than $\vec{F}_A$

iv. This develops a lesser pressure at point B, which is inside the liquid and on the convex side of the meniscus.

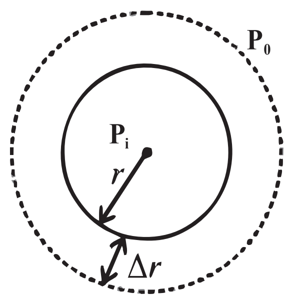
v. Thus, the pressure on the concave side i.e., outside the liquid, is greater than that on the convex side, i.e.,inside the liquid.

Q.31 Derive Laplace's law for spherical membrane.

Ans:

Let, P_0 - outside pressure
P_i - inside pressure

∴ Excess pressure = $(P_i - P_0)$



i. Let, the radius of the drop increases from r to r + Δr, where Δr is very small, so that the inside pressure remains almost constant.

ii. Initial surface area $(A_1) = 4\pi r^2$

Final surface area $(A_2) = 4\pi (r + \Delta r)^2$

$A_2 = 4\pi (r^2 + 2r\Delta r + \Delta r^2)$

$A_2 = 4\pi r^2 + 8\pi r \Delta r + 4\pi \Delta r^2$

As Δr is very small, Δr^2 is neglected (i.e. $4\pi \Delta r^2 \cong 0$)

∴ $A_2 = 4\pi r^2 + 8\pi r \Delta r$

iii. Increase in surface area $(dA) = (A_2 - A_1)$

$= 4\pi r^2 + 8\pi r \Delta r - 4\pi r^2$

Increase in surface area $(dA) = 8\pi r \Delta r$ ...(1)

iv. Work done to increase the surface area,

$dw = TdA$

$= T (8\pi r\Delta r)$ ...(2)

v. This work done is also equal to product of force and the distance Δr.

The increase in radius of bubble is Δr.

$dw = dF \Delta r$ ...(3)

vi. Excess force = Excess pressure × area

$dF = (P_i - P_0) 4\pi r^2$ ...(4)

vii. Substituting (4) in eq (3)

$dw = (P_i - P_0) 4\pi r^2 \Delta r$ ...(5)

viii. Form equation (2) and (5),

$T (8\pi r\Delta r) = (P_i - P_0) 4\pi r^2 \Delta r$

∴ $(P_i - P_0) = \frac{2T}{r}$

This is called Laplace's law of a spherical membrane.

For soap bubble: Soap bubble having two free surfaces in contanct with air.

∴ Total increase in surface area = $2(8\pi r\Delta r)$

and $dw = 16\pi r\Delta r\, T$

Work done due to excess pressure

$dw = (P_i - P_0) \times 4\pi r^2 \Delta r = 16\pi r \Delta r T$

∴ $(P_i - P_0) = \frac{4T}{r}$

Type - VIII

Numerical based on excess of pressure

Formula used

1. For liquid drop

$P_i - P_0 = \frac{2T}{r}$

2. For soap bubble

$P_i - P_0 = \frac{4T}{r}$

★ 1) What should be the diameter of a water drop so that the excess pressure inside it is 80 N/m? (Surface tension of water = 7.27 ×10^{-2} N/m)

Data: $P_i - P_0 = 80\ N/m^2$, $T = 7.27\times10^{-2}$ N/m

To find: d

Formula: $P_i - P_0 = \frac{2T}{r}$

Solution: For liquid drop

$P_i - P_0 = \frac{2T}{r}$

$r = \frac{2T}{P_i - P_0} = \frac{2\times7.27\times10^{-3}}{80}$

$= 1.817 \times 10^{-3}$ mm ≈ 1.8mm

∴ D = 2r = 3.6mm.

Ans : Diameter of the water drop so that excess pressure inside it is $80 N/m^2$ is 3.6mm.

★ 2) **An air bubble of radius 0.2 mm is situated just below the water surface. Calculate the gauge pressure. Surface tension of water = 7.2×10^{-2} N/m.**

Data: $r = 0.2mm = 2\times10^{-4}m$,
$T = 7.2\times10^{-2}N/m$

To Find: Gauge pressure

Formula: $P_i - P_0 = \frac{2T}{r}$

Solution: The guage pressure for air bubble inside water is excess of press inside

$$\text{Gauge Pressure} = P_i - P_0 = \frac{2T}{r}$$

$$\Delta P = \frac{2\times7.2\times10^{-2}}{2\times10^{-4}} = 720N/m^2$$

Ans : The gauge pressure on air bubble is $720N/m^2$.

Problem for Practice

1. What should be the pressure inside a small air bubble of 0.1 mm rdius, situated just below the surface?Surface tensionof water = $7.2 \times 10^{-2}Nm^{-1}$ and atmospheric pressure = 1.013×10^5 Nm^{-2}.

Ans : $1.027\times105Nm^{-2}$

2. Two soap bubbles have radii in the ratio 2:3. Compare the excess of pressure inside these bubbles. Also compare the work done in blowing these bubbles.

Ans: 3 : 2, 4 : 9

3. A raindrop of diameter 4 mm is about to fall on the ground. Calculate the pressure inside the raindrop.
[Surface tension of water T = 0.072 N/m, atmospheric pressur = 1.013×10^5 N/m^2].

Ans:

Capillary action

Note:- *A tube having a very fine bore ($\approx$ 1 mm) and open at both ends is called a capillary tube.*

Q.32 What is capillarity? Give some applications of capillarity.

Ans: Capillary action or capillarity:

The phenomenon of rise or fall of liquid level inside a capillary tube when it is dipped in the liquid is called capillary action or capillarity.

Applications of capillarity:

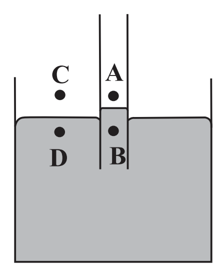
i. Oil rises up the wick of a lamp.
ii. Cloth rag sucks water.
iii. Water rises up the crevices in rocks.
iv. Sap and water rise up to the top most leaves in a tree.
v. Blotting paper absorbs ink.

Q.33 Explain capillary action.

Ans:

1) Explanation for capillary drop :

i. Consider a capillary tube dipped in a liquid which does not wet the surface (mercury).
ii. The shape of mercury meniscus in the capillary is upper convex.
iii. Let us consider four points as shown in the figure. Point A is just above the convex surface inside the capillary. B is just below the convex suface inside the capillary. C is just above the plane surface outside the capillary. D is just below the plane surface outside the capillary and is at the same horizontal level as that of B.



iv. Let P_A, P_B, P_C and P_D be the values of the pressures at the points A, B, C, and D respectively. Since, the pressure on the concave side is always greater than that on the convex side.

∴ $P_B > P_A$

v. As the points A and C are at the same level, the pressure at both these points is the same, and it is the atmospheric pressure.

