03

Kinetic Theory of Gases and Radiation

Syllabus

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3.1 Introduction

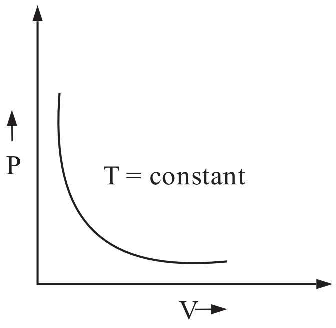
Boyle's law

i. It states that the volume of a given mass of a gas is inversely proportional to its pressure, provided the temperature remians constant.

ii. Mathematically,

$$V \propto \frac{1}{P} \text{ or } V = \frac{K}{P} \text{ or } PV = K$$

iii. If P_1 and V_1 are the initial values of pressure and volume and P_2 and V_2 are their final values, then according to Boyle's law.
$P_1V_1 = P_2V_2$.



P versus V graph

Charle's law

i. This law gives relationship between volume and temperature of gas at constant pressure.

ii. The volume of a fixed mass of gas at constant pressure is directly proportional to its absolute temperature.

iii. Mathematically, $V \alpha T$

$\therefore$ $\frac{V}{T}$ = Constant

iv. If V_1 and T_1 are the inital values of volume and temperature and V_2 and T_2 are their final values then,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Gay Lussac's law

i. This law gives relation between pressure and temperature of a gas.

ii. At constant volume, pressure of fixed mass of gas is directly proportional to its absolute temperature.

iii. Mathematically,

$\therefore$ $P \alpha T$

$\therefore$ $P = KT$

$\therefore$ $\frac{P}{T} = K = \text{Constant}$

iv. If P_1 and T_1 are the initial values of pressure and temperature and P_2 and V_2 are their final values then,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Avogadro’s hypothesis :

The number of molecules per unit volume is same for all gases at a fixed temperature and pressure.

The number of molecules in 22.4 liters of any gas are 6.02×10^{23}. This is known as Avogadro number and denoted as N_A.

The perfect gas equation is,

PV = nRT where, n is the number of moles and $R = N_A k_B$, called as a universal gas constant.

$R = 8.314\ \text{J mol}^{-1}\ \text{K}^{-1}$

$$n = \frac{M}{M_0} = \frac{N}{N_A}$$

where M is the mass of the gas containing N molecules. M_0 is the molar mass and N_A is Avogadro’s number.

The value of Boltzmann constant (k_B) in SI units is $1.38 \times 10^{-23}\ \text{JK}^{-1}$.

Importance of Avogadro's number.

i. To calculate the actual weight of one atom of an element.

Weight of one atom of an element

$$= \frac{\text{Atomic wight (in gram)}}{\text{Avogadro'number}}$$

ii. To calculate the actual weight of one molecule of a substance.

Weight of one molecule of a substance

$$= \frac{\text{Molecular weight (in gram)}}{\text{Avogadro'number}}$$

iii. To calculate the number of atoms present in given amount of an element.

Number of atoms present in m gram of an

$$\text{element} = \frac{\text{Avogadro's number}}{\text{Atomic weight}} \times m$$

iv. To calculate the number of molecules present in given amount of a substance.

Number of molecules present in m gram of a

$$\text{substance} = \frac{-\text{Avogadro's number}}{\text{Molecular weight}} \times m$$

v. To calculate the number of molecules present in given valume of the gas. At S.T.P., the 22.4 litres of every gas contain an Avogadro's number of molecules.

∴ Number of molecules present in v litres of a gas at S.T.P.

$$\text{S.T.P.} = \frac{\text{Avogadro's number}}{22.4} \times v$$

Graham's law of diffusion

It states that rate of diffusion of a gas is inversety proportional to the square root of its density.

Dalton's law of partial pressure

It states that the total pressure exerted by a mixture of non- reacting gases occupying a given volume is equal to the sum of the partial pressures which each gas would exert if it alone occupied the same volume at the given temperature.

Q.1 Prove that $PV = NK_BT$

Ans :

i. The three gas laws applied to a fixed mass m of an enclosed gas are,

a. Boyle’s law $V \propto \frac{1}{p}$ at constant T.

b. Charle’s law $V \propto T$ at constant V.

c. Gay-Lussac’s law $P \propto T$ at constant V.

ii. Combining the three laws,

$$PV \propto T \text{ i.e., } \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

iii. In terms of number of moles (n) of the gas,

$PV \propto nT$

∴ $\boxed{PV = nRT}$ ...(1)

Where, R = proportionality constant
= Universal gas constant

iv. But, number of moles (n)

$$= \frac{\text{mass of the gas (m)}}{\text{molar mass}(M_0)} = \frac{N}{N_A} \qquad ...(2)$$

Where, N = number of molecules,
N_A = Avogadro’s number
M_o = Mass of 1 mole of gas

v. The universal gas constant can also be expressed in terms of Boltzmann constant (k_B) as,

$R = N_A k_B$ ...(3)

Substituing equations (2) and (3) in equation (1), we get

$$\therefore \quad PV = \frac{N}{N_A} \times N_A k_B T = Nk_B T$$

Hence proved

Type - I

Numerical based on Ideal gas equation

Formulae Used

$PV = nRT$

★1) Two vessels A and B are filled with same gas where volume, temprature and pressure in vessel A is twice thevolume, temprature and pressure in vessel B. Calculate the ratio of number of molecules of gas in vessel A to that in vessel B.

Data: $P_A = 2P_B,\ T_A = 2T_B,\ V_A = 2V_B$

To find: Ratio of number of molecules($N_1 : N_2$)

Formulae: i. $PV = nRT$ ii. $n = \frac{N}{N_A}$

Solution:

i. $PV = nRT$

$$\therefore \quad PV = \frac{N}{N_A} RT \qquad \left[\because \ n = \frac{N}{N_A}\right]$$

$$N = \frac{PVN_A}{RT}$$

ii. For Vessel A

$$N_1 = \frac{P_A V_A N_A}{RT_A} \qquad ......(i)$$

For Vessel B

$$N_2 = \frac{P_B V_B N_A}{RT_A} \qquad ......(ii)$$

Dividing eg (i) and (ii)

we get

$$\frac{N_1}{N_2} = \frac{P_A V_A}{T_A} \times \frac{T_B}{P_B V_B}$$

$$= \frac{2P_B \times 2V_B \times T_B}{2T_B \times P_B \times V_B}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{2}{1}$$

Ans : The ratio of number of molecules is 2 : 1.

2) 16 g of oxygen occupy 0.025 m³ at 27° C. If the universal gas constant is 8.4 J / mol-K. Find the pressure exerted by it [Molecular weight of oxygen = 32]

Data: $m = 16$ g.

$M = 32,\ V = 0.025 m^3$

$$\therefore \quad n = \frac{m}{M} = \frac{16}{32} = \frac{1}{2} = 0.5$$

$T = 27^\circ C = 273 + 27 = 300K.$

$R = 8.4$ J/mol-K.

To Find: P

Formula : $PV = nRT$

Solution : $P = \frac{nRT}{V} = \frac{0.5 \times 8.4 \times 300}{0.025}$

$\therefore \quad P = 54000\ N/m^2$

$\therefore \quad P = 5.4 \times 10^4\ N/m^2$

Ans : Pressure exerted by O_2 is $5.4 \times 10^4\ N/m^2$

3) Two tanks of equal volume contain equal masses of oxygen and nitrogen at 127°C. find the ratio of -

i. number of molecules in two tanks

ii. pressure in two tanks

Data: $V_0 = V_N,\ m_o = m_N,$

$T_1 = T_2 = T = 127^\circ C = 273 + 127 = 400K$

$M_O = 32,\ M_N = 28$

$\frac{N_O}{N_N} = ?,\ \frac{P_O}{P_N} = ?$

Let, N_o = No. of molecules of oxygen

N_N = No. of molecules of nitrogen

n_o = No. of moles of oxygen

n_N = No. of moles of nitrogen

N_A = Avogadro number.

To Find : $\frac{N_O}{N_N} = ?,\ \frac{P_O}{P_N} = ?$

Formula : i. $n = \frac{N}{N_A}$ ii. $n = \frac{m}{M}$

Solution:

i. For Oxygen

$$n_{.o}\frac{N_o}{N_A}=\frac{m_o}{M_o}$$

$\therefore$ $$m_{.o}=\frac{N_o}{N_A}\times M_o \quad ...(i)$$

For Nitrog

$$n_n\frac{N_N}{N_A}=\frac{m_N}{M_N}$$

$\therefore$ $$m_{.N}=\frac{N_A}{N_A}\times M_N \quad ....(ii)$$

But, $m_0 = m_N$

Equating equation (i) and (ii),

$$\frac{N_o}{N_A}.Mo=\frac{N_N}{N_A}M_N$$

$\therefore$ $N_oM_o = N_N M_N$

$\therefore$ $$\frac{N_o}{N_N}=\frac{M_N}{M_o}$$

$\therefore$ $$\frac{N_o}{N_N}=\frac{28}{32}=\frac{7}{8}=7:8$$

ii. $PV = nRT$

$P_oV = n_o RT \quad ...(7)$

$P_NV = n_N RT \quad ...(8)$

Divide equation (7) by (8)

$\therefore$ $$\frac{P_oV}{P_NV}=\frac{n_oRT}{n_NRT}$$

$\therefore$ $$\frac{P_o}{P_N}=\frac{n_o}{n_N}$$

$\therefore$ $$\frac{P_o}{P_N}=\frac{m_o}{M_o}/\frac{m_N}{M_N}$$

$$=\frac{m_o}{M_o}\times\frac{M_N}{m_N}$$

But, $m_o = m_N$

$\therefore$ $$\frac{P_o}{P_N}=\frac{M_N}{M_o}=\frac{28}{32}$$

$\therefore$ $$\frac{P_o}{P_N}=\frac{7}{8}=7:8$$

Ans : Ratio of number of molecules is 7:8 and ratio of pressure by two gas is 7:8

Problem for Practice

1. 1 molar volume is the volume occupied by one mol of an ideal gas at S.T.P. Find the volume. R = 8.31 J/mol/K one litre $=10^{-3}m^3$.

Ans : 22.4 litres.

2. A vessel contains two non-reacting gases: neon (monoatomic) and oxygen (diatomic). The ratio of their partial pressures is 3:2. Estimate the ratio of (i) number of molecules and (ii) mass density of neon and oxygen in the vessel. Atomic mass of Ne = 20.2, molecular mass of $O_2 = 32.0$

Ans : (i)1.5, (ii) 0.947

3. A balloon partially filled with Helium has a volume of $30m^3$, at the earth's surface, where pressure is 76 cm of Hg and temperature is 27°C, what will be the increase in volume of gas if balloon rises to a height, where pressure is 7.6 cm of Hg and temperature is (–54°C)?

Ans : 189 m^3

3.2 Behaviour of a Gas

Q.2 Why is it necessary to make assumptions while studying behaviour of a gas?

Ans :

i. For any solid object, its motion can be described well with the help of Newton's laws of motion.

ii. Similarly, a gas enclosed in a container can be characterised by macroscopic state variables like pressure, volume and temperature.

iii. However, as gas molecules are always in random motion, it is difficult to understand behaviour of a gas in terms of motion of a single molecule.

iv. The number of molecules in the gas is so large ($=10^{23}$ per m^3) that motion of individual molecule cannot be related with macroscopic parameters P,V,T and energy E.

