06

Superposition of waves

Syllabus

6.1 Introduction

6.2 Progressive wave

Q.1 Define progressive wave

Ans : A wave in which the disturbance produced in the medium travels in a given direction continuously, without any damping and obstruction, from one particle to another, is a progressive wave or a travelling wave.

Example : The sound wave which is a pressure wave consisting of compressions and rarefactions travelling along the directions of propagation of the wave.

★ Q.2 State the characteristics of progressive waves.

Ans : Characteristics of progressive waves:

i. Each particle in a medium executes the same type of vibration. Particles vibrate about their mean position performing simple harmonic motion.

ii. All vibrating particles of the medium have the same amplitude, period and frequency.

iii. The phase (state of vibration of a particle), changes from one particle another.

iv. No particle remains permanently at rest. Each particle comes to rest momentarily while at the extreme positions of vibration.

v. The particles attain maximam velocity when they pass through their mean positions.

vi. During the propagation of wave, energy is transferred along the wave. There is no transfer of matter.

vii. The wave propagates through the medium with a certain velocity. This velocity depends upon properties of the medium.

viii. Progressive waves are of two types - transverse waves and longitudinal waves.

ix. In a transverse wave, vibrations of particles are perpendicular to the direction of propagation of wave and produces crests and troughs in their medium of travel. In longitudinal wave, vibrations of particles produce compressions and rarefactions along the direction of propagation of the wave.

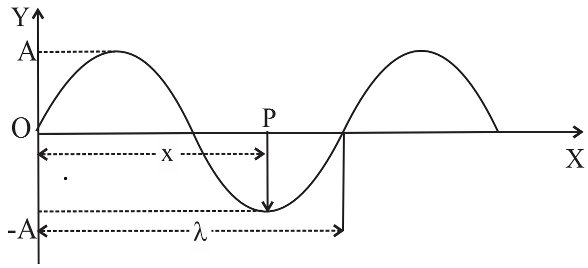
x. Both the transverse as well as the longitudinal mechanical waves can propagate through solids but only longitudinal waves can propagate through fluids.

Q.3 Obtain an equation of a SHP wave travelling in positive X-axis and express it in different forms. Also, write down the equation of a progressive wave travelling along the negative direction of X-axis.

Ans: Equation of a simple harmonic progressive wave:

i. Consider a simple harmonic progressive wave travelling in the positive direction of the X-axis.

ii. Suppose y represents the displacement of the particle and x represents its distance from the origin



iv. At any instant t, the displacement of the particle at the origin ($x = 0$) is given by,
$y = A \sin\omega t$, ...(1)
where, A is the amplitude, and ω is the angular velocity of SHM.

v. Consider a particle at point P situated at distance x from the origin O. The particle at point A lags in phase behind the particle at origin O. Let, this phase lagging be δ.

vi. The displacement of the particle at point A at time t is given by,
$y = A \sin(\omega t - \delta)$ ...(2)

vii. For distance λ corresponding phase lag is 2π. Hence, for distance x the phase lag is

$$\delta = \frac{2\pi x}{\lambda}$$

∴ Equation (ii) becomes,

$$y = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right) \quad ...(3)$$

Now, $\omega = \frac{2\pi}{T}$.

∴ $$y = A\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

∴ $$\boxed{y = A\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]} \quad ...(4)$$

Now $\frac{1}{T} = n$.

∴ $$\boxed{y = A\sin\left[2\pi\left(nt - \frac{x}{\lambda}\right)\right]} \quad ...(5)$$

∴ $$y = A\sin\left[2\pi n\left(t - \frac{x}{n\lambda}\right)\right]$$

But, $n\lambda = V$

∴ $$\boxed{y = A\sin\left[2\pi n\left(t - \frac{x}{v}\right)\right]} \quad ...(6)$$

and $$\boxed{y = A\sin\left[\frac{2\pi}{\lambda}(Vt - x)\right]} \quad ...(7)$$

These are equations of simple harmonic progressive waves in different forms.

viii Equation of simple harmonic progressive wave travelling in the negative direction of X-axis is given by writing (–x) for x. Hence, the equation is,

$$y = A\sin\left[2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

★ Q.4 A wave is represented by an equation y = A sin (Bx + Ct). Given that the constants A,B and C are positive, can you tell in which direction the wave is moving?

Ans: The wave is moving in direction of negative X-axis.

Key Points

i. Equation of simple harmonic progessive wave travelling in positive direction of X-axis
a. $y = \sin(\omega t - kx)$
b. $y = A\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$
c. $y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

ii. Equation of simple harmonic progressive wave travelling in negative direction of X-axis.
$y = A\sin(\omega t + kx)$

iii. Angular wave number or propogation constant
$$k = \frac{2\pi}{\lambda}$$
SI unit : rad/m

iv. Wave velocity $v = n\lambda$

v. Particle velocity $v_P = \omega\sqrt{A^2 - y^2}$ OR
$$v_P = \frac{dy}{dt}$$

vi. Intensity of wave
The wave intensity is defined as the average amount of flow in medium per unit time and

per unit its cross section area

$$I = \frac{\text{Energy}}{\text{Area} \times \text{time}} = \frac{\text{Power}}{\text{Area}}$$

$$\boxed{I = 2\pi nA\rho v}$$

Where n is frequency of wave
A is amplitude
ρ is density of medium
v is wave velocity