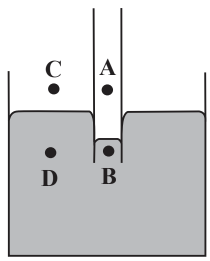
∴ $P_A = P_C$ ....(1)

Between the points C and D, the surface is plane.

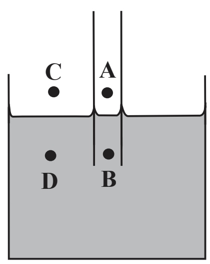
$\therefore$ $P_C = P_D = P_A$ ....(2)

$\therefore$ $P_B > P_D$

vi. But the points B and D are at the same horizontal level. Thus in order to maintain the same pressure, the mercury in the capillary rushes out of the capillary. Because of this, there is a drop in the level of mercury inside the capillary.



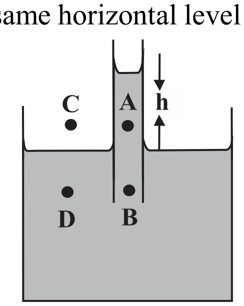
2) Explaination of capillary rise:



i. Suppose a capillary tube is dipped into water.

ii. Consider the situation before the movement of water inside the capillary. The shape of the surface of water in the capillary is concave.

iii. Let us consider four points as shown in the figure. Point A is just above the concave suface inside the capillary. B is just below the concave surface inside the capillary. C is just above the plane suface outside the capillary. D is just below the plane surface outside the capillary and is at the same horizontal level as that of B.



iv. Let P_A, P_B, P_C and P_D be the pressures at points A, B, C and D respectively.

v. Since pressure on concave side of liquid surface is greater than that on the convex side.

$\therefore$ $P_A > P_B$

vi. As the pressure is same on both sides of a plane surface,

$\therefore$ $P_C = P_D$

vii. The points A and C are at the same level, the pressure at both this points is the same.

$P_A = P_C = P_D$ = atmospheric pressure

$\therefore$ $P_D > P_B$

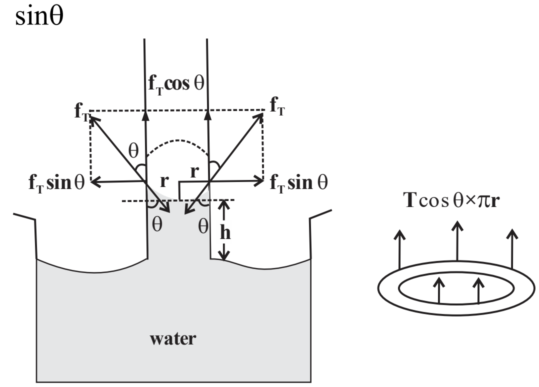
viii. But points B and D are at same horizontal level in the liquid. Therefore, in order to maintain the same pressure the liquid out of the capillary flows into the capillary tube and rises above the point B, till the pressure at B beocmes the same as that at D. Because of this, there is a rise in the level of liquid inside the capillary tube.

★ Q.34 Derive an expression for Capillary rise for a liquid having concave meniscus.

Ans:

i. Consider a case of rise of liquid inside a capillary tube as shown in figure. Let, r be the radius of the capillary h be the height to which the liquid rises inside the capillary. T be the surface tension of the liquid θ be the angle of contact for the liquid solid pair.

ii. Surface tension force (T), acting on unit length, is resolved into two component a vertical component Tcosθ and horizontal T sinθ



iii. All the horizontal components cancel each other, while the all vertical components acting on the circumference of liquid in side the capillary. So the vertical force acting on the liquid inside the capillary

$= 2\pi r T\cos\theta$ ...(1)

iv. Vertical force acting on the liquid inside the capillary balances weight of the liquid rise inside the capillary.

W= mg ... (2)

If ρ is the density of the liquid,

$m = \text{volume} \times \text{density} = \pi r^2 h\rho$

Equation (2) become,

$W = \pi r^2 h\rho g$ ... (3)

v. From equation (1) and (3)

Upward force = downward force.

$\therefore$ $2\pi r T\cos\theta = \pi r^2 h\rho g$.

$\therefore$ $T = \dfrac{rh\rho g}{2\cos\theta}$ OR $h = \dfrac{2T\cos\theta}{r\rho g}$

This formula is used to determine the surface tension of a liquid which partially or wholly wets the capillary in capillary tube experiment.

Type - IX

Numerical based on Capillary

Formula used

$$h = \frac{2T\cos\theta}{r\rho g}$$

★ **1) Calculate the rise of water inside a clean glass capillary tube of radius 0.1 mm, when inmmersed in water of surface tension 7×10^{-2} N/m. The angle of contact between water and glass is zero, density of water = 1000 kg/m³, g = 9.8 m/s².**

Data: $r = 0.1\text{mm} = 10^{-4}$ m,

$T = 7 \times 10^{-2}$ N/m, $\theta = 0°$,

$\rho = 1000$ kg/m³, g = 9.8 m/s²

To find: h

Formula: $h = \dfrac{2T\cos\theta}{r\rho g}$

Solution:

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$h = \frac{2\times(7\times10^{-2})\cos 0°}{10^{-4}\times10^{3}\times9.8}$$

$$= \frac{14\times10^{-1}}{9.8} = 0.143\text{m}$$

Ans : The rise of water inside the glass capillary is 0.143m.

2) A capillary tube of radius 5×10^{-4}m is immersed in a beaker filled with mercury. The mercury level inside the tube is found to be 8×10^{-3}m below the level of reservoir. Determine the angle of contact between mercury and glass. Surface tension of mercury is 0.465 N/m and its density is 13.6×10^3 kg/m³.(g = 9.8 m/s²)

Data: $r = 5\times10^{-4}$ m, $h = -8\times10^{-3}$m,

T = 0.465 N/m, g = 9.8m/s²,

$\rho = 13.6 \times 10^3$ kg/m³.