Hence, it is necessary to make certain assumptions while studying behaviour of a gas.

Q.3 State the assumptions of the kinetic theory of gases.

Ans: Assumptions of kinetic theory of gases :

A. Nature of gas molecules :

i. The molecules are assumed to be rigid and perfectly elastic spheres of very small diameter.

ii. The actual volume occupied by the molecules is very small as compared to the total volume of the gas.

iii. Inter molecular force between neighboring gas molecule's absent.

iv. The average kinetic energy of gas is directly proportional to the absolute temperature of gas.

v. On account of large number of molecules in the gas, the number of molecules per unit volume of the gas remains constant everywhere in the gas.

B. Random motion of gas molecules :

i. The collisions are perfectly elastic.

ii. The time of impact is very small.

iii. Between two successive collisions, the molecules follows a straight lines, path know as free path.

iv. The gas molecules can move with all possible velocities.

3.3 Ideal Gas and Real Gas

Q.4 Distingvish between Ideal gas and Real gas

Ans :

No.	Ideal Gas	Real Gas
i.	Ideal gas has no definite volume.	Real gas has definite volume.
ii.	There are no intermolecular forces in this cases.	Intermolecular forces are non-zero in this case.
iii.	An ideal gas cannot be liquefied	Real gas can be liquified and also solidified.
iv	Elastic collision between the particle	Non-elastic collision between the particle
v.	Size of particle is negligible in case of ideal gases	The particle of real gases have finite size.

★ Q.5 Mention the conditions under which real gas obeys ideal gas equation

Ans : A real gas obeys ideal gas equation when temperature is very high and pressure is very low.

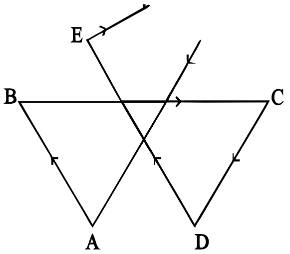
3.4 Mean Free Path

Q.6 Define : i. Free path ii. Mean free path

Ans:

i. **Free path :** The straight line path covered by the molecules between two successive collisions is called the free path.

Between two successive collisions, a molecule travels in a straight line with constant velocity as shown in figure.



ii. **Mean free path :**

The average distance covered by the molecule between two successive collisions is called the mean free path.

Mean free path is measured in Angstrom unit or meter.

Let, $\lambda_1, \lambda_2, \ldots\ldots \lambda_N$ be the free paths, then mean free path

$$\lambda_1 = \frac{\lambda_1 + \lambda_2 + \ldots + \lambda_N}{N}$$

where 'N' is the number of collisions.

Q.7 On what factor mean free path depends. Write a formula for mean free path.

Ans :

i. The mean free path (λ) varies

a. inversely with the density $\rho = \frac{N}{V}$ of the gas.

where, N = number of molecules,
V = volume of the gas

b. inversely with dquare of the diameter of molecule d^2 (because it depends on the cross - section of a molecule).

ii. Thus, from above proportionalities, it can be shown that,

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 (N/V)}$$

Type - II

Numerical based on mean free path

Formulae Used

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 (N/V)}$$

1) Obtain the mean free path of nitrogen molecule at 0°C and 1.0 atm pressure. The molecular diameter of nitrogen is 324 pm (assume that the gas is ideal).

Data: $T = 0°C = 273K$,
$P = 1.0atm = 1.01\times10^5 Pa$,
$d = 324\ pm = 324 \times 10^{-12}\ m$,
$k_B = 1.38 \times 10^{-23}\ J/K$

To find: Mean free path of $N_2(\lambda)$

Formulae: i. $PV = Nk_BT$ ii. $\lambda = \dfrac{1}{\sqrt{2}\pi d^2 \left(\dfrac{N}{V}\right)}$

Solution:

i. $PV = Nk_BT$

$$\frac{N}{V} = \frac{P}{k_B T}$$

Using this in formula (ii),

$$\lambda = \frac{k_B T}{\sqrt{2}\pi d^2 P}$$

$$= \frac{1.38\times10^{-23}\times273}{\sqrt{2}\pi(324\times10^{-12})^2\times1.01\times10^5}$$

$= 8.002 \times 10^{-4} \times 10^{-4} = 8 \times 10^{-8}$
$= 0.8 \times10^{-7}m$

$$\boxed{\textbf{Ans :} \text{Mean free path of } N_2 \text{ molecule is } 0.8\times10^{-7}\ m.}$$

Problem for Practice

1. If the mean free path of a molecule of an ideal gas having diameter 10^{-8} cm is 10^{-4} cm. Calculate the number of gas moleculer per unit volume .

   Ans : $22.5\times10^{18}/cm^3$

2. Calculate the diameter of one molecule of an ideal gas number dinsity $2\times10^8/cm^3$ and mean free path of molecule is 10^{-8} cm

   Ans : 3.35 mm

Q.8 Explain the terms:
i. Mean velocity (Average velocity)
ii. Mean square velocity,
iii. Root mean square velocity.

Ans:

i. Mean velocity (Average velocity):

It is defined as the arithmetic mean of velocities of all molecules of gas.

Consider 'N' molecules of an ideal gas enclosed in a container. Let $v_1, v_2 ...., v_N$ be their velocities, then their mean velocity

$$\bar{v} = \frac{v_1 + v_2 .... + v_n}{n}$$

S.I. Unit : m/s.

ii. Mean square velocity :

Mean square velocity is defined as the average of the square of the velocities of all molecules.

Let, $v_1, v_2, ..., v_n$ be the velocities of n molecules of a gas; then mean square velocity is,

$$\overline{v^2} = \frac{{v_1}^2 + {v_2}^2 + ... + {v_n}^2}{n}$$

S.I. Unit : m^2/s^2.

iii. Root mean square velocity(rms velocity):

The square root of the mean of the squares of the velocities of gas molecules is called rms velocity.

Let, $v_1, v_2, ... , v_n$ be the velocities of n molecules of a gas; then mean square velocity; is,

$$Vrms = \sqrt{\frac{{v_1}^2 + {v_2}^2 + ... + {v_n}^2}{n}} = \sqrt{\overline{v^2}}$$

S.I. Unit : m/s.

Type - III
Numerical based on rms velocity of Gas

Formulae Used

1. Mean speed

$$\bar{v} = \frac{v_1 + v_2 .... v_3 + ... v_N}{N}$$

2. Mean square speed

$$\overline{v^2} = \frac{v_1^2 + v_2^2 .... v_3^2 + ... v_N^2}{N}$$

3. R.M.S. Velocity

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{v_1^2 + v_2^2 .... v_3^2 + ... v_N^2}{N}}$$

1) The velocities of three molecules are 2 m/s, 3 m/s, and 4 m/s respectively. Find the mean velocity, mean square velocity and the root mean square velocity of the molecules.

Data : $v_1 = 2$ m/s, $v_2 = 3$ m/s $v_3 = 4$ m/s

To Find : $\overline{v}$, $\overline{v^2}$, V_{rms}

Formula :

i. $\overline{v} = \frac{v_1 + v_2 + v_3}{3}$

ii. $\overline{v} = \frac{v_1 + v_2 + v_3}{3}$

iii. $V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2}{3}}$

Solution:

i. Mean velocity is,

$$\overline{v} = \frac{v_1 + v_2 + v_3}{3} = \frac{2+3+4}{3} = 3 \text{ m/ s}$$

ii. Mean square velocity,

$$\overline{v} = \frac{v_1 + v_2 + v_3}{3} = \overline{c}^2 = \frac{4+9+16}{3}$$

$$= 9.667 \text{ m}^2/\text{s}^2$$

iii. RMS velocity is,

$$V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2}{3}} = \sqrt{\frac{4+9+16}{3}}$$

$$= 3.109 \text{ m/s}$$

Ans : The mean velocity is 3m/s mean square velocity is 9.667 m^2/s^2 rms velocity is 3.109 m/s

Problem for Practice

1. Find r.m.s. velocity of three molecules having velocities 10 km/s, 20 km/s, 30 km/s.

Ans : 21.60 km/s

2. The velocities of seven molecules are 1 km/s, 2 km/s, 3 km/s, 4 km/s, 5 km/s, 6 km/s, and 7 km/s respectively. Find the mean square velocity of the molecules.

Ans : 21.60 km/s

Ans : 20 km/s.

3.5 Pressure of Ideal Gas

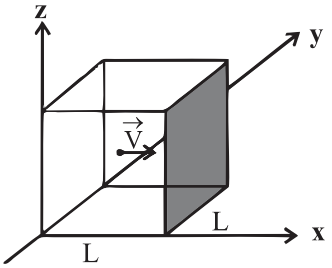
Q.9 On the basis of kinetic theory of gases derive on expression for the pressure exerted by by gas.

Ans :

i. Let there be n moles of an ideal gas enclosed in a cubical box of volume $V(=L^3)$ with sides of the box parallel to the co-ordinate axes, as shown in figure. The walls of the box are kept at a constant temperature T.

ii. The gas molecules are in continuous random motion, colliding with each other and hitting the walls of the box and bouncing back.

iii. As per one of the assumptions, we neglect intermolecular collisions with the walls and consider only elastic collisions with the walls.



iv. A typical molecule moving with the velocity $\vec{v}$, about to collide elastically with the shaded wall of the cube parallel to yz-plane.

v. During elastic collision, the component v_x of the velocity will get reversed, keeping v_y and v_z components unaltered.

vi. Hence the change in momentum of the particle is only in the x component of the momentum, ΔPx is given by, ΔPx = final momentum-initial momentum

$= (-mv_x) - (mv_x) = -2mv_x$ ...(1)

vii. Thus, the momentum transferred to the wall during collision is $+2mv_x$. The re-bounced molecule then goes to the opposite wall and collides with it and travels back towards the shaded wall again. This means that the molecule travels a distance of 2L in between two collisions. Where L is the length of the cubical box.

viii. The time for the molecule to travel back and forth to the shaded wall is

$$\Delta t = \frac{2L}{v_x}$$

ix. Average force exerted on the shaded wall by molecule 1 is given as, Average force = Average rate of change of momentum

∴ $$F_{X_1} = \frac{2mv_{x1}}{2L/v_{x1}} = \frac{mv_{x_1}^2}{L} \qquad ...(2)$$

where v_{x1} is the x component of the velocity of molecule 1.

xii. Considering other molecules 2,3,4... with the respective x components of velocities v_{x2}, v_{x3}, v_{x4} ...., the total average force on the wall is,

$$F_{avg} = \frac{m}{L}\left(v_{x1}^2+v_{x2}^2+v_{x3}^2+....\right) \qquad ...[From(2)]$$

∴ The average pressure

$$P = \frac{\text{Average force}}{\text{Area of shaded wall}} = \frac{m\left(v_{x1}^2+v_{x2}^2+...\right)}{L\times L^2}$$

......(3)

xiii. The average of the square of the x component of the velocities is given by,

$$\overleftarrow{v_x^2} = \frac{v_{x1}^2+v_{x2}^2+v_{x3}^2+...+v_N^2}{N}$$

Substituting in eg (3) we get

∴ $$P = \frac{mN\overline{v_x^2}}{V}$$

where $\overline{v_x^2}$ is the average over all possible values of v_x

xiv. Now, $\overline{v^2} = \overline{v_x^2}+\overline{v_y^2}+\overline{v_z^2}$

By symmetry, $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3}\overline{v^2}$

since the molecules have no preffered direction to move.