vii. At a distance r form a point source of power P the intensity is given by

$$I = \frac{P}{4\pi r^2}$$

$$\therefore \quad I \alpha \frac{1}{r^2}$$

viii. The human ear can hear sound of intensity upto 10^{-12} W/m which is threshold of intensity The maximum intensity of sound which can be tolerated by human ear is 1 W/m which is thereshold of pain

ix. Phase difference $= \frac{2\pi}{\lambda} \times$ Path diff

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$$

Phase difference $= \frac{2\pi}{T} \times$ time difference

$$\Delta\phi = \frac{2\pi}{T} \times \Delta t$$

Type – I
Numerical based on equation of SHP wave

Formulae used

1. Wave travelling in positive X-direction

$$y = A \sin 2\pi\left(nt - \frac{x}{\lambda}\right)$$

2. Propogation constant

$$k = \frac{2\pi}{\lambda}$$

3. Wave velocity $v = n\lambda$

4. Particle velocity

$$v_P = \frac{dy}{dt}$$

$$v_P = \omega\sqrt{A^2 - y^2}$$

★ 1) The amplitude of a wave is represented by $y = 0.2\sin 4\pi\left(\frac{t}{0.08} - \frac{x}{0.8}\right)$ SI units

Find

i. Wavelength ii. frequency and
iii. amplitude of the wave.

Data : $y = 0.2\sin 4\pi\left(\frac{t}{0.08} - \frac{x}{0.8}\right)$

To Find : i. λ ii. n iii. A

Formula : $y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

Solution:

The given equation can be written as,

$$y = 0.2 \sin 2\pi\left(\frac{t}{0.04} - \frac{x}{0.4}\right) \text{ SI units}$$

Comparing with $y = A\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$

We get,

$$\lambda = 0.4\text{m},\ n = \frac{1}{0.04} = 25\text{Hz},\ A = 0.2\text{ m}$$

Ans : Amplitude is 0.2 m, Waveelength is 0.4m and frequency is 25Hz

2) The equation of SHP wave is $y = 0.2\sin\frac{\pi}{8}[64t - x]$ in SI unit. Find wavelength frequency, amplitude and velocity of wave.

Data: $y = 0.2 \sin\frac{\pi}{8}[64t - x]$

To Find: i. λ ii. n iii. A iv. V

Formula: $y = A \sin 2\pi\left[nt - \frac{x}{\lambda}\right]$

Solution :

i. $$y = 0.2 \sin\frac{\pi}{8}[64t - x]$$

$$y = 0.2 \sin 2\pi\left[4t - \frac{x}{16}\right]$$

Comparing with

$$y = A \sin 2\pi\left[nt - \frac{x}{\lambda}\right]$$

We get,

$A = 0.2m$; $n = 4Hz$; $\lambda = 16m$

ii. $v = n\lambda$

$v = 4\times16 = 64$ m/s

Ans :	Amplitude = 0.2m, Frequency = 4Hz Wavelength = 16m, Wavevelocity = 64m / s

3) Write the equation of progressive wave with amplitude 0.02m, period 0.05 sec and travelling with velocity 20m/s along positive x-axis

Data : $A = 0.02m$, $T = 0.05$ sec, $v = 20$ m/s

To Find : Equation of wave

Formula : i. $v = n\lambda$ ii. $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$

Solution :

i. $v = n\lambda = \frac{1}{T}\times\lambda$ ...$\left(\because n = \frac{1}{T}\right)$

$\therefore$ $\lambda = V\times T = 20\times 0.05 = 1$ m

ii. $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$

$\therefore$ $y = 0.02\sin 2\pi\left[\frac{t}{0.05} - \frac{x}{1}\right]$ m

Ans :	Equation of progressive wave is $y = 0.02 \sin 2\pi\left[\frac{t}{0.05} - \frac{x}{1}\right]$ m

4) The equation of a simple harmonic progressive wave is given by, $y = 4 \sin \pi\left(\frac{t}{0.02} - \frac{x}{75}\right)$ in c.g.s. unit. Find the displacement and velocity of the particle at a distance of 50 cm from the origin and at the instant 0.1 second.

Data : $y = 4 \sin \pi\left(\frac{t}{0.02} - \frac{x}{75}\right)$ in c.g.s unit

$x = 50$ cm, $t = 0.1$ sec

To Find : i. y ii. v_p

Formula : i. $y = A \sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$

ii. $v_P = \omega\sqrt{A^2 - y^2}$

Solution :

i. $y = 4 \sin \pi\left(\frac{t}{0.02} - \frac{x}{75}\right)$ cm ....(1)

To find y, substituting t and x in equation (1), we get,

$$y = 4\sin \pi\left(\frac{0.1}{0.02} - \frac{50}{75}\right)$$

$$y = 4\sin \pi\left[\frac{1}{10}\times\frac{100}{2} - \frac{2}{3}\right]$$

$$= 4\sin\frac{13\pi}{3}$$

$$= 4\sin\left[4\pi + \frac{\pi}{3}\right] = 4 \sin\frac{\pi}{3}$$

$\therefore$ $y = 4\times\frac{\sqrt{3}}{2} = 2\sqrt{3} = 2\times 1.732$

$\therefore$ $y = 3.463$ cm.

ii. Velocity of particle v is given by,

$$v = \omega\sqrt{A^2 - y^2}$$

$\therefore$ $v = \frac{2\pi}{T}\sqrt{A^2 - y^2}$

$$= \frac{2\times3.14}{0.04}\sqrt{4^2 - (3.463)^2}$$

$$v = \frac{2\times3.143\times100}{4}\sqrt{16-12}$$

$$= \frac{3.143\times100\times2}{2} = 314.3 \text{ cm/s} = 3.143 \text{ m/s}$$

Ans :	Displacement of particle is 3.463 cm and velocity of particle is 3.143 m/s

5) A transverse wave of amplitude 0.01 m and frequency 500 Hz is travelling along a stretched string with a speed of 200 m/s. Find the displacement of a particle at a distance of 0.7 m from the origin after 0.01 second.