To find: Angle of contact (θ)

Formula: $T = \dfrac{hr\rho g}{2\cos\theta}$

Solution: $T = \dfrac{hr\rho g}{2\cos\theta}$

$$\cos\theta = \frac{hr\rho g}{2T}$$

$$= \frac{-\overset{4}{\cancel{8}}\times10^{-3}\times5\times10^{-4}\times13.6\times10^{3}\times9.8}{\underset{1}{\cancel{2}}\times0.465}$$

$\therefore$ $-\cos\theta = \dfrac{4\times5\times13.6\times9.8}{0.465}\times10^{-4}$

$\therefore$ $-\cos\theta = 0.5735$

$\therefore$ $\cos(\pi-\theta) = 0.52732$

$\therefore$ $180° - \theta = 55°2'$

$\therefore$ $\theta = 124°58'$

Ans : The angle of contact between mercury and glass is nearly 124°58'

3) Calculate the density of paraffin oil, if a glass capillary of diameter 0.25 mm dipped in paraffin oil of the surface tension 0.0245 N/m rises a height of 4 cm. [Angle of contact

of paraffin oil with glass is 28° and g = 9.8 m/s²]

Data: d = 0.25 mm

$\therefore$ $r = \frac{d}{2} = \frac{0.25}{2}\ 0.125\ mm = 0.125 \times 10^{-3} m$

$T = 0.0245\ N/m,\ h = 4\ cm = 0.04\ m$

$\theta = 28^0,\ g = 9.8\ m/s^2,$

To find : ρ

Formula: $T = \frac{rh\rho g}{2\cos\theta}$

Solution: $T = \frac{rh\rho g}{2\cos\theta}$

$\therefore$ $\rho = \frac{2T\cos\theta}{rhg}$

$= \frac{2 \times 0.0245 \times \cos 28^{\circ}}{0.125 \times 10^{-3} \times 0.04 \times 9.8}$

$\therefore$ $= \frac{2 \times 0.0245 \times 0.8829}{0.125 \times 10^{-3} \times 0.04 \times 9.8}$

$\therefore$ $\rho = 882.9\ kg/m^3$

Ans : The density of praffin oil is $882.9\ kg/m^3$.

4) Water rises to height 3.2 cm in glass capillary tube. Find height to which same water rises in another capillary having half area of cross section.

Data: $h_1 = 3.2\ cm, A_2 = \frac{1}{2} A_2 = A_1$

To Find: h_2

Formula : $T = \frac{hr\rho g}{2\cos\theta}$

$rh = \frac{2T\cos\theta}{\rho g}$

As T, θ, ρ, g are constant

$rh = \text{constant}$

$h_1 r_1 = h_2 r_2$

Solution: As $A_2 = \frac{1}{2} A_1$

$\therefore$ $\pi r_2^2 = \frac{1}{2} \pi r_1^2$

$\therefore$ $r_2 = \frac{r_1}{\sqrt{2}}$

$\therefore$ $r_1 = r_2\sqrt{2}$

For same liquid, $h_1 r_1 = h_2 r_2$

$h_1 = \frac{h_1 r_2}{r_2} = \frac{h_1 r_2 \sqrt{2}}{r_2}$

$h_2 = 3.2 \times \sqrt{2} = 4.525\ cm$

Ans : In another capillary water rises to 4.525 cm

Problem for Practice

1. A capillary tube of 0.5 mm bore stands vertically in a wide vessel containing a liquid of surface tension 60 dynes/cm. The liquid wets the tube and has specific gravity of 0.8. Calculate the rise of liquid in the tube.
Ans:6.12cm

2. A liquid rises to a height of 4.5 cm in a glass capillary tube of radius 0.01cm. What will be the height of liquid column in a glass capillary tube of radius 0.02cm ?
Ans:2.25 cm

3. A capillary tube of uniform bore is dipped vertically in water, which rises by 7 cm in the tube. Find the radius of the capillary if the surface tension of water is 70 dynes/cm.
Ans: 0.02041 cm

4. Calculate the depression of mercury in a tube of diameter 0.4 cm when it is held vertically in mercury in a trough, with one end of the tube open to atmosphere (density of mercury = $13.6\ g / cm^3$, surface tension of mercury = 490 dyn/cm, angle of contact for mercury with glass = 140^0).
Ans: 0.2816 cm.

5. A capillary tube of radius 0.5 mm is dipped vertically in a liquid of surface tension 0.04 N/m and relative density 0.8 gm/cc. Calculate the height of capillary rise, if the angle of contact is 10^o.(Given $g = 9.8\ m/s^2$)
Ans: 2.01×10^{-2} m

2.5 Fluids in motion

Q.35 Define following

1. **Hydrodynamics**
2. **Steady flow**
3. **Flow line**
4. **Streamline**
5. **Flow tube**

Ans:

1. Hydrodynamics:

The branch of Physics which deals with the study of properties of fluids in motion is called hydrodynamics.

2) **Steady flow:**

Flow in which measurable property, such as pressure or velocity of the fluid at a given point is constant over time is called steady flow.

3) **Flow line:**

Path of an individual particle in a moving fluid is called as flow line.

4) **Streamline:**

The curve whose tangent at any point in the flow is in the direction of the velocity of the flow at that point is called streamlines. Streamlines and flow lines are indentical for a steady flow.

5) **Flow Tube:**

An imaginary bundle of flow line bound by an imaginary wall is called a flow tube. For a steady flow, the fluid cannot cross the walls of a flow tube. Fluids in adjacent flow tubes cannot mix.

Q.36 Define Streamline flow.

Ans: Streamline flow:

Flow of liquid over plane surface or through a tube so long as the velocity of fluid is less 'than a certain limiting value (critical velocity) is called the streamline flow or laminar flow.

Note: Streamline flow

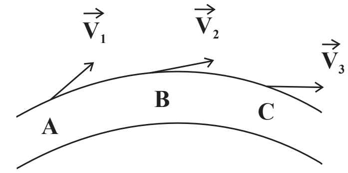
i. Consider a liquid flowing along the path ABC in layers as shown in figure.

ii. Let v_1, v_2 and v_3 be the velocities of a particle, of the liquid at points A, B and C respectively.

iii. If the flow of the liquid is steady, orderly or streamlined, then each new particle of the liquid arriving at 'A' will have the same velocity $\vec{v}_1$.

iv. This velocity is directed along the tangent to curve ABC at the point 'A'.

v. A particle arriving at 'B' will always have the same velocity $\vec{v}_2$. This velocity mayor may not be equal to $\vec{v}_1$.



vi. To sum up, every particle of the liquid follows the path of its preceding particle with exactly the same velocity in the streamline flow of a liquid.

vii. In the streamline flow of a liquid, the velocity of every particle crossing a particular point is the same.

viii. Velocity of particles in different layers is different.

ix. Particles do not move from one layer to the other layer. Adjacent particles remain confined to a single layer.

Q.37 What is Turbulent flow?

Ans: Turbulent flow:

The irregular and unsteady flow of a fluid when its velocity increases beyond critical velocity is called turbulent flow.

Example: The air in hurricane or storms, motion of water in high water fall etc.

Q.38 Distinguish between streamline flow and turbulent flow.

Ans:

No	Streamline flow	Turbulent flow
i.	The smooth flow of a fluid, with velocity smaller than certain critical velocity (limiting value of velocity) is called streamline flow or laminar flow of a fluid.	The irregular and unsteady flow of a fluid when its velocity increases beyond critical velocity is called turbulent flow.
ii.	In a streamline flow, velocity of a fluid at a given point is always constant.	In a turbulent flow, the velocity of a fluid at any point does not remain constant.
iii.	Two streamlines can never intersect, i.e., they are always parallel.	In a turbulent flow, at some points, the fluid may have rotational motion which gives rise to eddies.