Therefore, average pressure

$$\boxed{P = \frac{1}{3}\frac{N}{V}m\overline{v^2}} \qquad ...(3)$$

Type - IV

Numerical based on pressure exerted by gas

Formulae Used

$$P = \frac{1}{3}\rho\,\overline{v}^2 = \frac{1}{3}\rho\,v_{rms}^2$$

$$P = \frac{1}{3}\frac{M}{V}v_{rms}^2$$

$$P = \frac{1}{3}\frac{NM}{V}v_{rms}^2$$

★ 1) A gas in a cylinder is at pressure P. If the masses of all the molecules are made one third of their original value and their speeds are doubles, then find the resultant pressure.

Data: $P_1 = P$, $m_2 = \frac{m_1}{3}$, $v_2 = 2v_1$

To find: P_2

Formula: $P = \frac{1}{3}\frac{N}{V}m\overline{v^2}$

Solution:

i. As $v_2 = 2v_1$

∴ $\overline{v_2^2} = 4\overline{v_1^2}$

ii. $P_1 = \frac{1}{3}\frac{N}{V}m_1\overline{v_1^2}$

and

$P_2 = \frac{1}{3}\frac{N}{V}m_2\overline{v_2^2}$

Dividing P_1 and P_2 we get

∴ $$\frac{P_1}{P_2} = \left(\frac{m_1}{m_2}\right)\left(\frac{\overline{v_1^2}}{\overline{v_2^2}}\right) = 3\times\frac{1}{4}$$

∴ $$P_2 = \frac{4}{3}P.$$

Ans: The resultant pressure of gas is $\frac{4}{3}$ P.

2) Find the number of molecules in 1 cm³ of oxygen at N.T.P., if the mass of oxygen molecule is 5. 313 × 10^{-26}kg. The rms velocity of oxygen molecule at N.T.P is 462 m/s. (1 atmospheric pressure = 1.013 × 10^5 N/m²)

Data: $V = 1\ cm^2 = 1 \times 10^{-6}\ m^3$,
$V_{rms} = 462 m/s$, $m = 5.313 \times 10^{-26} kg$,
$P = 1 atm = 1.013 \times 10^5\ N/m^2$.

To Find: **N**

Formula: Pressure exerted by gas,

$$P=\frac{1}{3}\frac{mN}{V}V^2_{rms}$$

$$\therefore \quad N=\frac{3PV}{mV^2_{rms}}$$

Solution:

$$N=\frac{3PV}{mV^2_{rms}}$$

$$=\frac{3\times(1.013\times10^5)\times(1\times10^{-6})}{(5.313\times10^{-26})\times(462)^2}$$

$= 2.68\times10^{19}$ molecules.

Ans : The number of molecules are 2.68×10^{19}.

★3) **If the density of oxygen is 1.44 kg/m³ at a pressure of 10^5 N/m², find the root mean square velocity of oxygen molecules.**

Data: $\rho = 1.44$ kg/m³, $P = 10^5$ N/m²

To find: v_{rms}

Formula: $P=\frac{1}{3}\rho v^2_{rms}$

Solution:

$$P=\frac{1}{3}\rho v^2_{rms}$$

$$\therefore \quad v_{rms}=\sqrt{\frac{3P}{\rho}}=\sqrt{\frac{3\times10^5}{1.44}}=456.44\text{m / s}$$

Ans : The root mean square velocity of oxygen molecule is 456.44m / s

Problem for Practice

1. Find the mass of 15 cc of a gas if the RMS velocity of the molecules of gas at N.T.P. is 400 m/s. Take normal atmospheric pressure as 1.013×10^5 N/m².

Ans : 2.85×10^{-5} kg.

2. The mass of Hg molecule is 3×10^{-27} kg. If 10^{24} molecules strike 10 cm² of area of a surface at an angle of 45° with the normal with a speed of 10^4m/s, what pressure will the surface experience?

Ans: 4230 N/m²

3. Find the R.M.S. velocity of H_2 molecules at N.T.P. [Given: Density of H_2 = 0.09 kg/m³, P = 10^5 N/m²]

Ans:1825.7 m/s

4. Find the number of molecules in 1 cm³ of oxygen at N.T. P, if mass of a oxygen molecules is 5.25×10^{-28} kg and r.m.s. velocity of oxygen molecule at N.T.P. is 426m/s. [Take pressure at N.T.P. $=10^5$ N/m²]

Ans:3.130×10^{21} molecules.

3.6 Root Mean Sqaure (rms) Speed

Q.10 Deduce Boyle's law on the basis of kinetic theory of gases.

Ans:

i. **Boyle's law:** It states that at constant temperature, the pressure exerted by the given mass of a gas is inversely proportional to its volume.

i.e. $P\propto\frac{1}{V}$

∴ PV = Constant.

ii. Consider a gas having N molecules each of mass m.

Let, P - Pressure of the gas
V - Volume of the gas
T - Temperature of the gas

From kinetic theory of gases,

$$P=\frac{1}{3}\frac{Nm}{V}V^2_{rms}$$

$$\therefore \quad PV=\frac{2}{3}N\left(\frac{1}{2}mV^2_{rms}\right)$$

$\frac{1}{2}mV^2_{rms}$ = average K.E. of a gas molecule,

iv. According to assumptions of kinetic theory, K.E. is constant at constant temperature.

$$\therefore \quad \frac{1}{2}mV^2_{rms}=\text{constant}$$

$$\therefore \quad \frac{2}{3}N\left(\frac{1}{2}mV^2\right)=\text{constant}$$

∴ PV = constant

$$\therefore \quad P\propto\frac{1}{V}$$

In this way, Boyle's law can be deduced from the kinetic theory of gases.

Q.11 Obtain the expression, $V_{rms}=\sqrt{\frac{3RT}{M}}$.

Ans:

i. Consider one mole of a gas.

P - Pressure of the gas.
V - Volume of the gas.
T - temperature of the gas
V_{rms} - rms velocity of the gas.

ii. Pressure exerted by gas is,

$$P = \frac{1}{3}\frac{M}{V}V^2_{rms}.$$

$\therefore$ $PV = \frac{1}{3}MV^2_{rms}$

From ideal gas equation,

PV = RT

$\therefore$ $RT = \frac{1}{3}MV^2_{rms}$

$\therefore$ $V_{rms} = \sqrt{\frac{3RT}{M}}$

iii. For one mole of the gas, M represents molecular weight. Also ,

$\therefore$ $V_{rms} \propto \sqrt{T}$

i.e. The rms velocity of the gas molecule is directly proportional to the square root of its absolute temperature. This is consistent with the assumption of the kinetic theory.

★ Q.12 What will happen to the mean square speed of the molecules of a gas if the temperature of the gas increases?

Ans : The mean square speed of the molecules of the gas is

$$\overline{v^2} = \frac{3RT}{M_0}$$

$\therefore$ $\overline{v^2} \propto T$

Hence, the mean square of molecules of the gas increases in same proportion with the increase in temperature.

Type - V
Numerical based on rms speed

Formulae Used

$$v_{rms} = \sqrt{\frac{3RT}{m_0}} \text{ OR } \overline{v^2} = \frac{3RT}{m_0}$$

★ 1) Calculate the ratio of mean square speeds of molecules of a gas at 30K and 120K.

Data: $T_1 = 30K$, $T_2 = 120K$

To find: $\left(\frac{\overline{v_1^2}}{\overline{v_2^2}}\right)$

Formula: $\overline{v^2} = \frac{3RT}{M_0}$ As R and M_o is constant

$\therefore$ $\overline{v^2}\,\alpha\,T$

Solution: $\overline{v^2}\,\alpha\,T$

$\therefore$ $\frac{\overline{v_1^2}}{\overline{v_2^2}} = \frac{T_1}{T_2} = \frac{30}{120} = \frac{1}{4}$

Ans : The ratio of mean square speed is 1 : 4

★ 2) Show that rms velocity of an oxygen molecule is $\sqrt{2}$ time that of a sulfur dioxide molecule at S.T.P.

To show that : $v_{O_2} = \sqrt{2}v_{SO_2}$

Proof : $M_{O_2} = 32,\ M_{SO_2} = 64$

Now, r.m.s. velocity,

$$V_{rms} = \sqrt{\frac{3RT}{M_0}}$$

$\therefore$ $v_{O_2} = \sqrt{\frac{3RT}{32}}$ ....(i)

$v_{SO_2} = \sqrt{\frac{3RT}{64}}$ ....(ii)

Dividing (i) and (ii)

$\therefore$ $\frac{v_{O_2}}{v_{SO_2}} = \sqrt{\frac{3RT}{32}\times\frac{64}{3RT}} = \sqrt{2}$

$\therefore$ $v_{O_2} = \sqrt{2}v_{SO_2}$

Hence proved

★ 3) At what temperature will oxygen molecules have same rms speed as helium molecules at S.T.P.? (Moleclar masses of oxygen and helium are 32 and 4 respectively)

Data: $v_{O_2} = v_{He}$, $T_{He} = 273K$, $M_{O_2} = 32$, $M_{He} = 4$,

To find: T_{O_2}

Formula: $v_{rms} = \sqrt{\frac{3RT}{M_0}}$

Solution: $v_{rms} = \sqrt{\frac{3RT}{M_0}}$

For Helium

$$v_{He} = \sqrt{\frac{3R \times 273}{4}} \quad ....(i)$$

For Oxygen

$$v_{O_2} = \sqrt{\frac{3R \times T_{O_2}}{32}} \quad ....(ii)$$

According to given condition

$\because \quad v_{O_2} = v_{H_e}$

$$\therefore \quad \sqrt{\frac{3R \times T_{O_2}}{32}} = \sqrt{\frac{3R \times 273}{4}}$$

From (i) and (ii)

$$\therefore \quad T_{O_2} = \frac{32 \times 273}{4} = 2184K$$

Ans : 2184K rms speed of O_2 is same as that of he at NTP

★ **4) Compare the rms speed of hydrogen molecules at 127°C with rms speed of oxygen molecules at 27°C given that molecular masses of hydrogen and oxygen are 2 and 32 respectively.**

Data: $T_{H_2} = 127°C = 127 + 273 = 400K$

$M_{H_2} = 2$

For oxygen,

$T_{O_2} = 27°C = 27 + 273 = 300K$

$M_{O_2} = 32$

To find: $v_{H_2} : v_{O_2}$

Formula: $v_{rms} = \sqrt{\frac{3RT}{M_0}}$

Solution:

$$v_{rms} = \sqrt{\frac{3RT}{M_0}}$$

For H_2 molecule

$$V_{H_2} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}} \quad ......(i)$$

For O_2 molecule

$$V_{O_2} = \sqrt{\frac{3RT_{O_2}}{M_{O_2}}} \quad ......(i)$$

Dividing (i) and (ii)

$$\therefore \quad \frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{T_{H_2}}{M_{H_2}} \times \frac{M_{O_2}}{T_{O_2}}}$$

$$\therefore \quad \frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{400}{2} \times \frac{32}{300}} = \frac{8}{\sqrt{3}}$$

$$\therefore \quad v_{H_2} : v_{O_2} = 8 : \sqrt{3}$$

Ans : The ratio of rms speed of hydrogen and oxygen molecule is $8 : \sqrt{3}$

Problem for Practice

1. Compute the rms velocity of oxygen molecules at 127^0 C. (Given: density of oxygen at N.T.P.=1.43 kg/m^3, 1 atmospheric pressure = 1.13×10^5 N/m^2)

Ans : 558 m/s

2. Find the temperature at which the RMS speed of a gas would be four times it's value at N.T.P.

Ans : 4368 K

3. At what temperature will the rms velocity of the molecules of a gas be double its value at N.T.P.?

Ans : 1092 K or 819^0 C

4. At what temperature will helium molecules have the same rms velocity as hydrogen molecules at N.T.P.? Take molecular weight of Hydrogen and Helium as 2 and 4 respectively.