Data : $A = 0.01$ m, $n = 500$ Hz, $v = 200$ m/s

$x = 0.7\ m,\ t = 0.01\ s$

To Find : y

Formula : $y = A \sin 2\pi\left(nt - \frac{x}{\lambda}\right)$

Solution: $v = n\lambda$

$\therefore \quad \lambda = \frac{v}{n} = \frac{200}{500} = 0.4\ m$

$y = A \sin 2\pi\left(nt - \frac{x}{\lambda}\right)$

$y = 0.01 \sin 2\pi\left(500 \times 0.01 - \frac{0.7}{0.4}\right)$

$y = 0.01 \sin 2\pi\left(5 - \frac{7}{4}\right)$

$y = 0.01 \sin \cancel{2}^{1}\, \pi \times \frac{13}{\cancel{4}_{2}}$

$y = 0.01 \sin \frac{13\pi}{2}$

$\therefore \quad y = 0.01 \sin\left(6\pi + \frac{\pi}{2}\right)$

$y = 0.01 \sin \frac{\pi}{2} = 0.01 \times 1$

$\boxed{y = 0.01\ m}$

Ans : Displacement of particle is 0.01m

6) A progressive wave of frequency 50Hz is travelling with a velocity 350 m/s through a medium. Find :

i. the phase difference between two particles separated by 7 m,

ii. the change in phase at a given point in time interval 0.005 second.

Data: $n = 50$ Hz, $v = 350$ m/s, $\Delta x = 7$m, $\Delta t = 0.005$ sec

To Find : i. $(\Delta\phi)$, When $\Delta x = 7$m

ii. $(\Delta\phi)$, When $\Delta t = 0.005$ sec

Formula : i. $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

ii. $\Delta\phi = \frac{2\pi}{T} \times \Delta t = 2\pi n \Delta t$ iii. $v = n\lambda$

Solution :

i. $v = n\lambda$

$\lambda = \frac{v}{n} = \frac{350}{50} = 7m$

ii. $(\Delta\phi)_1 = \frac{2\pi}{\lambda} \times \Delta x$

$= \frac{2\pi}{7} \times 7 = 2\ \pi$ rad.

ii. $\Delta\phi = \frac{2\pi}{T} \times \Delta t = 2\pi n \Delta t$

$= 2\pi \times 50 \times 0.005$

$= 0.5\pi = \frac{\pi}{2}^{c}$

Ans : i. Phase difference between two points seperated by 7m is 2π rad.
ii.Phase difference between after 0.005 sec is $\frac{\pi}{2}$ rad.

★ 7) A wave of frequency 500 Hz is travelling with a speed of 350 m/s.

i. What is the phase difference between two displacements at a certain point at times 1.0 ms apart?

ii. What will be the smallest distance between two points which are 45° out of phase at an instant of time?

Data: $n = 500$Hz, $v = 350$m/s,

$\Delta t = 1.0\ ms = 10^{-3}$s,

$\Delta\phi = 45° = \left(\frac{\pi}{4}\right)^{c}$

To find: i. $(\Delta\phi)$ when $\Delta t = 1$ms

ii. (Δx) when $\Delta\phi = 45°$

Formulae: i. $v = n\lambda$ ii. $\Delta\phi = \frac{2\pi}{T}\Delta t$

iii. $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta\phi$

Solution:

i. $v = n\lambda = \frac{350}{500} = \frac{7}{10}$ m

ii. $\Delta\phi = \frac{2\pi}{T}\Delta t = 2\pi n \Delta t \quad \ldots\left(\because n = \frac{1}{T}\right)$

$= 2 \times \pi \times 500 \times 10^{-3} = \pi$ rad

iii. $\Delta x = \frac{\lambda}{2\pi} \times \Delta\phi = \frac{\lambda}{2\pi} \times \frac{\pi}{4} = \frac{\lambda}{8} = \frac{7}{10 \times 8}$

$= 0.0875\ m = 8.75\ cm$

Ans : i. Phase difference between two displacements at an instant is π rad.
ii. Smallest distance between the two points is 8.75cm

★ **8) Two sound waves travel at a speed of 330 m/s. If their frequencies are also identical and are equal to 540 Hz, what will be the phase difference between the waves at points 3.5 m from one source and 3m from the other if the sources are in phase?**

Data: $v = 330$ m/s, $n = 540$ Hz, $x_1 = 3.5$ m, $x_2 = 3$ m

To find: $(\Delta\phi)$

Formulae: i. $\lambda = \frac{v}{n}$ ii. $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$

Solution:

i. $$\lambda = \frac{v}{n} = \frac{330}{540} = \frac{11}{18} \text{m}$$

ii. $$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{11} \times (3.5 - 3) \times 18$$

$$= \frac{2 \times \pi \times 18}{11} \times \frac{1}{2} = \frac{18\pi}{11}$$

$$= 1.64\pi$$

Ans : The phase difference between the waves is $1.64\ \pi$

Problem For Practice

1. The equation of a plane progrressive wave is $y = 10 \sin 2\pi (t - 0.005x)$ where y and x are in cm and t in seconds. Calculate : the (i) Amplitude (ii) frequency (iii) wavelength (iv) velocity of the wave.