2.6 Critical velocity and Reynold's number

Q.39 Define the term critical velocity.

Ans: The certain limiting velocity of liquid below which the flow of liquid is streamline is called critical velocity.

Q.40 What is Reynold's number? OR
A. Define Reynold's number.
B. Explain how Reynold's number is used to determine the nature of flow of liquid.

Ans:

A. Reynold's number:

i. It is a pure number which determines the nature of flow of a liquid through a pipe.

ii. It is denoted by R_n.

$$R_n = \frac{v_c \rho d}{\eta}$$

where,
v_c = critical velocity
ρ = density of liquid
d = diameter of tube
η = coefficient of viscosity

iii. It has no unit and dimensions.

B. Nature of flow:

i. Generally, when the value of R_n lies between 0 and 1000, the flow of liquid is streamline or laminar.

ii. When N lies between 1000 and 2000, then laminar flow may change to turbulent flow.

iii. If values of R_n is greater than 2000, the flow of liquid is turbulent.

Q.41 Define
1. Viscosity
2. Viscous drag (force)

Ans:

1. Viscosity:
The property of fluid by virtue of which the relative motion between different layers of a fluid experience a dragging force is called viscosity.

2. Viscous drag (force):
The dragging force experienced by the relative motion between different layers of a fluid is called viscous force or viscous drag.

Q.42 What is velocity gradient. Give its units and dimension.

Ans: Velocity gradient:
The rate of change of velocity with distance measured from the stationary layer is called the velocity gradient.
If v and v + dv are the velocities of layers of liquid at distances x and x + dx respectively from the bottom then,

$$\text{velocity gradilent} = \frac{v + dv - v}{x + dx - x} = \frac{dv}{dx}$$

Units and dimensions:
SI unit : s^{-1}
Dimensions: $[M^0L^0T^{-1}]$

Q.43 State and explain Newton's law of viscosity.

Ans: Newton's law of viscosity:
For streamline flow, the viscous force acting on any layers is directly proportional to
i. area of the layer
ii. velocity gradient

Explanation:

i. Let A be the area of layer and $\left|\frac{dv}{dx}\right|$ be the velocity gradient, then the viscous force F is given by,

$F \propto A$ ...(1)

$F \propto \frac{dv}{dx}$ ...(2)

Combining equations (1) and (2) we have,

$$F \propto A\left(\frac{dv}{dx}\right)$$

$$\therefore \quad F = \eta A\left(\frac{dv}{dx}\right)$$

where η = constant called coefficient of viscosity of the liquid which depends upon the nature of the liquid.

Q.44 Define coefficient of viscosity. State S.I. and C.G.S. units of coefficient of viscosity.

Ans: Coefficient of viscosity:
From Newton's law of viscosity,

$$F = \eta A \frac{dv}{dx}$$

$\therefore \quad \eta = \dfrac{F}{\dfrac{dv}{dx}.A}$

If $A = 1\ m^2$, $\dfrac{dv}{dx} = 1\ s^{-1}$

then $F = \eta$

Hence, the coefficient of viscosity is defined as the viscous force per unit area per unit velocity gradient.

Units:

i. Unit of η: Ns/m^2 or decapoise in SI system and dyne s/cm^2 or poise in CGS system.

ii. Dimensions of n are $[M^1L^{-1}T^{-1}]$

Type - X
Numerical based on Newton's law of Viscosity

Formula used

1. Velocity gradient

$$v_g = \frac{dv}{dx}$$

2. $F = \eta A \dfrac{dv}{dx}$

1) A horizontal force of 1N is required to move a metal plate of area 10^{-2} m/s, with the velocity of 2×10^{-2} m/swhen it rests on a layer of oil 1.5×10^{-3} m thick. Find the coefficient of viscosity of oil.

Data: $F = 1N, A = 10^{-2}m^2$,
$dv = 2\times10^{-2}m/s$,
$dx = 1.5\times10^{-3}m$

To find: η

Formula: $F = \eta = A\dfrac{dv}{dx}$

Solution: $\eta = \dfrac{F}{A\dfrac{dv}{dx}} = \dfrac{F\times dx}{Adv}$

$$\eta = \frac{1\times1.5\times10^{-3}}{10^{-2}\times2\times10^{-2}} = 7.5\frac{Ns}{m^2}$$

Ans: The coefficient of viscosity of oil is $7.5Ns/m^2$

Problem for Practice

1. A flat square plate of side 20cm moves over another similar plate with a thin layer of 0.4 cm of a liquid between them. If a force of one kg wt moves one of the plates uniformly with a velocity of 1 ms^{-1}, calculate the coefficient of viscosity of the liquid.

Ans:0.05cm

2. A circular metal plate of radius 5cm, rests on a layer of caster oil 2 mm thick, whose coefficient of viscosity is 15.5 poise. Calculate the horizontal force required to move the plate with a speed of 5 cm s^{-1}.

Ans:3.04×10^4 dyne

2.7 Stroke's law

Q.45 State Stoke's law

Ans: Statement of Stoke's law: The viscous force acting on a small sphere falling through a medium is directly proportional to the radius (r) of the sphere, its velocity (v) through fluid and coefficient of viscosity (η) of the fluid.

Mathematically, $F \propto \eta rv$

$\therefore \quad F = k\eta\ rv = 6\pi\eta rv$

where k is the constant of proportionality. Numerically, it is equal to 6π.

Type - XI
Numerical based on Stroke's law

Formula used

$F = 6\pi\eta rv$

★ 1) A steel ball with radius 0.3mm is falling with velocity of 2 m/s at a time t, through a tube filled with glycerin, having coefficient of viscosity 0.833 Ns/m^2. Determine viscous force acting on the steel ball at that time.

Data: $r = 0.3mm = 0.3 \times 10^{-3}m$, $v = 2$ m/s,
$\eta = 0.833Ns/m^2$.

To find: F

Formula: $F = 6\pi\eta rv$

Solution: $F = 6\pi\eta rv$

$F = 6\times3.142\times0.833\times0.3\times10^{-3}\times2$

$\therefore \quad F = 9.421\times10^{-3}$ N

Ans: Viscous force acting on the steel is 9.421×10^{-3} N

★ 2) **Calculate the viscous force acting on a rain drop of diameter 1 mm, falling with a uniform velocity 2 m/s through air. The coefficient of viscosity of air is 1.8×10^{-5} Ns/m^2.**

Data: $d = 1\text{ mm} = 10^{-3}\text{ m}$

$\therefore$ $r = 5 \times 10^{-4}$m,

$v = 2$m/s, $\eta = 1.8 \times 10^{-5}$ Ns/m^2

To find: F_s

Formula: $F = 6\pi\eta rv$

Solution:

$F = 6\pi\eta rv$

$F = 6 \times 3.142 \times 1.8 \times 10^{-5} \times 5 \times 10^{-4} \times 2$

$\therefore$ $F = 3.393 \times 10^{-7}$N.