Ans : 546 K or 273^0 C

3.7 Interpretation of Temperature in Kinetic Theory

Q.14. Show that the K.E. per unit volume of the gas is $\frac{3}{2}$P, where P is the pressure of the gas.

Ans: Consider a gas having N molecules, each of mass m. Let, P and V be the pressure and volume of the gas respectively.

From kinetic theory of gases,

$$P = \frac{1}{3}\frac{Nm}{V}V^2_{rms}$$

$\therefore$ $$PV = \frac{2}{3}N\left(\frac{1}{2}m\,V^2_{rms}\right)$$

$\therefore$ $$N\left(\frac{1}{2}m\,V^2_{rms}\right) = \frac{3}{2}PV$$

But, $\frac{1}{2}mV^2_{rms}$ = avg. K.E. of a gas molecule.

$\therefore$ Total K.E. of a gas = $N\left(\frac{1}{2}mC^2\right) = \frac{3}{2}PV$

$\therefore$ K.E. per unit volume of the gas = $\frac{3}{2}P$

Q.15 Obtain expression for

i. kinetic energy of a gas

ii. kinetic energy per unit volume

iii. kinetic energy per mole or kilomole

iv. kinetic energy per molecule

v. kinetic energy per unit mass.

Ans:

i. **Kinetic energy of a gas,**

$\therefore$ $$P = \frac{1}{3}\frac{M}{V}V^2_{rms}$$

$\therefore$ $$3PV = MV^2_{rms}$$

$\therefore$ $$\frac{3}{2}PV = \frac{1}{2}Nm\,V^2_{rms}$$

But, $\frac{1}{2}Nm\,V^2_{rms}$ = kinetic energy of a gas

$\therefore$ kinetic energy of a gas = ${}^{3}\!/_{2}\,PV$

ii. Kinetic energy per unit volume

$$= \frac{\text{kinetic energy of a gas}}{\text{volume of a gas}}$$

Energy density of a gas $= \frac{{}^{3}\!/_{2}\,PV}{V} = \frac{3}{2}P$

iii. **Kinetic energy per mole or kilomole**

For an ideal gas, PV = nRT.

Kinetic energy of a gas = ${}^{3}\!/_{2}\,PV$

Kinetic energy of a gas = ${}^{3}\!/_{2}\,nRT$

Kinetic energy per mole or kilomole = ${}^{3}\!/_{2}\,RT$

iv. Kinetic energy per molecule

$$= \frac{\text{kinetic energy per kilomole or mole}}{\text{Avogadros number } N_0}$$

$$= \frac{3}{2}\frac{RT}{N_A}$$

But, $\frac{R}{N_A} = K_B$

$\therefore$ where, K_B is called Boltzmann constant.

Kinetic energy per molecule = $\frac{3}{2}K_BT$

v. Kinetic energy per unit mass

$$= \frac{\text{kinetic energy per kilomole (or mole)}}{\text{molecular weight}}$$

$$= \frac{{}^{3}\!/_{2}\,RT}{M} = \frac{3RT}{2M}$$

★ Q.16 Explain, on the basis of the kinetic theory of gases, how the pressure of a gas changes if its volume is reduced at constant temperature.

Ans : Pressure exerted by gas is given by

$$P = \frac{1}{3}\frac{Nm}{V}\overline{V}^2$$

$$P = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}m\overline{V}^2\right)$$

Now $\frac{1}{2}m\overline{V}^2$ = constat at given temperature

$\therefore$ $$P = \frac{\text{constant}}{V}$$

$\therefore$ $$P \;\alpha\; \frac{1}{V}$$

Hence at constant temperature, pressure of gas is increased if volume is reduced

Type - V
Numerical based on K.E of gas

Formulae Used

1. Kinetic energy of gas = $\frac{3}{2}PV$
2. Kinetic energy per unit volume = $\frac{3}{2}P$
3. Kinetic energy per mole = ${}^{3}\!/_{2}\,RT$
4. K.E. per unit mass of gas = $\frac{3}{2}\frac{RT}{M}$
5. Kinetic energy per molecule $= \frac{3}{2}K_BT = \frac{3}{2}\frac{RT}{N}$

★ 1) Find kinetic energy of 5 litre of a gas at S.T.P. Given standard pressure is 1.013

$\times\ 10^5$ N/m^2.

Data: $V = 5L = 5\times10^{-3}m^3$

$P = 1.013\times10^5 N/m^2$, $T = 273K$

To find: K.E.

Formula: $K.E. = \frac{3}{2}PV$

Solution: $K.E. = \frac{3}{2}PV$

$$K.E. = \frac{3}{2}\times1.013\times10^5\times5\times10^{-3}$$

$$= 7.5975\times10^2 J$$

Ans : The kinetic energy of the gas is 7.5975×10^2 J

2) Calculate the average molecular kinetic energy i. per kmol ii. per kg iii. per molecule. of oxygen at 127°C, given that molecular weight of oxygen is 32,R is 8.31Jmol^{-1} K^{-1}and Avogadro's number N_A is 6.02×10^{23} molecules mol^{-1}

Data : $T = 127°C = 127 + 273 = 400K$

M = 32Molar mass = 32 kg kmol

$R = 8.31\ J\ mol^{-1}\ K^{-1}$

$N_A = 6.02\times10^{23}$ molecules mol^{-1}

To find: i. KE/kmol

ii. KE/kg

iii. KE/molecule

Formulae: i. $KE/kmol = \frac{3}{2}RT\times10^3$

ii. $KE/kg == \frac{3RT}{2M}\times10^3$

iii. $KE/molecule = \frac{3RT}{2N_A}$

Solution:

i. $KE/kmol = \frac{3}{2}RT\times10^3$

$$KE/kmol = \frac{3}{2}\times8.31\times400\times10^3$$

$$= 4.986\times10^6 J$$

ii. $KE/kg = \frac{3RT}{2M}\times10^3$

$$KE/kg = \frac{3}{2}\times\frac{8.31\times400}{32}\times10^3$$

$$= \frac{3\times8.31}{2\times8}\times10^5 = 1.5\times1.04\times10^5$$

$$= 1.56\times10^5 J$$

iii. $KE/molecule = \frac{3RT}{2N_A}$

$$KE/molecule = \frac{3}{2}\times\frac{8.31\times400}{6.02\times10^{23}}$$

$$= \frac{6\times8.31}{6.02}\times10^{-21}$$

$$= 8.281\times10^{-21} J$$

Ans : i. KE / kmol of gas is 4.987

ii. E / kg of gas is 1.56×10^5J.

iii. KE / molecule of gas is 8.281×10^{-21}J.

3) At 300 K, what is the rms speed of Helium atom? [mass of He stom is 4u, 1u = 1.66×10^{-27}kg; k_B=1.38×10^{-23}J/K]

Data: $T = 300K$, $m = 4\times1.66\times10^{-27}kg$,

$k_B = 1.38\times10^{-23} J/K$.

To find: v_{rms}

Formula: i. $K.E. = \frac{1}{2}m\overline{v^2}$ .....(i)

ii. $K.E. = \frac{3}{2}k_BT$ ......(ii)

Comparing (i) and(ii),

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$$

$$\therefore \quad \overline{v^2} = \frac{3k_BT}{m}$$

$$\therefore \quad v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_BT}{m}}$$

Solution:

$$V_{rms}\sqrt{\frac{3k_BT}{m}}$$

$$V_{rms} = \sqrt{\frac{3\times1.38\times10^{-23}\times300}{4\times1.66\times10^{-37}}}$$

$$= \sqrt{\frac{12.42\times10^{-23}}{6.64}}$$

$$= 1.368 \times 10^3 = 1368 \text{ m/s}$$

Ans : The rms speed of He^- atom is 1368m / s

Problem for Practic

1. Find the K.E. per kg of nitrogen molecules at 127^0C. (Molecular weight of nitrogen = 28, R = 8320 J/mol K)

Ans : 1.783 × 10^5 J/kg.

2. The kinetic energy of 1kg of oxygen at 300K is 1.169 × 10^5 J. Find the kinetic energy of 4 kg of oxygen at 400 K.

Ans :6.235 × 10^5 J

3. The kinetic energy of 1 kg of oxygen at 300K is 1.356 × 10^6 J. Find the kinetic energy of 4 kg of oxygen at 400 K.

Ans :7.232 × 10^6J

3.8 Law of Equipartition of Energy

Q.17 State and explain law of equipartition of energy. (chapter 3)

Ans: **Statement:** For a gas in thermal equilibirium at a temperature T, the average energy for molecule associated with each quadratic term is $\frac{1}{2}k_BT$.

Explanation:

i. Translational kinetic energy associated with single molecule is given in terms of x,y and z components as, $K.E.=\frac{1}{2}mv_x^2+\frac{1}{2}mv_y^2+\frac{1}{2}mv_z^2$

ii. For a gas at temperature T, the average value of kinetic energy is denoted as $\langle K.E.\rangle$.

$$\therefore \quad \langle K.E.\rangle=\left\langle\frac{1}{2}mv_x^2\right\rangle+\left\langle\frac{1}{2}mv_y^2\right\rangle+\left\langle\frac{1}{2}mv_z^2\right\rangle$$

iii. But, average value of kinetic energy of a gas molecule at temperature T is given as, $\langle K.E.\rangle=\frac{3}{2}kBT$

$$\therefore \quad \left\langle\frac{1}{2}mv_x^2\right\rangle=\left\langle\frac{1}{2}mv_y^2\right\rangle+\left\langle\frac{1}{2}mv_z^2\right\rangle=\frac{3}{2}k_BT$$

iv. Since there is no preffered direction amongst three components,

$$\left\langle\frac{1}{2}mv_x^2\right\rangle=\left\langle\frac{1}{2}mv_y^2\right\rangle+\left\langle\frac{1}{2}mv_z^2\right\rangle=\frac{1}{2}k_BT$$

v. Thus, the mean energy associated with every component of translational kinetic energy in x,y and z directions is $\frac{1}{2}k_BT$

Q.18 Define degree of freedom of a system. Also on what factors do the degree of freedom depends?

Ans: Degrees of freedom of a system are defined as the total number of co-ordinates or independent quantities required to describe the position and configuration of the system completly. Degrees of freedom of any gas depend upon :

i. Atomicity of the gas i.e., whether the gas is monatomic, diatomic or triatomic

ii Geometric structure of the gas molecule i.e., whether the gas is rigid, non-rigid or linear or non-linear

iii. Temperature of gas.