Ans : i. 10cm, ii. 1Hz, iii. 200cm, iv. 200cms^{-1}

2. A wave travelling along a string is given by $y(x, t) = 0.005 \sin (80x - 3t)$ where the numerical are in SI units, Symbols have their usual meanings. Calculate : (i) Frequency of the wave (ii) Velocity of the wave (iii) Amplitude of particle velocity.

Ans :i. 0.48 Hz, ii. 7.5 cms^{-1}, iii. 0.015 ms^{-1}

3. A wave on a string is described by $y(x,t)=0.005 \sin (6.28x - 314t)$, in which all quantities are in SI units. Calculate its (i) amplitude and (ii) wavelength

Ans : i. 0.005m, ii. 1m

4. The equation of transdverse wave travelling in a rope is given by $y = 10 \sin \pi (0.01x - 2.00t)$ Where y and x are in cm and t in second. Find the amplitude, frequency, velocity and wavelength of the wave.

Ans : 10cm, 1Hz, 200 cms^{-1}, 200cm

5. The equation of simple harmonic progressive wave is given by $Y = 0.05 \sin \pi\left(20t - \frac{x}{6}\right)$, where all quantities are in S.I. units, Calculate the displacement of a particle at 5 m from origin and at the instant 0.1 second.

Ans :-0.025 m

6. A wave travelling along a string is described by $y(x,t)=0.005 \sin (80.0x-3.0t)$ in which the numerical constants are in SI units (0.005m, 80.0 rad m–1, and 3.0 rad s–1) Calculate (i) the amplitude,(ii) the wavelength, and (iii) the period and frequency of the wave. Also calculate the displacement y of the wave at a distance x = 30.0 cm and time t = 20s

Ans : i. 0.005 m, ii. 7.85 m, iii. 0.48 Hz

7. A simple harmonic wave is expressed by equations, $y = 7\times10^{-6} \sin\left[800\pi t - \frac{\pi}{42.5}x\right]$ Where y and x are in cm and t in seconds. Calculate the following: (i) amplitude(ii) frequency (iii) wave length (iv) wave velocity and (v) phase difference between two particles separated by 17.0 cm

Ans : 7×10^{-6} cm

8. Find the displacement of an air particle 3.5m from the origin of disturbance at t = 0.05s, when a wave of amplitude 0.2 mm and and frequency 500 Hz travels along it with a velocity 350 ms^{-1}

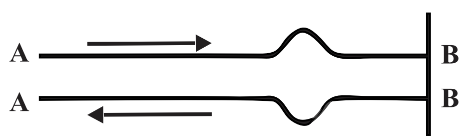
Ans:0

6.3 Reflection of waves

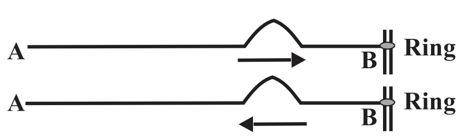
Q.5 Explain the reflection of transverse waves from the surface of :
i. denser medium
ii. rarer medium.

Ans:

i. When transverse waves are reflected from the surface of a denser medium (rigid wall):
a. The wave velocity is reversed.
b. As particles of denser medium are not free to oscillate, the particle velocity is reversed.
Therefore, phase changes by π radians.
When crest is incident, due to phase change of π radians trough is formed. Thus, crest is reflected as trough and trough is reflected as crest.



ii. When transverse waves are reflected from the surface of a rarer medium,
a. The wave velocity is reversed.
b. As particles of rarer medium are free to oscillate, the particle velocity is not reversed.
Hence, there is no change of phase due to reflection.
Therefore, crest is reflected as crest and trough is reflected as trough.

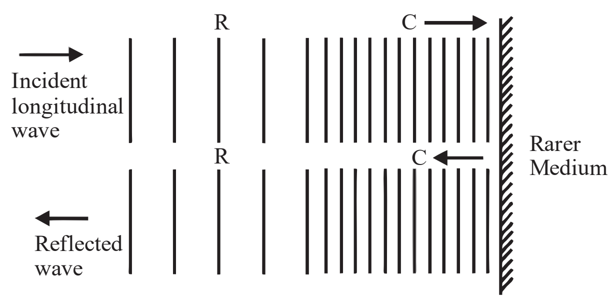


7. Explain the reflection of longitudinal waves (sound waves) from the surface of
i. denser medium
ii. rarer medium.

Ans

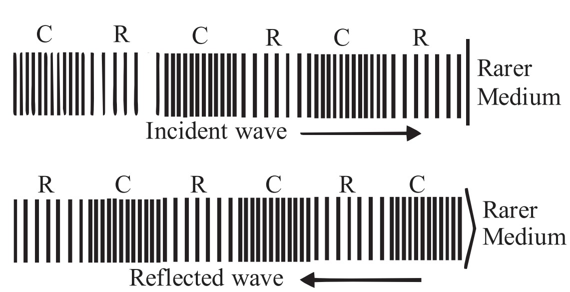
i. When longitudinal sound waves are reflected from the surface of a denser medium (i.e., rigid wall) :
a. The wave velocity is reversed.
b. As the particles of a denser medium are not free to oscillate, the particle velocity is also reversed. Therefore, the reversal of particle velocity produces a change of phase of π radians.