Ans : The viscous force acting on the rain drop is 3.393×10^{-7} N.

Problem for Practice

1. A rain drop of radius 0.3 mm falls through air with a terminal velocity of 1 ms^{-1}. The viscosity of air is 10×10^{-5} poise. Find the viscous force on the rain drop.

Ans:1.018×10^{-2} dyne.

Q.46 A. Define terminal velocity.

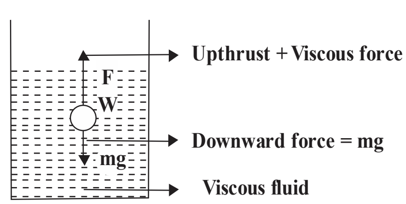
B. Obtain an expression for the terminal velocity of a small sphere falling under gravity through a viscous fluid.

Ans:

A. **Terminal velocity:** The constant maximum velocity acquired by a body falling through a viscous liquid is called as terminal velocity.

B. **Expression for terminal velocity:**

i. Consider a sphere of radius (r) and density (ρ) falling under gravity through a liquid of density (σ) and coefficient of viscosity (η) as shown in figure.



ii. Forces acting on the sphere during f downward

a. Viscous force = $F_v = 6\,\pi\eta rv$ (directed upwards)

b. Weight of the sphere, (F_g)

$$mg = \frac{4}{3}\pi r^3\rho g \text{ (directed downwards)}$$

b. Upward thrust as Buoyant force (F_u)

$$F_u = \frac{4}{3}\pi r^3\sigma g \text{ (directed upwards)}$$

iii. As the downward velocity increases, the viscous force increases. A stage is reached, when sphere attains terminal velocity.

iv. When the sphere attains the terminal velocity the total downward force acting on the sphere is balanced by the total upward force acting on the sphere.

Total downward force = Total upward force

Weight of sphere (mg) =

Viscous force + Buoyant force due to medium

$$= \frac{4}{3}\pi r^3\rho g = 6\,\pi\eta rv + \frac{4}{3}\pi r^3\sigma g$$

$$6\,\pi\eta rv = \left(\frac{4}{3}\pi r^3\rho g\right) - \left(\frac{4}{3}\pi r^3\sigma g\right)$$

$$6\,\pi\eta rv = \left(\frac{4}{3}\right)\pi r^3 g(\rho - \sigma)$$

$$v = \left(\frac{4}{3}\right)\pi r^3 g(\rho - \sigma) \times \frac{1}{6\pi\eta r}$$

$$v = \frac{2}{9}\,\frac{r^2 g(\rho - \sigma)}{\eta} \qquad \text{... (1)}$$

This is the expression for terminal velocity of the sphere.

Note: *For air medium, σ is neglected as compared to ρ.*

$$\text{Hence } v = \frac{2r^2\rho g}{9\eta}$$

Type -XII

Numerical based on Terminal velocity

Formula used

$$V = \frac{2}{q}\frac{r^2}{\eta}(\rho - \sigma)g$$

★ 1) **A spherical drop of oil falls at a constant speed of 4 cm/s in steady air. Calculate the radius of the drop. The density of the oil is 0.9 g/cm³, density of air is 1.0 g/cm³ and the coefficient of viscosity of air is 1.8 ×10⁻⁴ poise.(g = 980 cm/s²)**

Data: $v = 4cm/s$, $\eta = 1.8\times10^{-4}$poise.
$\sigma = 0.9\ g/cm^3$, $\rho = 1\ g/cm^3$, $g = 980cm/s^2$

To find: r

Formula: $\eta = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{v}$

Solution:

$$\eta = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{v},$$

$$r = \sqrt{\frac{9\eta v}{2(\rho-\sigma)g}}$$

$$= \sqrt{\frac{9\times1.8\times10^{-4}\times4}{2(1-0.9)980}}$$

$$= \sqrt{\frac{9\times1.8\times10^{-4}\times \not{4}^{\,2}}{2\times0.1\times \not{980}_{\,490}}} = \sqrt{\frac{9\times18\times10^{-4}}{49\times10}}$$

$$= \frac{3\times3}{7}\sqrt{\frac{2}{10}}\times10^{-2}$$

$= 0.574\times10^{-2}$m

∴ r = 0.00574 cm

Ans : Radius of the drop is 0.00574cm

★ 2) **With what terminal velocity will an air bubble 0.4 mm in diameter rise in a liquid of viscosity 0.1 Ns/m² and specific gravity 0.9? Density of air is 1.29kg/m³.**

Data: $d = 0.4mm = 4\times10^{-4}m$, $r = 2\times10^{-4}m$,
$\eta = 0.1\ Ns/m^2$, specific gravity = 0.9
$\rho = 1.29kg/m^3$, $g = 9.8ms^{-2}$

To find: v

Formula: $v = \frac{2}{9}\frac{r^2 g(\rho-\sigma)}{\eta}$

Solution: Specific gravity

$$= \frac{\text{Density of liquid}}{\text{Density of water}} = 0.9,$$

∴ $\sigma = 0.9\times1000 = 900kg/m^3$,

$$v = \frac{2}{9}\frac{r^2 g(\rho-\sigma)}{\eta}$$

$$v = \frac{2}{9}\frac{(2\times10^{-4})^2\times9.8\times(1.29-900)}{0.1}$$

$$v = \frac{2}{9}\times4\times10^{-8}\times98\times(-898.71)$$

$$= \frac{784\times(-898.71)\times10^{-8}}{9}$$

$= -0.782\times10^{-3}$m/s

Negative sign indicates that the bubble is rising up inside the liquid.

Ans : The terminal velocity for the air bubble is -0.782×10^{-3} m/s. Negative sign indicate bubble is rising up.

Problem for Practice

1. Eight rain drops of radius 1 mm each falling down with terminal velocity of $5cms^{-1}$ coalesce to form a bigger drop. Find the terminal velocity of the bigger drop.

Ans:20 cms⁻¹

2. Find the terminal velocity of a steel ball 2mm in diameter falling through glycerine. Relative density of steel = 8, relative density of glycerine = 1.3 and viscosity of glycerine = 8.3 poise.

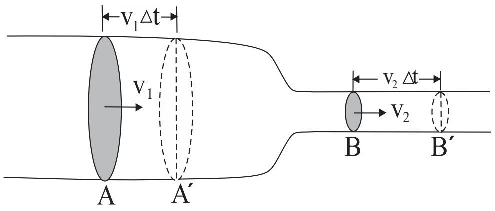
Ans:1.758 cm s⁻¹

2.8 Equation of Continuity

Q.47 Obtain the equation of continuity for the incompressible non-viscous fluid having a steady flow through a pipe.