Q.19 For a diatomic atom, describe the energy associated with each degree of freedom.

i. Translational kinetic energy associated with 3 dof is,

$$E\text{ (translational)} = \frac{1}{2}mv_x^2+\frac{1}{2}mv_y^2+\frac{1}{2}mv_z^2$$

ii. A diatomic atom also possesses two aditional rotaional dof. If the molecule is considered along X-axis and I_y and I_z are moments of inertia about Y and Z axes with ω_y and ω_z the respective angular speeds, the rotational kinetic energy will be

$$E\text{ (rotational)} = \frac{1}{2}I_y\omega_y^2+\frac{1}{2}I_z\omega_z^2$$

iii. If the molecule is considered to be rigid, the total energy due to translational and rotational dof is,

$$E=\frac{1}{2}mv_x^2+\frac{1}{2}mv_y^2+\frac{1}{2}mv_z^2+\frac{1}{2}I_y\omega_y^2+\frac{1}{2}I_z\omega_z^2$$

iv. But diatomic molecules which are non-rigid, possess additional vibrational dof. This vibrational motion adds energy associated with the vibrations to the total energy of molecule.

$$\therefore \quad E = E(\text{translational}) + E(\text{rotational}) + E(\text{vibrational})$$

v. The term E(vibrational) consists of the kinetic energy and potential energy term.

$$\therefore \quad E(\text{vibrational}) = \frac{1}{2}mu^2 + \frac{1}{2}kr^2$$

vi. Hence for a non-rigid diatomic gas in thermal equilibrium at a temperature T, the mean kinetic energy associated with the translational motion of molecule along the three directions is $3 \times \frac{1}{2}k_BT$, the mean kinetic energy associated with the rotaional motions about two perpendicular axes is $2 \times \frac{1}{2}k_BT$, and total vibrational energy $2 \times \frac{1}{2}k_BT$, corresponding to kinetic and potential energy terms.

3.9 Specific Heat Capacity

Note :

Q.34 Why do we generally consider two specific heats of a gas?

Ans.

i. In case of gas, slight change in temperature causes considerable change in pressure as well as volume of the gas.

ii. Therefore, two principal specific heats are defined for a gas i.e., specific heat capacity at constant volume (s_V) and specific heat capacity at constant pressure (s_p).

Q.35 Define the principal and molar specific heat capacities of a gas at constant volume and constant pressure.

Ans.

i. Principal specific heat capacity of a gas at constant volume : The principal specific heat capacity of a gas at constant volume (s_v) is defined as the quantity of heat absorbed or released to increase or decrease the temperature of unit mass of the gas through one kelvin (or 1 ^{0}C), when its volume is kept constant.

ii. Principal specific heat capacity of a gas at constant pressure : The principal specific heat capacity of a gas at constant pressure (s_p) is defined as the quantity of heat absorbed or released to increase or decrease the temperature of unit mass of the gas through one kelvin (or 1 ^{0}C), when its pressure is kept constant.

iii. Molar specific heat capacity of a gas at constant volume : The molar specific heat capacity of a gas at constant volume (C_v) is defined as the quantity of heat absorbed or released to increase or decrease the temperature of one mole of the gas through one kelvin (or 1 ^{0}C), when its volume is kept constant.

iv. Molar specific heat capacity of a gas at constant pressure : The molar specific heat capacity of a gas at constant pressure (C_p) is defined as the quantity of heat absorbed or released to increase or decrease the temperature of one mole of the gas through one kelvin (or 1 ^{0}C), when its pressure is kept constant.

Q.36 Explain why specific heat capacity of a gas at constant pressure is greater than that at constant volume. OR Why Cp > Cv.

Ans.

i. When the gas is heated at constant volume, there is no work done against external pressure.

ii. Here entire heat supplied is used to increase temperature of the gas.

iii. When the gas is heated at constant pressure, volume of the gas changes.

iv. So some part of the supplied heat is used by the gas to expand against external pressure. and remaining part of heat supplied is used to raise the temperature.

v. Due to this, for the same rise in temperature, the heat to be supplied at constant pressure is greater than that for heating at constant volume.

∴ $C_p > C_v$

Note : *Relation between molar specific heat and principle specific heat.*

Molar specific heat capcity = Molecular weight × Principle specific heat capacity

$$C_p = \mu \times s_p$$

$$C_v = \mu \times s_v$$

Q.20 Derive the Mayer's relation for molar as well as principal specific heats.

Ans:

i. Consider one mole of an ideal gas that is enclosed in a cylinder by light, frictionless airtight piston.

ii. Let P, V and T be the pressure, volume and temperature respectively of the gas

iii. If the gas is heated so that its temperature rises by dT, but the volume remains constant, then the amount of heat supplied to the gas (dQ_1) is used to increase the internal energy of the gas (dE). Since volume of the gas is constant, no work is done in moving the piston

∴ $dQ_1 = dE = C_v dT$ ...(1)

where C_v is the molar specific heat of the gas at constant volume.

iv. On the other hand, if the gas is heated to the same temperature, at constant pressure, volume of the gas increases by an amount say dV. The amount of heated supplied to the gas is used to increase the internal energy of the gas as well as to move the piston backwards to allow expansion of gas.The work done to move the piston dW = PdV.

∴ $dQ_2 = dE + dW = C_p dT$ ...(2)

Where, C_p is the molar specific heat of the gas at constant pressure.

v. From equations (1) and (2),

∴ $C_p dT = C_v dT + dW$

∴ $(C_p - C_v)\ dT = PdV$ ...(3)

vi. For one mole of gas,

$PV = RT$

∴ $P\,dV = R\,dT$, since pressure is constant.

Substituting in equation (3), we get

$(C_p - C_v) + dT = RdT$

∴ $\boxed{C_p - C_v = R}$ ...(4)

This is known as Mayer's relation between C_P and C_v.

vii. Also $C_p = M_0S_p$ and $C_v = M_0S_v$, where M_0 is the molar mass of the gas and S_p and S_v are respective principal specific heats. Thus, $M_0S_P - M_0S_v = R/J$

where, J is mechanical equivalent of heat.

∴ $$S_p - S_v = \frac{R}{M_0 J}$$

Q.21 Describe specific heat capacities of gases.

a. Monoatomic Gases

b. Diatomic Gases

c. Polyatomic Gases

Ans: a. Monoatomic Gases :

i. The molecule of monoatomic gas has only three translational degrees of freedom.

∴ The average K.E. of a molecule at temperature T is $\frac{3}{2}K_BT$ and the total internal energy per mole of gas is

$$U = \frac{3}{2}K_BT \times N_A = \frac{3}{2}RT$$

ii. If C_V is the molar specific heat at constant volume, then

$$C_V \text{ (monoatomic gas)} = \frac{dU}{dT} = \frac{3}{2}R$$

iii. For an ideal gas, $C_p - C_V = R$

where, C_P is the molar specific heat at constant pressure.

∴ $C_P = C_V + R$

$$= \frac{3}{2}R + R = \frac{5}{2}R$$

and the ratio of specific heats

$$\gamma = \frac{C_p}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

b. Diatomic Gases :

A diatomic molecule is considered as a

rigid rotator, has 5 degrees of freedom like dumbbell.

Using the law of equipartition of energy, the total internal energy of a mole of diatomic gas is,

$$U = \frac{5}{2}K_BT \times N_A = \frac{5}{2}RT$$

For molar specific heats at constant pressure and constant volume for rigid diatomic molecule are given as

$$C_p = \frac{7}{2}R \text{ and } C_v = \frac{5}{2}R$$

$\therefore$ $\gamma(\text{for rigid diatomic}) = \frac{7}{5}$

If the diatomic molecule is **not rigid**, then it has additional vibrational mode.

$$\therefore \quad U = \left(\frac{5}{2}K_BT + K_BT\right)N_A$$

$$= \frac{7}{2}RT$$

$$C_V = \frac{7}{2}R, \quad C_p = \frac{9}{2}R \text{ and } \gamma = \frac{9}{7}$$

c. **Polyatomic Gases :**

A polyatomic molecule has 3 translational, 3 rotational degrees of freedom and a certain number (f) of vibrational modes.

By the law of equipartition of energy, one mole of such a gas has

$$U = \left(\frac{3}{2}K_BT + \frac{3}{2}K_BT + f\,K_BT\right)N_A$$

i.e. $C_V = (3 + f)\,R$ and $C_P = (4 + f)\,R$

$$\therefore \quad \gamma = \frac{4+f}{3+f}$$

The relation $C_P - C_V = R$ is true for any ideal gas, whether mono, di or polyatomic.

★Q. 22 State law of equipartition of energy and hence calculate molar specific heat of mono and di- atomic gases at constant volume and constant pressure.

Ans: Refer to 17 and Q. 21

Type - VII

Numerical based on specific heat capacity

Formula used

1. $C_p - C_v = R$
2. $S_p - S_v = \frac{R}{M_0J}$

1) Given the values of the two principal specific heats, S_P= 3400 cal kg^{-1}K^{-1} and S_V= 2400 cal kg^{1} K^{-1} for the hydrogen gas, find the value of J if the universal gas constant R = 8300 J Kg^{-1}K^{-1}.

Data: S_P= 3400 cal kg^{-1}K^{-1}
S_V= 2400 cal kg^{-1}K^{-1} , M_0= 2,
R = 8300 J Kg^{-1}K^{-1}

To find: J

Formula: $S_P - S_V = \frac{R}{M_0J}$

Solution: $S_P - S_V = \frac{R}{M_0J}$

$$3400 - 2400 = \frac{8300}{2 \times J}$$

$$J = \frac{8300}{2 \times 1000}$$

$$J = \frac{8.3}{2} = 4.15 \text{ J/cal}$$

Ans : The value of J is 4.15 J / cal.

★2) The difference between the two molar specific heats of a gas is 8000 J kg^{-1} K^{-1}. If the ratio of the two specific heats is 1.65, calculate the two molar specific heats.

Data: $\frac{C_P}{C_V} = 1.65$

$\therefore$ $C_P = 1.65C_V$
$C_P - C_V = 8000$

To find: i. C_p ii. C_V

Solution : $C_p - C_V$ = 8000

$\therefore$ $1.65C_V - C_V$ = 8000

$\therefore$ $0.65C_V$ = 8000

$$\therefore \quad C_V = \frac{8000}{0.65}$$

$\therefore$ $C_V = 1.231 \times 10^4$ = 12310 J kg^{-1}K^{-1}

Now, $C_P = 8000 + C_V$ = 8000 + 12310

$= 20310\ \text{J kg}^{-1}\text{K}^{-1}$

Ans : The value C_V is12310 J kg^{-1}K^{-1}and C_P is 20310 J kg^{-1}K^{-1}.

Problem for Practice

1. The difference between two specific heat of gas is 5000J/kgK and ratio of specitic heat is 1.6 Find the two specific heat

Ans : 8333.3 J/kg. K

: 13333.3 J/kg K

2. Calculate the gas constant for 1g of gas it $C_p = 0.245$ Cal/g ºC and $C_v = 0.165$ Cal/g ºC and $J = 4.2 \times 10^7$ erg/cal.

Ans : 3.36 × 10^6 erg/g ºC

3. The specific heat of argon at constant pressure is 0.127 J/gmk and ratio of specifiz heat is 1.667 Calculate the value of J one litre of argon weight is 17.86 g at NTP

Ans : 4.01 J/cal

4. If the difference between the two specitic heats of triatomic gas is 500 J/kg K then find the principal specibic heat of gas at constant pressure $\left(Y = \frac{4}{3}\right)$

Ans : 2000 J/kg K

3.10 Absorption, Reflection and Transmission of Heat Radiation

Q.23 Define coefficient of absorption, reflection and transmission.