When both velocities i.e. particle and wave velocity are in same direction, a compression is formed. As both the velocities are reversed after reflection, they will be directed away from the denser medium after reflection. Hence, compression is reflected as compression, and similarly rarefaction is reflected as rarefaction.



ii. When longitudinal waves are reflected from the surface of a rarer medium :
a. The wave velocity is reversed.
b. As the particles of rarer medium are free to oscillate, the particle velocity is not reversed. Hence, there is no change of phase due to reflection.

When compression is incident, both the velocities are directed towards the rarer medium. After reflection only wave velocity is reversed, and therefore, two velocities are opposite after reflection. Hence, the compression is reflected as rarefaction and rarefaction is reflected as compression.



6.4 Superposition of waves

Q.7 State and explain the principle of superposition of the waves.

Ans: Principle of superposition of waves :

When two or more waves, travelling through a medium, arrive at a point simultaneously, then each wave produces its own displacement at that point independently of the others. Hence the resultant displacement at that point is the vector sum of the displacements due to all individual waves.

Let, y_1, y_2, y_3 .... y_n be the individual displacements due to 'n' waves respectively.

By principle of Superposition, resultant displacement is,

$$\left(\vec{Y}\right) = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ... + \vec{y}_n$$

For two waves

$$\left(\vec{Y}\right) = \vec{y}_1 + \vec{y}_2$$

Constructive interference :

When the two sound waves meeting at a point are in phase i.e. compression of one wave coincides with compression of another wave or rarefaction of one wave coincides with rarefaction of another wave, then resultant intensity of sound will be maximum at that point. This effect is called constructive interference.

Destructive interference :

When the two sound waves meeting at a point are out of phase i.e. compression of one wave coincides with rarefaction of another wave or vice versa, then resultant intensity of sound will be minimum at that point. This effect is called destructive interference.

Q.8 Find the amplitude of the resultant wave produced due to interference of two waves given as $y_1 = A_1 \sin\omega t$, $y_2 = A_2 \sin(\omega t + \phi)$

Ans : Expression for amplitude of resultant wave:

i. Consider two waves having the same frequency but different amplitudes A_1 and A_2. Let these waves differ in phase by ϕ.

ii. The displacement of each wave at x = 0 is given as $y_1 - A_1 \sin\omega t$ and $y_2 = A_2 \sin(\omega t + \phi)$

iii. According to the principle of superposition of waves, the resultant displacement at x = 0 is

$y = y_1 + y_2$

$\therefore \quad y = A_1 \sin\omega t + A_2 \sin(\omega t + \phi)$

$$y = A_1 \sin\omega t + A_2 \sin\omega t \cos\phi + A_2 \cos\omega t \sin\phi$$

$$y = (A_1 + A_2\cos\phi)\sin\omega t + A_2\sin\phi\cos\omega t$$

Let, $A_1 + A_2\cos\phi = A\cos\theta$ ...(1)

$A_2\sin\phi = A\sin\theta$ ...(2)

$\therefore \quad y = A\cos\theta\sin\omega t + A\sin\theta\cos\omega t$

$\therefore \quad y = A\sin(\omega t + \theta)$

iv. This is the equation for displacement of the resultant wave. It has the same frequency as that of the interfering waves.

v. The resultant amplitude A is given by squaring and adding equation (1) and (2)

$$A^2\cos^2\theta + A^2\sin^2\theta = (A_1 + A_2\cos\phi)^2 + A_2^2\sin^2\phi$$

$$A^2 = A_2^2 + 2A_1A_2\cos\phi + A_2^2\cos^2\phi + A_2^2\sin^2\phi$$

$$\therefore \quad A = \sqrt{A_1^2 + 2A_1A_2\cos\phi + A_2^2}$$

Case-1: Condition for maximum amplitude :

vi. When $\phi = 0$, i.e. the waves are in phase, the resultant amplitude is

$$A = \sqrt{A_1^2 + 2A_1A_2\cos 0 + A_2^2}$$

$$= \sqrt{(A_1 + A_2)^2} = A_1 + A_2$$

The resultant amplitude is maximum, when $\phi = 0$

If the amplitudes of the waves are equal i.e., $A_1 = A_2 = A$. then the resultant amplitude is 2A

Thus, the maximum amplitude is the sum of the two amplitudes when the phase difference between the two waves is zero.

Case-2: Condition for minimum amplitude :

vii. When $\phi = \pi$, i.e., the waves are out of phase, the resultant amplitude is

$$A = \sqrt{A_1^2 + 2A_1A_2\cos\pi + A_2^2}$$

$$= \sqrt{(A_1 - A_2)^2} = |A_1 - A_2|$$

The resultant amplitude is minimum, when

$\phi = \pi$

If the amplitudes of the waves are equal i.e., $A_1 = A_2 = A$. then the resultant amplitude is zwro.

Thus, the minimum amplitude is the difference of the two amplitudes when the phase difference between the two waves is π.