Ans: Equaiton of continuity:

Consider a non-viscous and incompressible liquid flowing steadily between the sections A and B of a pipe of varying cross-section. Let a_1 be the area of cross-section, v_1 fluid velocity, ρ_1 fluid density at section A; and the values of corresponding quantities at section B be a_2, v_2 and ρ_2 .



Equation of continuity

As m = Volume × density

= Area of cross-section × length × density

∴ Mass of fluid that flows throgh section A in time Δt,

$m_1 = a_1 v_1 \Delta t \rho_1$

Mass of fluid that flows through section B in time Δt,

$m_2 = a_2 v_2 \Delta t \rho_2$

By conservation of mass,

$m_1 = m_2$

or

$a_1 v_1 \Delta t \rho_1 = a_2 v_2 \Delta t \rho_2$

As the fluid is incompressible, so $\rho_1 = \rho_2$, and hence

$a_1 v_1 = a_2 v_2$ or av = constant.

This is the equaiton of continuity. It states that during the streamlined flow of the non-viscous and incompressible fluid through a pipe of varying cross-section, the product of area of cross-section and the normal fluid velocity (av) remains contant throughout the flow.

Q.48 Why doews velocity increase when water flowing in broader pipe enters a narrow pipe?

Ans:

i. According to equation of continuity, in steady flow of liquid,

Av = constant

where A = area of cross section

v = velocity of the liquid flow

ii. This means, the volume of the liquid flowing through a pipe at any cross sectional area is constant.

iii. When water flowing in a broader pipe enters a narrow pipe, the area of cross section of the water decreases, therefore the velocity of water increases.

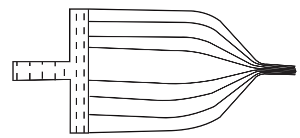
Type -XIII
Numerical based on equation of Continuity

Formula used

Av = constant

$A_1v_1 = A_2v_2$

1) As shown in the given figure, a piston of cross-sectional area 2 cm² pushes the liquid out of a tube whose area at the outlet in 40 mm². The piston is pushed at a rate if 2cm/s. Determine the speed at which the fluid leaves the tube.



Data: $A_1 = 2\ cm^2 = 2 \times 10^{-4}\ m^2$,

$v_1 = 2\ cm/s = 2 \times 10^{-2}\ m/s$

$A_2 = 40\ mm^2 = 40 \times 10^{-6}\ m^2$.

To find: Speed at which fluid leaves the tube (v_2).

Formula: $A_1v_1 = A_2v_2$

Solution:

$A_1v_1 = A_2v_2$

$$v_2 = \frac{A_1v_1}{A_2} = \frac{2\times10^{-4}\times2\times10^{-2}}{40\times10^{-6}} = 0.1\ m/s$$

Ans : The speed at which fluid leaves the tube is 0.1m/s

2) The speed of water is 2m/s through a pipe of internal diameter 10 cm. What should be the internal diameter of nozzle of the pipe if the speed of water at nozzle is 4 m/s?

Data: $v_1 = 2$ m/s, $d_1 = 10$ cm = 0.1 m, $v_1 = 4$ m/s

To find: Internal diameter of nozzle (d_2)

Formula: $A_2v_2 = A_1v_1$

Solution:

$A_2v_2 = A_1v_1$

$$A2\ (4) = \frac{\pi(0.1)^2}{4}\times(2)$$

$$\therefore \quad \frac{\pi d_2^2}{4}\ (4) = \frac{\pi(0.1)^2}{4} \times 2$$

$\therefore \quad d_2 = \sqrt{\frac{(0.1)^2}{2}} = \frac{1}{\sqrt{2}} \times 10^{-1} = 0.707 \times 10^{-1}$

$\therefore \quad d_2 = 7.07 \times 10^{-2}$m.

Ans :	The internal diameter of the nozzele is 7.07×10^{-2}m

Problem for Practice

1. Water flows through a horizontal pipe whose internal diameter is 2.0cm, at a speed of 1.0 ms^{-1}. What should be the diameter of the nozzle, if the water is to emerge at a speed of 4.0 ms^{-1}?

Ans: 1.0 cm.

2. In a normal adult, the average speed of the blood through the aorta (which has a radius of 0.9 cm) is 0.33 ms^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.5 cm. Calculate the speed of blood through the arteries.

Ans: 0.036 ms^{-1}

2.9 Bernoulli's equation

Q.49 State Bernoulli's principle and derive Bernoulli's equation.

Ans:

i. **Statement:** The work done per unit volume os a fluid by the surounding fluid is equal to the sum of the chanes in kinetic and potential energies per unit volume that occur during the flow.

ii. Consider an ideal fluid flowing through a tube of varying cross section and height. Consider an element of fluid lying between cross sections P and R.

iii. Let,

a. v_1, v_2 = speeds of the fluid at the lower end P and the upper end R respectively.

b. A_1, A_2 = cross section areas of the fluid at the lower end P and upper end R respectively.

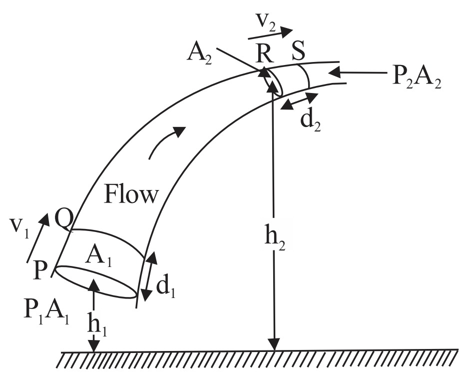
c. P_1, P_2 = pressures of the fluid at the lower end P and upper end R respectively.

d. d_1, d_2 = distances travelled by the fluid at the lower end P and the upper end R during the time interval dt with velocities v_1 and v_2 respectively.

e. ρ = density of the fluid flowing through the tube.

f. h_1, h_2 = mean heights of section P and R from ground or reference level.

g. P_1A_1, P_2A_2 = forces acting on the fluid at section P and R respectively.