Ans: When the radiant energy is incident on the surface of any body, it is used up in three different ways. Part of this energy is absorbed by the body, some part is reflected, and remaining part is transmitted by the body

i. Coefficient of absorption (a) :
It is defined as the ratio of the quantity of radiant heat absorbed by the body to the total quantity of radiant heat incident on it. $a = \frac{Q_a}{Q}$

ii. Coefficient of reflection (r) :
The ratio of the quantity of radiant heat reflected by the body to the total quantity of radiant heat incident on. $r = \frac{Q_r}{Q}$

iii. Coefficient of transmission (t) :
It is defined as the ratio of the quantity of radiant heat transmitted by the body to the total quantity of radiant heat incident on it. $t = \frac{Q_t}{Q}$

Q.24 Show that a + r + t = 1.

Ans:

i. Let, Q be the quantity of radiant heat incident on the surface of the body,

ii. Q_a , Q_r, and Q_t be the quantities of heat absorbed, reflected and transmitted by the body respectively.

Then, $Q_a + Q_r + Q_t = Q$

$\therefore \quad \frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = 1$

$\therefore \quad a + r + t = 1$

⋆ Q.25 Define :
i. Diathermanous substancesand
ii. Athermanous substances.

Ans:

i. Diathermanous substances :
The substances which can transmit the radiant heat incident upon their surfaces are called diathermanous substances [$t \neq 0$].
Examples: glass, quartz, gases etc.

ii. Athermanous substances :
The substances which cannot transmit the radiant heat incident upon their surfaces are called athermanous substances [$t = 0$].
For athermanous body $(a + r) = 1$
Examples : wood, iron, copper.

Note : For total heat incident on an object $a + r + t_r = 1$ If $r = a = 0$, then $t_r = 1$. This implies, all the object i.e. the object is completely transparent to the radiation. Such an object is said to be perfect transmitter.

3.11 Perfect Blackbody

Q.26 What is a perfectly black body? State its

properties.

Ans: **A perfectly black body :**

A body which can absorb all the quantity of radiant heat incident on it is called a perfectly black body. For perfectly black body,

a = 1, or t = 0 and r = 0

Properties of perfectly black body:

i. A perfectly black body absorbs all the radiant heat incident upon it.(i.e., a = 1)

ii. The coefficient of emission of a perfectly black body is 1; it is very good emitter of heat.

iii. A perfectly black body does not reflect or transmit the radiant heat incident upon it. (i.e., t = 0, r = 0)

iv. A perfectly black body does not exist in nature.For practical purposes lamp black is treated as a perfectly black body as its coefficient of absorption is 0.98.

★ Q.27 What is a perfect blackbody? How can it be realized in practice?

Ans :

i. A body, which absorbs the entire radiant energy incident on it, is called an ideal or perfect blackbody.

ii. Perfectly blackbody does not exist in nature. However, for practical purpose, lamp black can be treated as perfectly blackbody.

iii. A blackbody is most nearly realized in practice by the use of a small hole in the wall of a uniform temperature enclosure.

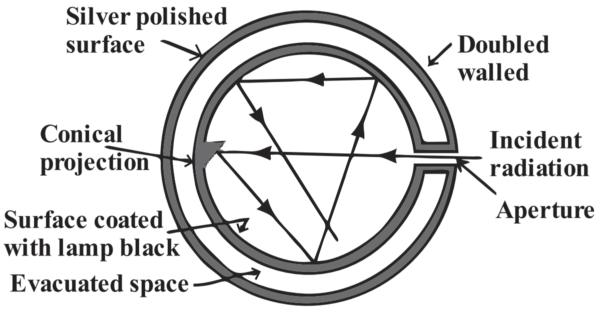
iv. Such an enclosure has its walls maintained at the same temperature. Thus, the radiation coming out of the hole is approximately blackbody radiation.

v. In practice, Ferry's absorber behaves as a perfectly blackbody.

Q.28 With a neat and labelled diagram, explain Ferry's perfectly black body. [Oct. 2014]

Arrangement of perfectly black body :

Ans: The arrangement described below can be considered as perfectly black body.



i. It consists of a double walled hollow sphere having tiny hole or aperture, through which radiant heat can enter.

ii. The space between the walls is evacuated and outer surface of the sphere is silvered.

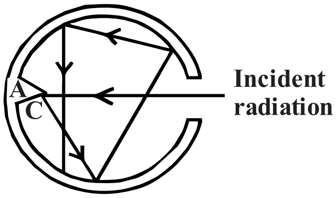
iii. The inner surface of sphere is coated with lamp black.

iv. There is a conical projection on the inner surface of sphere opposite the aperture. The projection ensures that a ray travelling along the axis of the aperture is not incident normally on the surface and is therefore not reflected back along the same path.

v. A heat ray entering the sphere through the aperture suffers multiple reflections and is almost completely absorbed inside. Thus, the aperture behaves like a perfect blackbody.

Q.29 State the use of conical projection in artificial perfectly black body.

Ans: If there has been no conical projection ‘C’ in the black body, a part of the radiations which are incidenting normally on the area ‘A’ would have been reflected back. Due to the presence of conical projection almost all the radiations will get reflected in different directions as shown in the fig.



Q.30 Write a short note on cavity radiator.

Ans :

i. Cavity radiator consists of a block of material with internal cavity.

ii. The inner and outer surfaces are connected by a small hole.

iii. The radiation falling on the block that enters through the hole, cannot escaps back form it. Hence, the cavity acts as a blackbody.

iv. When the block is heated to high temperature, thermal radiation is emitted. This is called cavity radiation and it resembles the radiation emitted by a blackbody.

v. Its nature depends only on the temperature of the cavity walls and not on the shaps and size of the cavity or the material of the cavity walls.

3.12 Emission of Heat Radiation

Q.31 State and explain prevost' theoery of echange of heat

Ans: All bodies at all temperatures above 0 K (absolute zero temperature) radiate thermal energy and at the same time, they absorb radiation received from the surroundings.

i. The average translational kinetic energy determines the temperature of the body.

ii. According to theory of heat ecvhange, all the bodies above 0 K absorb radiations from the surroundings.

iii. For a body, the absorbed raduation (being energy (increases the kinetic energy of the constituent atoms oscillating about their mean positions.

iv. The absorbed radiation therefore causes a rise in the temoerature of the body.

v. The body inself also radiates, therefore its energy decreases causing lowering of temperature.

vi. If a body radiates more than it absorbs, its temperature decreases and if the body absorbs more than it radiates, than the temperature of the body increases.

vii. When the rate of absorption of radiatin is same as the rate of emission of radiation, the temperature of the body remains constant and the body is said to be in thermal equilibrium with its surroundings. this means, all bodies radiate as well as absorb radiation also at room tempeature, but their rate of emission and rate of absorption are same, hence their temperature remains constant.

It can be inferred form this, that hot bodies would radiate more than cooler bodies.

Q.32 Explain the term :

i. Emissive power

ii. Emissivity

iii. Absorptive power

Ans: i. Emissive power (E) :

The quantity of radiant energy emitted per unit time per unit surface of a body at a given temperature is called its emissive power at that temperature. Every body radiates energy at all temperature except at the absolute zero temperature. If

Q = amount of radiantenergy emitted,

A = surface area of body,

t = time for which body radiates energy.

Formula : $R = \frac{Q}{At}$

SI unit : J/m^{2}s or W/m^2

Dimensions : $[M^1L^0T^{-3}]$

ii. Coefficient of emission (emissivity) (e):

It is defined as the ratio of emissive power of the body at a given temperature to the emissive power of a perfectly black body at the same temperature.

Coefficient of emission is also called as emissivity.

Formula : $e = \frac{R}{R_b}$

where,

R = Emissive power of ordinary body at given temperature.

R_b= Emissive power of perfectly black body at same given temperature.

For perfectly black body, e = 1

For perfectly reflector, e = 0

For ordinary bodies, e < 1

Good absorber is good emitter of heat.

iii. Absorptive power :

It is defined as the amount of radiant energy absorbed per unit area per unit time by a surface at that temperature. A body which absorbs all radiation of all wavelengths would be called ‘Perfectly black body’.

Q.33 State the factors on which emissive power depends.

Ans: Emissive power of a body depends on :

i. temperature of the body
ii. nature of the body
iii. nature of the surroundings

Note: Emissive power of a perfectly black body is always greater than any other body at same temperature.The emissive power of some surfaces are, Lampblack-98%. Aluminium paint 33%, platinum-11%, copper-5%, silver-3%.

Type - VIII
Numerical based on Emissive power

Formula used

1. $a + r + t = 1$

2. $R = \dfrac{Q}{At}$

3. $e = \dfrac{R}{R_B}$

4. $a = \dfrac{Q_a}{Q}$

5. $r = \dfrac{Q_r}{Q}$

6. $t = \dfrac{Q_t}{Q}$

1) Radiant energy is incident on a body at the rate of 100 J/s. If the absorption coefficient of the body is 0.8 and the reflection coefficient is 0.15, find the radiant energy absorbed, reflected and transmitted by the body in 10 seconds.

Data: $Q = 100$ J/s
$a = 0.8$
$r = 0.15$

To Find : Q_a, Q_r, Q_t in 10 seconds

Formula : **i.** $a = \dfrac{Qa}{Q}$ ii. $r = \dfrac{Qr}{Q}$

iii. $t = \dfrac{Qt}{Q}$ iv. $a + r + t = 1$

Solution :

i. Radiant energy incident in 10 s.

$Q = 100 \text{ J/s} \times 10 \text{ s} = 1000 \text{ J}$

ii. We have, $a + r + t = 1$

$\therefore$ $0.8 + 0.15 + t = 1$

$\therefore$ $0.95 + t = 1$

$\therefore$ $t = 0.05$

iii. $Q_a = Q \times a = 1000 \times 0.8 = 800$ J

iv. $Q_r = Q \times r = 1000 \times 0.15 = 150$ J

v. $Q_t = Q \times t = 1000 \times 0.05 = 50$ J

Ans : 800J of energy is absorbed, 150J of energy is reflected and 50J of energy is transmitted in 10 seconds.

★2) The emissive power of a sphere of area 0.02 m² is 0.5 kcal s⁻¹ m⁻². What is the amount of heat radiated by the spherical surface in 20 second?

Data: $R = 0.5 \text{kcal} s^{-1} m^{-2}$, $t = 20$sec,
$A = 0.02 \text{ m}^2$

To find: Q

Formula: $R = \dfrac{Q}{At}$

Solution : $Q = RAt$
$Q = 0.5 \times 0.02 \times 20 = 0.2$ kcal

Ans : The amount of head radiated by the spherical surface is 0.2kcal.

3) A metal cube of each side 2 cm long emits 0.16 kcal of heat in 100 second. Calculate emissive power in mechanical unit at that temperature.[J= 4200/kcal]

Data: $L = 2 \text{ cm} = 2 \times 10^{-2}$m,
$t = 100$ s, $J = 4200$ J/kcal,
$Q = 0.16$ kcal
$A = 6L^2 = 6 \times (2\times10^{-2})^2 = 24 \times 10^{-4} \text{ m}^2$

To Find: R

Formula: Emissive power,

$R = \dfrac{Q}{At} \times J$ ...(in mechanical unit)

Solution: $R = \dfrac{Q}{At} \times J$

$\therefore$ $R = \dfrac{0.16 \times 4200}{24 \times 10^{-4} \times 100} = \dfrac{0.16 \times 42}{24} \times 10^4$

$\therefore$ $R = \dfrac{16 \times 42 \times 100}{24} = 2800 \text{ J/m}^2\text{s}$

Ans : Emissive power of body is 2800 J/m^{2}s

Problem for Practive

1. Coefficient of absorption of a body is 0.54 and coefficient of reflection is 0.16. The amount of heat incident on a body is 4000 J. Calculate amount of heat transmitted through a body.