Key Point

i. When two waves having displacement

$y_1 = A_1 \sin\omega t$

$y_2 = A_2 \sin(\omega t + \phi)$

Then resultant amplitude is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

ii. Intensity of wave α (Amplitude)^2

$\therefore$ $I \alpha A^2$

$\therefore$ $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

iii. For constructive interference

a. Amplitude is maximum

$A_{max} = A_1 + A_2$

b. Intensity is maximum

$I_{max} \ \alpha \ (A_1 + A_2)^2$

c. Particles are in same phase

iv. For destrctive interference

a. Amplitude is minimum

A_{min} - $|A_1 - A_2|$

b. Intensity is minimum

$I_{min} \ \alpha \ (A_1 - A_2)^2$

c. Particles are in opposite phase

v. $$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2$$

INTEXT QUESTION

The displacements of two sinusoidal waves propagating through a string are given by the following equations

$y_1 = 4 \sin(20x - 30t)$

$y_2 = 4 \sin(25x - 40t)$

where x and y are in centimeter and t is in second

i. Calculate the phase difference between these two waves at the points x = 5cm and t = 2 s.

ii. When these two waves interfare, what are the maximum and minimum values of the intensity?

Data: $y_1 = 4 \sin(20x - 30t)$ cm and $y_2 = 4 \sin(25x - 40t)$ cm

To Find : i. $\Delta\phi$ ii. I_{max} iii. I_{min}

Formula: i. $\Delta\phi = |\phi_2 - \phi_1|$

ii. $I_{max} = (A_1 + A_2)^2$

iii. $I_{min} = (A_1 - A_2)^2$

Solution:

i. **When x = 5 cm and t = 2 s,**

$y_1 = 4 \sin(20\times5 - 30\times2)$

$= 4 \sin(100 - 60) = 4 \sin 40$

$\therefore$ $\phi_1 = 40$ rad

$y_2 = 4 \sin(125 - 80) = 4 \sin 45$

$\phi_2 = 45$

$\therefore$ Phase difference,

$\Delta\phi = |\phi_2 - \phi_1| = |45 - 40| = 5$ rad

ii. $A_1 = 4$ cm and $A_2 = 4$ cm,

For maximum Intensity

$\therefore$ $I_{max} = (A_1 + A_2)^2 = (4 + 4)^2 = 64$

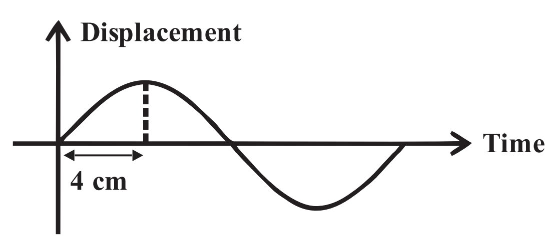
For minimum Intensity

$\therefore$ $I_{min} = (A_1 - A_2)^2 = (4 - 4)^2 = 0$

Ans :	i.	The phase difference is 5 rad.
	ii.	The maximum itensity is 64 while the minimum intensity is 0.

A progressive wave travels on a stretched string. A particle on this string takes 4.0 ms to move from its mean position to one of its extreme positions. The distance between two consecutive points on the string which are at their mean positions (at a certain time instant) is 2.0 cm. Find the frequency, wavelength and speed of the wave.

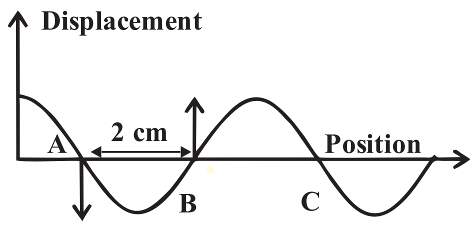
Solution:



i. A particles takes 4.0×10^{-3}s to trave form its mean position to extreme position. This is a quarter of the complete oscillation. Hence, the particle will take $4 \times 4.0 \times 10^{-3}s = 16 \times 10^{-3}s$ to complete one oscillation.

$\therefore$ Frequency, $n = \frac{1}{T} = \left(\frac{1}{16}\right) \times 10^{-3} = 62.5\text{Hz}$

ii. Two consecutive particles at their mean positions, one will be moving upwards while the other will be moving downwards. The distance between two consecutive particles moving in the same direction will be 2×2 cm = 4 cm.
Wavelength, $\lambda = 4\text{cm} = 0.04\text{m}$



iii. Speed of wave, $v = n \times \lambda$
$= 62.5 \times 0.04 = 2.5\text{m/s}$

Ans :	i.	Frequency of the wave is 62.5Hz.
	ii.	Wavelength of the wave is 0.04m.
	iii.	Speed of the wave is 2.5m / s

6.5 Stationary Waves

Q.9 What are stationary waves ?

Ans: Stationary Waves : When two identical progressive waves travelling along the same path in opposite directions, interfere with each other, by superposition of waves resultant wave obtained in the form of loops, is called stationary waves.

These waves do not travel in any direction, and they do not transfer energy through the medium.

Q.10 Explain analytically how stationary waves are formed. Show that nodes and antinodes are equispaced.