Flow of fluid through a tub e of varying cross section and height

iv. The volume dV of the fluid passing through any cross section during time interval dt is the same

i.e., $dV = A_1d_1 = A_2d_2$ ....(1)

v. Since, the fluid is ideal there is no internal friction in the fluid. The only non gravitational force working on the fluid element is due to the pressure of the surrounding fluid.

vi. Therefore, the net work W done on the element by the surrounding fluid during the flow from P to r is,

$W = P_1A_1d_1 - P_2A_2d_2$ ....(2)

The negative sign indicates that the force at R opposes the displacement of the fluid.

vii. Substituting equation(1) in equation (2), we get,

$W = P_1dV - P_2dV$

$\therefore \quad W = (P_1 - P_2)dV$ ....(3)

viii. At the begining of the time interval dt,

a. mass of the fluid between P and Q = ρA_1d_1,

b. Kinetic energy of the fluid between P and

$Q = \frac{1}{2}\rho(A_1d_1)v_1^2$

c Potential energy of the fluid between P and Q = mgh1 = $\rho\ dVgh_1$

ix. At the end of the time inteval dt,

a. mass of the fluid between R and S = $\rho\ A_2d_2$

b. Kinetic energy of the fluid between R and

$$S = \frac{1}{2}\rho(A_2d_2)v_2^2$$

c. Potential energy of the fluid between R and S = $mgh_2 = \rho\ dVgh_2$

x. The net change in the kinetic energy $\Delta K.E$, during time interval dt is,

$$\Delta K.E = \frac{1}{2}\rho(A_2d_2)v_2^2 - \frac{1}{2}\rho(A_1d_1)v_1^2$$

$\therefore$ $$\Delta K.E = \frac{1}{2}\rho(dV)v_2^2 - \frac{1}{2}\rho(dV)v_1^2$$

$\therefore$ $$\Delta K.E = \frac{1}{2}\rho dV\left(v_2^2 - v_1^2\right) \quad ....(4)$$

xi. The net change in the gravitational potential energy during time interval dt is,

$$\Delta P.E = \rho dVgh_2 - \rho dVgh_1$$

$\therefore$ $$\Delta P.E = \rho dVg\left(h_2 - h_1\right) \quad ....(5)$$

xii. As the work done W is due to forces other than the conservative force of gravity, it equals the change in the total mechanical energy

$\therefore$ $W = \Delta K.E + \Delta P.E$.

Sustituting equation (3), (4) and (5) in the above equation,

$(P_1 - P_2)\ dV$

$$= \frac{1}{2}\ \rho dV\left(v_2^2 - v_1^2\right) + \rho dVg\left(h_2 - h_1\right)$$

$$(P_1 - P_2) = \frac{1}{2}\ \rho\left(v_2^2 - v_1^2\right) + \rho g\left(h_2 - h_1\right)$$

....(6)

This is known as Bernoulli's equation.

xiii. Bernoulli's equation can be writtenb as ,

$$P_1 + \frac{1}{2}\ \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\ \rho v_2^2 + \rho gh_2$$

Or $P + \frac{1}{2}\ \rho v^2 + \rho gh = \text{constant}.$

Q.50 Why does the speed of a liquid increase and its pressure decrease when a liquid passes through constriction in a horizontal pipe?

Ans:

i. As per equation of continuity, when the liquid flows through a constriction, the area of cross-section of the liquid decreases, therefore the velocity of the liquid increases.

ii. According to Bernoulli's theorem, the sum of pressure energy, potential energy and kinetic energy per unit mass is constant at all cross-section in the stream line flow of an ideal liquid.

$\therefore$ $$P + \frac{1}{2}\ \rho v^2 + \rho gh = \text{constant.}$$

iii. If the liquid is flowing through a horizontal tube, the two ends of the tube are at the same level.

iv. Therefore, there is no gravitational head (level difference) i.e., h = 0

$\therefore$ $$P + \frac{1}{2}\ \rho v^2 = \text{cosntant} \quad ...(1)$$

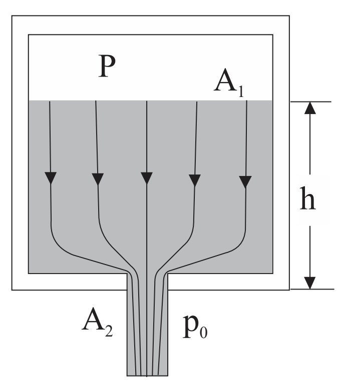
v. This shows that since the velocity of liquid increases, its pressure decreases when passing through a constriction in a horizontal pipe.

Q.51 What is the basis of the Bernoulli's principle?

Ans: Principle of conservation of energy forms the basis of Bernoulli's priciple

Q.52 Derive the equation of the speed of a liquid flowing through an orifice at a depth 'h' below the free surface.

Ans:



i. Consider a liquid of density 'ρ' filled in a tank of large cross-sectional area A_1 having and orifice of cross-sectional area A_2 at the bottom as shown in figure. Such that $A_2 << A_1$

ii. The liquid flows out of the tank through the orifice. Let v_1 and v_2 be the speeds of the liquid at A_1 and A_2 respectively.

iii. The height of the free surface above the orifice is h. Inlet and outlet, are exposed to the atmosphere.

∴ Pressure at positions at points A_1 and A_2 =Atmospheric pressure.

iv. From Bernoulli's equation we have,

$$P_0 + \frac{1}{2}\rho v_1^2 + \rho gh = P_0 + \frac{1}{2}\rho v_2^2 \quad ....(1)$$

v. Using equation of continuity, we have,

$A_1v_1 = A_2v_2$

∴ $$v_1 = \frac{A_2}{A_1}v_2 \quad ....(2)$$

vi. Substituting equation (2) in equation (1), we get,

$$\frac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 + \rho gh = \frac{1}{2}\rho v_2^2$$

$$\left(\frac{A_2}{A_1}\right)^2 v_2^2 + 2gh = v_2^2$$

$$2gh = v_2^2 - \left(\frac{A_2}{A_1}\right)^2 v^2$$

$$\left[1-\left(\frac{A_2}{A_1}\right)^2\right]v_2^2 = 2gh$$

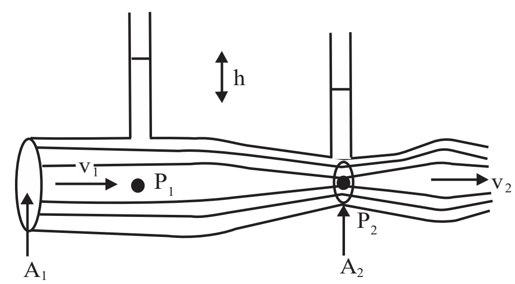
vii. Since, $A_2 << A_1$, the above equation reduces to , $v_2 = \sqrt{2gh}$ ....(3)

The above equation gives the speed of a liquid flowing out throgh an orifice at a depth 'h' below the free surface. It is the same as that of a particle falling freely through the height 'h' under gravity and is known as Torricelli's law.

Q.53 Explain the working of venturi-tube.

Ans: Venturi-tube:

i. The venturi-tube is a device which is used to measure the flow speed of incomprssible fluid.