Ans : 1200 J

2. The coefficient of absorption and coefficient of reflection of a thin uniform plate are 0.75 and 0.20 respectively. If 200 kcal of heat is incident on the surface of the plate, find the quantity of heat transmitted.

Ans:10 kcal

3. Calculate the heat radiated in one minute by a metal sphere of diameter 5 cm coated with lamp black when it is maintained at 227^0 C. Express your answer in kcal.

(Given : $\sigma = 5.7 \times 10^{-8}$ W / m^{2}K^4)

Ans: 0.3999 kcal

4. The emissive power of a sphere of area 0.02 m^2 is 0.5 k cal/s-m^2. What is the amount of heat radiated by the spherical surface is 20 seconds?

Ans : 0.2 k cal.

5. The energy of 6000 J is radiated in 5 minutes by a body of surface area 100 cm^2. Find the emissive power of the body.

Ans :2000 J/m^2-s

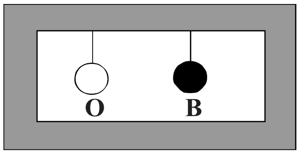
3.13 Kirchhoff's Law of Heat Radiation and its Theoretical Proof

Q.34 State Kirchhoff's law. Give its theoretical proof :

Ans: Kirchhoff's law of radiation :

It states that the coefficient of absorption of a body at a given wavelength is equal to its coefficient of emission at the same wavelength at any given temperature. i.e., a = e.

Theoretical proof :



i. Consider two bodies, ordinary body (O) and black body (B) suspended in uniform temperature enclosure.
Let, A - surface area of each body.

ii. After a sufficient interval of time, O and B shall attain the same temperature as that of the temperature of enclosure. Even then, every body will continue the exchange of heat with its surroundings.

iii. Let, R - emissive power of ordinary body.
R_b - emissive power of black body.
Q - amount of radiant energy incident per unit time per unit surface area of each body.
a - coefficient of absorption
e - coefficient of emission

iv. For ordinary body :
Quantity of radiant energy absorbed per unit time = aAQ
Quantity of radiant energy emits per unit time = RA

v. However, its temperature remains constant,

$\therefore$ $\begin{pmatrix}\text{Energy absorbed}\\ \text{per unit time}\end{pmatrix} = \begin{pmatrix}\text{Energy emitted}\\ \text{per unit time}\end{pmatrix}$

$\therefore$ aQA = RA

$\therefore$ aQ = R ...(1)

vi. For black body :
Quantity of radiant energy absorbed per unit time = AQ
Quantity of radiant energy emits per unit time = R_bA

$\therefore$ $\begin{pmatrix}\text{Energy absorbed}\\ \text{per unit time}\end{pmatrix} = \begin{pmatrix}\text{Energy emitted}\\ \text{per unit time}\end{pmatrix}$

$\therefore$ AQ = R_bA

$\therefore$ Q = R_b ... (2)

vii. From equations (1) and (2), we get,

$$\frac{aQ}{Q} = \frac{R}{R_b} \qquad \left(\text{but, } \frac{R}{R_b} = e\right)$$

$\therefore$ a = e

$\therefore$ Coefficient of absorption = Coefficient of emission Thus, Kirchhoff's law is proved.

3.14 Spectral Distribution of Blackbody

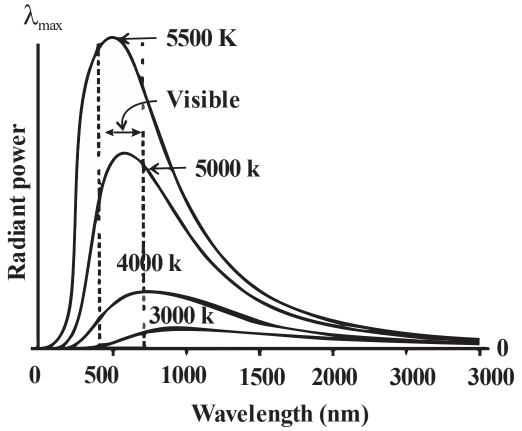
Radiation

Q.35 Explain spectral distribution of blackbody radiation.

Ans :

i. The rate of emission per unit area or power per unit area of a surface is defined as a function of the wavelength λ of the emitted radiation.

ii. Scientists studied the energy distribution of blackbody radiation as a function of wavelength.



iii. By Keeping the source of radiation at different temperatures they measured the radiant power corresponding to different wavelengths.The measurements were represented graphically in the form of curves showing variation of radiant power per unit area as a function of wavelength λ at different constant temperatures as shown in Figure.

Q.36 State Wien's displacement law.

\Ans :

i. Statement : The wavelength for which emissive power of a blackbody is maximum is inversely proportional to the absolute temperature of the blackbody.

ii. This us Wien's displacement law, i.e.,

$$\lambda_{max} \propto \frac{1}{T} \quad \text{or} \quad \lambda_{max} \propto \frac{b}{T}$$

$$\lambda_{max} T = b$$

Where b is called the Wien's constant and its value is 2.897×10^{-3} m K.

Type - IX
Numerical based on Wien's displacement law

Formula used

$$\lambda_{max} = \frac{b}{T}$$

★1) Calculate the value of λ_{max} for solar radiation assuming that surface temperature of Sun is 5800K. ($b = 2.897 \times 10^{-3}$m K) In which part of the electromagnetic spectrum, does this value lie?

Data: T = 5800K, $b = 2.897 \times 10^{-3}$mK

To find: i. (λ_{max})

ii. The region in electomagnetic spectrum to which the radiation belongs.

Formula: $\lambda_{max} = \frac{b}{T}$

Solution: $\lambda_{max} = \frac{b}{T}$

$$\lambda_{max} = \frac{2.897 \times 10^{-3}}{5800}$$

$$= 4.996 \times 10^{-1} \times 10^{-6} = 4.996 \times 10^{-7}\text{m}$$

$$= 4996 \text{ Å}$$

Ans : The value of λ_{max} is 4996 Å. This value lies in the visible rigion of the electrom- agnetic spectrum.

★2) Find the temperature of a blackbody if its spectrum has a peak at

i. λ_{max} = 700 nm (visible),

ii. λ_{max} = 3 cm (microwave region) and

iii. λ_{max} = 3m (FM radio waves) (Take Wien's constant $b = 2.897 \times 10^{-3}$ mK).

Data: $(\lambda_{max})_1 = 700\text{nm} = 700 \times 10^{-9}\text{m} = 7 \times 10^{-7}\text{m}$

$(\lambda_{max})_2 = 3 \text{ cm} = 3 \times 10^{-2}\text{m}$,

$(\lambda_{max})_3 = 3\text{m}$

$b = 2.897 \times 10^{-3}$ mK

To find: Temprature of blackbody for peaks

Formula: $\lambda_{max} = \frac{b}{T}$

Solution: $\lambda_{max} = \frac{b}{T}$, $T = \frac{b}{\lambda_{max}}$

For visible region

$$T_1 = \frac{b}{(\lambda_{max})} \quad ......(i)$$

i. $T_1 = \frac{2.897\times10^{-3}}{7\times10^{-7}} = 0.4138\times10^{4} = 4138K$

For microwave region

$$T_2 = \frac{b}{(\lambda_{max})} \quad ......(ii)$$

ii. $T_2 = \frac{2.897\times10^{-3}}{3\times10^{-2}} = 0.966\times10^{-1}K = 0.966K$

For FM radio waves

$$T_3 = \frac{b}{(\lambda_{max})} \quad ......(iii)$$

iii. $T_3 = \frac{2.897\times10^{-3}}{3} = 0.966\times10^{-3}K$

Ans : The respective temperatures of blackbody are $4138K, 0.0966K$ and $0.966\times10^{-3}K$

3) Earth's mean temprature can be assumed to be 280 K. How will the curve of blackbody radiation look like for this temprature? Find out λ_{max}. In which part of the electromagnetic spectrum, does this value lie?

Data: T = 280K

To find: i. (λ_{max})

ii. Curve of black body radiation

Formula: i. $\lambda_{max} = \frac{b}{T}$

Solution:

i. $\lambda_{max} = \frac{b}{T}$

$$\lambda_{max} = \frac{2.897\times10^{-3}}{280}$$

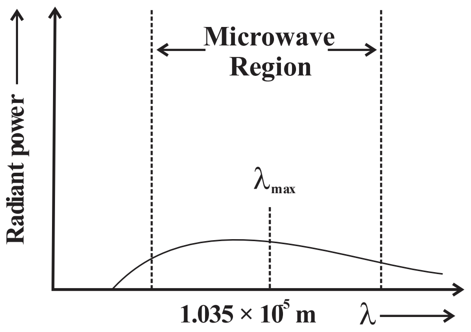
$(\because b=2.897\times10^{-3}$ mK)

$= 0.01035\times10^{-3}$

$= 1.035 \times 10^{-5}$m

ii. This radiation belongs to microwave region or electromagnetic spectrum.

The curve of blackbody radiation for the earth.



Ans : The value of λ_{max} is 1.035×10^{-5} m.This radiation belongs to microwave region of electromagnetic spectrum.

Problem for Practice

1. Find wavelength at which body radiates maximum energy if it's temperature is 227^0C (wien's constant b = 2.898×10^{-3} mK)

Ans: 5.796×10^{-6}m

3.15 Stefan-Boltzmann Law of Radiation

Q.37 State and explain Stefan's law of radiation.

Ans: Stefan's law of radiation :

The radiant energy emitted per unit time per unit area by a perfectly black body is directly proportional to the fourth power of its absolute temperature.

Let, E_b – quantity of radiant energy emitted per unit area by the perfectly black body,

T – absolute temperature,

According to Stefan's law.

$E_b \propto T^4$

∴ $E_b = \sigma T^4$,

where, σ – Stefan's constant

$\sigma = 5.67 \times 10^{-8}$ J / m² sK⁴.

OR $\sigma = 5.67 \times 10^{-5}$ erg / cm²sK⁴

Q.38 Give Expression for loss of heat per unit time by an ordinary body

Ans:

i. Consider a perfectly black body.
T - absolute temperature of body
T_0 - absolute temperature of surrounding.

ii. Energy radiated per unit time per unit surface area by the perfectly black body = σT^4.

iii. Energy absorbed per unit time per unit surface area by the perfectly black body from the surroundings = σT_0^4

iv. The net loss of heat by the perfectly black body per unit time per unit surface area = $\sigma(T^4 - T_0^4)$.

v. Let, A - surface area of the perfectly black body.

∴ The loss of heat of the perfectly black body per unit time = $\sigma A(T^4 - T_0^4)$.

vi. Let, e - emissivity of an ordinary body.

∴ The loss of heat of the ordinary body per unit time = $\sigma Ae(T^4 - T_0^4)$.