Ans:

i. Stationary waves are formed due to superposition of two exactly identical waves travelling through a medium in opposite directions.

ii. two consider progressive waves having same amplitude (a), wavelength (λ) and period (T) but travelling in opposite directions along the string is given by

$$y_1 = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad ...(1)$$

$$y_2 = a\sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) \quad ...(2)$$

iii. The resultant displacement at a point is given by, $y = y_1 + y_2$

$$\therefore \quad y = a\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] + a\sin\left[2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

$$\therefore \quad y = a\left[\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

Using trigonometric relation,

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$$

we can write,

$$y = 2a\sin\left(\frac{2\pi t}{T}\right).\cos\left(\frac{2\pi x}{\lambda}\right), \quad \textbf{OR}$$

$$y = A\sin\left(\frac{2\pi t}{T}\right) = A\sin\omega t \quad ...(3)$$

where, $A = 2a\cos\left(\frac{2\pi x}{\lambda}\right)$

iv. Equation (3) shows that the resultant motion is also SHM with the same period (T) but with the amplitude (A), which varies with x, i.e., the position of the particle.

v. There is absence of the term λ/x, appearing in the equation for a progressive wave. This shows that the resultant wave is not travelling in forward or backward direction. So, it is a stationary wave.

vi. **Nodes :** When A = 0, particles at these points are permanently at rest. They are called nodes.
We have A = 0

$$\therefore \quad 2a\cos\left(\frac{2\pi x}{\lambda}\right) = 0$$

$$\cos\left(\frac{2\pi x}{\lambda}\right) = 0$$

$$\therefore \quad \frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, ...$$

$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, ...$

∴ The distance between two successive nodes $= \left(\frac{5\lambda}{4} - \frac{3\lambda}{4}\right) = \frac{\lambda}{2}$

vii. **Antinodes :** When A = ± 2a, the particles at these points vibrate with maximum amplitude. They are called antinodes.

We have, $A = \pm 2a\, 2a\cos\left(\frac{2\pi x}{\lambda}\right) = \pm 2a$

∴ $\cos\left(\frac{2\pi x}{\lambda}\right) = \pm 1$

∴ $\frac{2\pi x}{\lambda} = 0, \pi, 2\pi, 3\pi, 4\pi, ...$

$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, ...$

viii. The distance between two successive antinodes, $= \left(\frac{3\lambda}{2} - \lambda\right) = \frac{\lambda}{2}$

ix. The distance of a node from its adjacent antinode $= \frac{3\lambda}{2} - \frac{5\lambda}{4} = \frac{\lambda}{4}$

Thus, in stationary waves, the nodes and antinodes are alternately situated & equispaced.

★ Q.11 Derive an expression for equation of stationary wave on a stretched string.

Ans :

i. Stationary waves are formed due to superposition of two exactly identical waves travelling through a medium in opposite directions.

ii. two consider progressive waves having same amplitude (a), wavelength (λ) and period (T) but travelling in opposite directions along the string is given by

$$y_1 = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad ...(1)$$

$$y_2 = a\sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) \quad ...(2)$$

iii. The resultant displacement at a point is given by, $y = y_1 + y_2$

∴ $y = a\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] + a\sin\left[2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$

∴ $y = a\left[\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$

Using trigonometric relation,

$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

we can write,

$y = 2a\sin\left(\frac{2\pi t}{T}\right).\cos\left(\frac{2\pi x}{\lambda}\right)$, **OR**

$$y = A\sin\left(\frac{2\pi t}{T}\right) = A\sin\omega t \quad ...(3)$$

where, $A = 2a\cos\left(\frac{2\pi x}{\lambda}\right)$

iv. Equation (3) shows that the resultant motion is also SHM with the same period (T) but with the amplitude (A), which varies with x, i.e., the position of the particle.

v. There is absence of the term λ/x, appearing in the equation for a progressive wave. This shows that the resultant wave is not travelling in forward or backward direction. So, it is a stationary wave.

★ Q.12 State the characteristics of stationary waves

Ans : Characteristics of stationary waves :

i. Stationary waves are produced due to superposition of two identical waves (either transverse or longitudinal waves) traveling through a medium along the same path in opposite directions.

ii. When a transverse stationary wave is produced on a string, some points on the string are motionless. The points which do not move are called **nodes**.

iii. There are some points on the string which oscillate with greatest amplitude (say A). They are called **antinodes**.

Points between the nodes and antinodes vibrate with values of amplitudes between 0 and A.

iv. The distance between two consecutive nodes is $\frac{\lambda}{2}$ and the distance between two consecutive antinodes is $\frac{\lambda}{2}$

v. Nodes and antinodes are produced alternately. The distance between a node and an adjacent antinode is $\frac{\lambda}{4}$

vi. The amplitude of vibration varies periodically in space. All points vibrate with the same frequency.

vii. Though all the particles (except those at the nodes) possess energy, there is no propagation of energy. The wave is localized and its velocity is zero. Therefore, we call it a stationary wave or standing wave.

viii. All the particles between adjacent nodes (i.e., in one loop) vibrate in phase. There is no progressive change of phase from one particle to another particle. All the particles in the same loop are in the same phase of oscillation, which reverses for the adjacent loop.

Q.13 Distinguish between progressive waves and stationary waves.