Venturi-tube

ii. It has a constriction in the tube. As the fluid passes through the constriction, its speed increases in accordance with the equation of continuity. The pressure thus decreases as required by the Bernoulli's equation.

iii. The fluid of density ρ flows through the Venturi tube. A_1, A_2 = areas of cross section at wider part and at constriction respectively.

v_1, v_2 = speeds of the fluids at A_1 and A_2 respectively.

P_1, P_2 = pressures on the fluid at A_1 and A_2 respectively.

From Bernoulli's equation,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \quad ....(1)$$

$$(P_1 - P_2) = \frac{1}{2}(v_2^2 - v_1^2) \quad ....(2)$$

iv. Two vertical tubes are connected to the Venturi tube at A1 and A2. If the differnce in height of the liquid levels in the tubes is h, we have,

$(P_1 - P_2) = \rho gh$ ....(3)

Substituting equation(3) in equation(2) we get,

$2gh = v_2^2 - v_1^2$

v. Using equation of continuity, the rate of flow of liquid passing through a cross section can be calculated by knowing the areas A_1 and A_2

Type - XIV]

Numerical based on Bernouli's principle

Formulae used

1.

2.

1) **The pressure of water inside the closed pipe is 3×10^5 N/m². This pressure reduces to 2×10^5N/m². on opening the valve of the pipe. Calculate the speed of water flowing through the pipe. (Density of water = 1000 kg/m³).**

Data: $P_1 = 3\times10^5$ N/m², $P_2 = 2 \times 10^5$ N/m², $\rho_w = 1000$kg/m³, $v_1 = 0$

To find: Speed of water(v_2)

Formula: $P_1\text{-}P_2=\frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$

Solution:

$$P_1\text{-}P_2=\frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$3\times10^{-5}\text{-}2\times10^{-5}=\frac{1}{2}\times10^3\times\left(v_2^2 - 0\right)$$

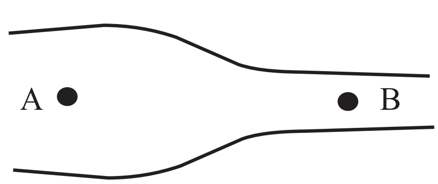
$$\therefore \quad v_2^2 = \frac{2\times10^5}{10^3}$$

$$\therefore \quad v_2 = \sqrt{2}\times10$$

$$= 14.14 \text{ m/s}$$

Ans :	Speed of water flowing through the pipe is 14.14 m / s.

2) **The given figure shows a streamline flow of a non-viscous liquid having density 1000 kg/m³. The cross sectional area at point A is 2 cm² and at point B is 1 mm². The speed of liquid at the point A is 5 cm/s. Both points A and B are at teh same horizontal level. Calculate the difference in pressure at A and B.**



Data: $\rho = 1000$kg/m³, $A_1 = 2$cm² $= 2\times10^{-4}$m², $A_2 = 1$mm² $= 10^{-6}$ m², $v_1 = 5$cm/s $= 5\times10^{-2}$ m/s, $h_1\text{-}h_2=0$

To find: Pressure differene between A and B ($P_1\text{-}P_2$)

Formula:

i. $A_1v_1=A_2v_2$

ii. $(P_1\text{-}P_2)=\frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$

Solution:

$$A_1v_1=A_2v_2$$

$$\therefore \quad v_2 = \frac{A_1v_1}{A_2} = \frac{2\times10^{-4}\times5\times10^{-2}}{10^{-6}} = 10\text{m / s}$$

$$(P_1\text{-}P_2)=\frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

$$(P_1\text{-}P_2) = \frac{1}{2}\rho(v_2^2 - v_1^2) \quad ....(\because h_2 - h_1 = 0)$$

$$\therefore \quad P_1\text{-}P_2 = \frac{1}{2}\times1000\times(100 - 0.0025)$$

$$= 500\times99.9975$$

$$\therefore \quad P_1\text{-}P_2 = 49998.75 \text{ Pa} = 4.99 \times 10^4\text{Pa}$$

Ans :	The difference in pressure at A and B is 4.99×10^4 pa

3) **Doors of a dam are 20 m below the surface of water in the dam. If one door is opened, what will be the speed of the water that flows out of the door?(g = 9.8m/s²)**

Data: h = 20m, g = 9.8 m/s²

To find: Speed of water (v)

Formula: $v = \sqrt{2gh}$

Solution:

$$v = \sqrt{2gh}$$

$$v = \sqrt{2\times9.8\times20} = \sqrt{392} = 14\sqrt{2}$$

$$= 14 \times 1.414$$

$$v = 19.796 \text{ m/s}$$

Ans :	Speed of water when it flows out of dam's door is 19.796m / s.

4) **With what velocity does water flow out of an orifice in a tank with gauge pressure 4×10^5 N/m² before the flow starts? Density of water = 1000kg/m³.**

Data: $P = 4\times10^{-5}$N/m², $\rho = 1000$ kg/m³

To find: Velocity of flow of water(v)

Formulae: i. $h = \frac{P}{\rho g}$ ii. $v = \sqrt{2gh}$

Solution:

$$h = \frac{P}{\rho g}$$

$$h = \frac{4 \times 10^5}{1000 \times 10} = 40m$$

$$v = \sqrt{2gh}$$

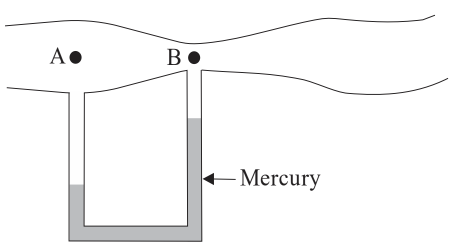
$$v = \sqrt{2 \times 10 \times 40}$$

$$= 20\sqrt{2} = 20 \times 1.414$$

$\therefore$ $v = 28.28$ m/s

Ans :	Velocity of the water flowing out is 28.28m/s

5) **Water flows through a tube as shown in the given figure. Find the difference in mercury level, if the speed of flow of water at point A is 2m/s and at point B is 5 m/s. (g = 9.8 m/s)**



Data: v_1=2m/s, v_2 = 5m/s

To find: Difference in mercury level (h)

Formula: $2gh = v_2^2 - v_1^2$

Solution:

$$2gh = v_2^2 - v_1^2$$

$$h = \frac{v_2^2 - v_1^2}{2g} = \frac{25-4}{2 \times 9.8} = \frac{21}{19.6} = \frac{210}{196}$$

$\therefore$ $h = \frac{30}{28} = \frac{15}{14} = 1.07m$

Ans :	The difference in mercury level is 1.07m.

Problem for Practice

1. A cylinder of height 20 m is completely filled with water. Find the velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom. Given g = $10ms^{-2}$.

Ans: $20ms^{-1}$

2. A boat strikes an under water rock which punctures a hole 5 cm in diameter in its hull which is 1.5 m below the water line. At what rate in litre per second does water enter?

Ans: 10.65 litres s^{-1}

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