Type - X

Numerical based on Stefan Boltzmann Law

Formula Used

For Back body

1. $Q = \sigma AtT^4$

2. $P = \frac{Q}{At} = \sigma T^4$

3. Rate of loss of energy per unit time

$$\frac{dQ}{dt} = \sigma A(T^4 - T_0^4)$$

For ordinary body

1. $Q = \sigma eAtT^4$

2. $P = \frac{Q}{At} = \sigma eT^4$

3. Rate of loss of heat of ordinary body per unit time

$$\frac{dQ}{dt} = \sigma Ae(T^4 - T_0^4)$$

1) Calculate the energy radiated in one minute by a blackbody of surface area 200 cm^2 at 127°C(σ=5.7×10^{-8}Jm^{-2}s^{-1}K^{-4})

Data: $A = 200cm^2 = 200\times10^{-4}m^2$,
$T = 127°C = (127+273)K = 400K$,
$t = 1$ min $= 60$ s;
$s = 5.7\times10^{-8}$ J m^{-2}s^{-1}K^{-4}

To find: Q

Formula: $Q = \sigma AtT^4$

Solution: $Q = \sigma AtT^4$

$Q = \sigma AtT^4$
$= 5.7 \times 10^{-8} \times 200 \times 10^{-4} \times 60 \times (400)^4$
$= 5.7 \times 10^{-8} \times 1.2 \times 10^4 \times 10^{-4} \times 4^4 \times 10^8$
$= 5.7 \times 1.2 \times 256 = 1751.04$ J

Ans : Energy radiated in one minute is1751.04 J.

2) Calculate the energy radiated in one minute by a blackbody of surface area 100 cm^2 when it is maintained at 227°C.

Data: $A = 100cm^3 = 100 \times 10^{-4}m^2 = 10^{-2}m^2$;
$T = 227°C = 227 + 273 = 500K$
$\sigma = 5.67 \times 10^{-8}$J m^{-2}s^{-1}K^{-4}
$t = 1min = 60s$

To find: Energy radiated(Q)

Formula: $Q = \sigma AtT^4$

Solution: $Q = \sigma AtT^4$

$Q = 5.67 \times 10^{-8} \times 10^{-2} \times 60 \times (500)^4$
$= 5.67 \times 10^{-8} \times 10^{-2} \times 60 \times 5^4 \times 10^8$
$= 5.67 \times 0.6 \times 625 = 2126.25$J.

Ans : Energy radiated in one minute is 2126.25 J.

3) A small-blackened solid copper sphere of radius 2.5 cm is placed in an evacuated chamber.The temperature of the chamber is maintained at 100°C. At what rate energy must be supplied to the copper sphere to maintain its temperature at 110°C?(Take Stefan's constant σ to be 5.67×10^{-8}Js^{-1}m^{-2}K^{-4} and treat the sphere as blakbody.)

Data: $r = 2.5cm$,
$T_0 = 100°C = 100+273 = 373K$
$T = 110°C = 110+273 = 383K$
$\sigma = 5.67\times10^{-8}Js^{-1}m^{-2}K^{-4}$

To find: Rate of energy supplied $\left(\frac{dQ}{dt}\right)$

Formulae: i. $A = 4\pi r^2$ ii. $\frac{dQ}{dt} = \sigma A\left(T^4 - T_0^4\right)$

Solution:

i. $A = 4\pi r^2$
$A = 4\times3.142\times(2.5)^2\times10^{-4}$

$= 4\times3.142\times6.25\times10^{-4}$
$= 3.142\times25\times10^{-4}$
$= 7.855 \times 10^{-3}m^2$

ii. $$\frac{dQ}{dt} = \sigma A\left(T^4 - T_0^4\right)$$

$$\frac{dQ}{dt} = 5.67\times10^{-8}\times7.855\times10^{-3}\times(383^4 - 373^4)$$

$= 44.54\times(2.152\times10^{10}-1.936\times10^{10})\times10^{-11}$
$= 44.54 \times 0.216\times10^{-1} = 0.962W$

Ans : Rate of energy supplied is 0.962W

4) Energy is emitted from a hole in an electric furnace at the rate of 20W, when the temprature of the furnace is 727°C. What is the area of the hole? (Take Stefan's constant σ to be $5.7\times10^{-8}Js^{-1}m^{-2}K^{-4}$)

Data: $\frac{dQ}{dt} = 20W$,
$T = 727°C = 727 + 273 = 1000K$
$\sigma = 5.67 \times 10^{-8}Jm^{-2}s^{-1}K^{-4}$

To find: Area of hole (A)

Formula: $\frac{dQ}{dt} = e\sigma AT^4$

Solution: Consider given hole to be perfect blackbody

∴ e = 1

From formula,

$20 = 5.7\times10^{-8}\times A\times(1000)^4$

$$A = \frac{20}{5.7\times10^{-8}\times10^{12}} = \frac{2}{5.7}\times10^{-3}$$

$$= 0.35\times10^{-3}m^3$$

Ans : The area of the hole is $3.5\times10^{-4}m^2$

5) Compare the rate of loss of heat from a metal sphere at 827°C with the rate of loss of heat from the same sphere at 427°C, if the temprature of the surrounding is 27°C.

Data: $T_1 = 827°C = 827 + 273 = 1100K$,
$T_2 = 427°C = 427+273 = 700K$,
$T_0 = 27°C = 27 + 273 = 300K$

To find: The ratio of rate of loss of heat (R_1: R_2)

Formula: $R = \frac{dQ}{dt} = e\sigma A(T^4 - T_0^4)$

Solution: $R_1 = \left(\frac{dQ}{dt}\right)_1 = e\sigma A(T_1^4 - T_0^4)$

$$R_2 = \left(\frac{dQ}{dt}\right)_2 = e\sigma A(T_2^4 - T_0^4)$$

$$\frac{R_1}{R_2} = \frac{(T_1^4 - T_0^4)}{(T_2^4 - T_0^4)} = \frac{(11)^4 - (3)^4}{(7)^4 - (3)^4}$$

$$= \frac{14641 - 81}{2401 - 81} = \frac{182}{29}$$

Ans : The ratio of rate of loss of heat R1 : R2 is 182 : 29

6) Compare the rates of emission of heat by a blackbody maintained at 727°C and at 227°C, if the blakbodies are surrounded by an enclosure (black) at 27°C. What would be the ratio of their rates of loss of heat?

Data: $T_0 = 27°C=27+273=300K$
$T_1 = 727°C=727+273=1000K$
$T_2 = 227°C=227+273=500K$

To find: Ratio of rate of loss of heat(R_1 : R_2)

Forrmula: $R = \frac{dQ}{dt} = e\sigma A(T^4 - T_0^4)$

Solution:

From formula,

$$R_1 = \left(\frac{dQ}{dt}\right)_1 = e\sigma A(T_1^4 - T_0^4) \quad ...00.(1)$$

$$R_2 = \left(\frac{dQ}{dt}\right)_2 = e\sigma A(T_2^4 - T_0^4) \quad ...(2)$$

Dividing equation (1) by equation (2),

$$\frac{R_1}{R_2} = \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4} = \frac{1000^4 - 300^4}{500^4 - 300^4}$$

$$= \frac{10^4 - 3^4}{5^4 - 3^4} = \frac{9919}{544} = 18.23$$

Ans : The ratio of the rate of energy radiated is 18.23 : 1.

7) A 60 watt filament lamp loses all its energy by radiation from its surface. The emissivity of the surface is 0.5. The area of the surface is 5×10^{-5} m². Find the temperature of the filament($\sigma = 5.67\times10^{-8}$ $Jm^{-2}s^{-1}K^{-4}$)

Data: $\frac{dQ}{dt} = 60\ W = 60\ J\ s^{-1}$, $e = 0.5$,

$A = 5\times10^{-5}\ m^2$, $\sigma = 5.67\times10^{-8}\ J/m^2sK^4$

To find: Temperature of the filament (T)

Formula: $\frac{dQ}{dt} = e\sigma AT^4$

Solution: $\frac{dQ}{dt} = e\sigma AT^4$

$$60 = 0.5\times5.67\times10^{-8}\times5\times10^{-5}\times T^4$$

$$\therefore \quad T^4 = \frac{60\times10^{13}}{5.67\times2.5}$$

$$\therefore \quad T^4 = 4.23\times10^{13}$$

$$\therefore \quad T = (42.3\times10^{12})^{1/4}$$

$$= 2.55\times10^3 = 2550K$$

Ans : The temprature of filament is 2550K.

8) Assuming that the temperature at the surface of the Sun is 6000 K, find out the size of a virtual star (in terms of the size of Sun) whose surface temperature is 3000 K and the power radiated by the Sun. Treat both, the Sun and virtual star as a blackbody.

Data: $T_{sun} = 6000K$, $T_{star} = 3000K$,

$P_{star} = 25\times P_{sun}$

To find: Radius of virtual star (r_{star})

Formulae: i. $A = 4\pi r_{sun}^2$ ii. $\left(\frac{dQ}{dt}\right) = \sigma AT^4$

Solution:

i. $A_{sun} = 4\pi r_{sun}^2$ and $A_{star} = 4\pi r_{star}^2$

ii. $$P_{sun} = \left(\frac{dQ}{dt}\right)_{sun} = \sigma A_{sun} T_{sun}^4 = \sigma 4\pi r_{sun}^2 T_{sun}^4$$

$$P_{star} = \left(\frac{dQ}{dt}\right)_{star} = \sigma A_{star} T_{star}^4 = \sigma 4\pi r_{star}^2 T_{star}^4$$

$$\therefore \quad \frac{P_{star}}{P_{sun}} = \frac{\sigma 4\pi r_{star}^2 T_{star}^4}{\sigma 4\pi r_{sun}^2 T_{sun}^4}$$

$$\therefore \quad 25 = \frac{(3000)^4\times r_{star}^2}{(6000)^4\times r_{sun}^2}$$

$$\therefore \quad \frac{r_{star}^2}{r_{sun}^2} = 25\times\frac{6000^4}{3000^4} = 25\times\left(\frac{6000}{3000}\right)^4$$

$$= 25\times(2)^4 = 25\times16$$

$$\therefore \quad r_{star} = \sqrt{25\times16}\times r_{sun} = 5\times4\times r_{sun}$$

$$\therefore \quad r_{star} = 20\times r_{sun}$$

Ans : The radius of the virtual star is $20\times r_{sun}$

Problem for Practice

1. Calculate the energy radiated in one minute by a perfectly black sphere of radius 5 cm when it is maintained at 127 °C.

   Ans : 2749 J

2. Compare the rates of emission of heat by black body at 727°C.

   Ans : 16 : 1

3. Lamp of 100 watt loses radiant energy from the surface area 2 cm² and coefficient of emission 0.5. What will be temperature of the surface? [$\sigma = 5.67\times10^{-8}$ watt /m²K⁴]

   Ans : 2049 K

4. A body of surface area 10 cm² and temperature 727°C emits 300 J of energy per minute. Find its emissivity.

   Ans : 0.08818

5. Compare the rates of loss of heat by the body at temperatures 527°C and 127°C. The temperature of the surrounding is 27°C.

   Ans : 22.94

6. Two spheres of same material having diameters in the ratio 2 : 1 are maintained at same temperature and kept in same enclosure. Compare their rates of emission.

   Ans : 4 : 1

7. Compare the rates of loss of heat by black body at 827^0C and at 427^0C.

Ans : 6.275 : 1

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