Ans:

No	Progressive waves	Stationary waves
i.	Progressive waves are propagated in the forward direction.	Stationary waves are not propagated in any direction.
ii.	Progressive waves are produced due to a disturbance created in the medium.	Stationary waves are formed due to superposition of two exactly identical waves travelling through a medium in opposite directions.
iii.	Progressive waves transfer energy through a medium from particle to particle.	Stationary waves do not transfer energy through a medium.
iv	Every particle vibrates with uniform amplitude	Amplitude increases from node to antinode.
v	Every particle lags behind the previous particle in phase.	All the particles in one loop are in the same phase, while particles in two successive loops are out of phase by π.
vi	The particle of the medium begins to vibrate as the wave approaches it.	Particles at nodes are permanently at rest.

Type - II
Numerical based on formation of stationary wave

Formula used :

1. Distance between two successive nodes or Antinodes is $\frac{\lambda}{2}$
2. Distance between node and adjacent antinode is $\frac{\lambda}{4}$
3. $v = n\lambda$

★ **1) Find the distance between two successive nodes in a stationary wave on a string vibrating with frequency 64 Hz. The velocity of progressive wave that resulted in the stationary wave is 48 ms^{-1} .**

Data: $v = 48ms^{-1}$, $n = 64$ Hz

To find: Distance between two successive nodes

Formula: i. $v = n\lambda$
ii. Distance between two successive nodes $\frac{\lambda}{2}$

Solution:

i. $v = n\lambda$

$$\lambda = \frac{v}{n} = \frac{48}{64} = 0.75m$$

ii. Distance between two successive nodes is $= \frac{\lambda}{2} = \frac{0.75}{2} = 0.375m$

Ans : Distance between two succesive nodes is 0.375m

★ **2) A sound wave in a certain fluid medium is reflected at an obstacle to form a standing wave. The distance between two successive nodes is 3.75 cm. If the**

velocity of sound is 1500 m/s, find the frequency.

Data: Distance between two successive node

$$\left(\frac{\lambda}{2}\right) = 3.75\text{cm} = 3.75\times10^{-2}\text{ m}$$

To find: n

Formula: $v = n\lambda$

$\therefore \quad n = \frac{v}{\lambda}$

Solution:

i. $\frac{\lambda}{2} = 3.75 \times 10^{-2}$ m

$\therefore \quad \lambda = 7.5\times10^{-2}$ m

ii. $n = \frac{v}{\lambda}$

$$n = \frac{1500}{7.5\times10^{-2}} = 200\times10^{2} = 20\text{kHz}$$

Ans :	The frequency of the sound wave is 20 kHz

★ 3) **Two sources of sound are separated by a distance 4 m. They both emit sound with the same amplitude and frequency (330 Hz), but they are 180° out of phase. At what points between the two sources, will the sound intensity be maximum?**

Soution

i. $v = n\lambda$

$$\lambda = \frac{v}{n} = \frac{330}{330}$$

$\therefore \quad \lambda = 1$m

ii. Exactly at the centre of line joining the two centres path difference is zero. i.e intensity is maximum

As sources are out of phase, two maxima will be obtained at $\frac{\lambda}{4}$ and $\frac{\lambda}{2}$ at the either side of centre

$\therefore$ Antinode is formes at

$$\pm\frac{\lambda}{4}, \pm\frac{3\lambda}{4}, \pm\frac{5\lambda}{4}, \pm\frac{7\lambda}{4}$$

$\therefore$ Sound intensity is maximum at ±0.25 m, ±0.75m, ±1.25m and ±1.75m from point at the centre.

Problem for Practice

1. Find the distance between two successive nodes in a stationary wave on a string vibratins at 128 Hz. The velocity of progressive wave is 64m/s

Ans : – 0.25

2. If distance between node and adjacent antinode is 4 cm and velocity of transverse wave in string is 1000 m/s. Find frequency of sound wave

Ans : 625 kHz

6.6 Free and Forces Vibrations

Q.14 Explain free vibrations give their examples.

Ans:

i. **Free vibration :** When a body, capable of oscillating, is displaced from its stable equilibrium position and released, it makes oscillations which are called free vibrations and the frequency of vibrations is called its natural frequency.

ii. The natural frequency of vibration depends on the dimensions, mass, elastic properties and mode of vibration of the vibrating body.

iii. If there is no resisting force acting on the vibrating body, vibrations are truly free and the amplitude will be constant.

But due to frictional resistance of surrounding medium, body continuously lose energy and due to which amplitude goes on decreasing and body comes to rest.

Examples :

a. Vibrations of air column in a pipe when sounded tuning fork is held over open end.
b. Vibrations of string.
c. Oscillations of simple pendulum.
d. When a prong of a tuning fork is struck on a rubber pad, the prongs vibrates with a single (natural) frequency.

Q.15 Explain forced vibrations give their

examples.

Ans:

i. **Forced vibrations :** The vibrations of body under action of external periodic force in which body vibrates with frequency equal to frequency of external periodic force (driving frequency) other than its natural frequency are called forced vibrations.

ii. The amplitude of forced vibrations depends upon difference between the frequency of external periodic force and natural frequency of vibration of body.

iii. It also depends on amplitude of applied force and damping force.

iv. If amplitude of forced vibrations is small then the difference between frequencies of vibration of body becomes large and vice versa.

Examples :

a. If a vibrating tuning fork is held with its stem in contact with a table top, the table top performs forced vibrations with the frequency equal to that of the fork.

b. In oscillations of pendulum in a clock, amplitude of oscillations is constant.

Q.16 Give the differe1nce between free vibrations and forced vibrations.

Ans:

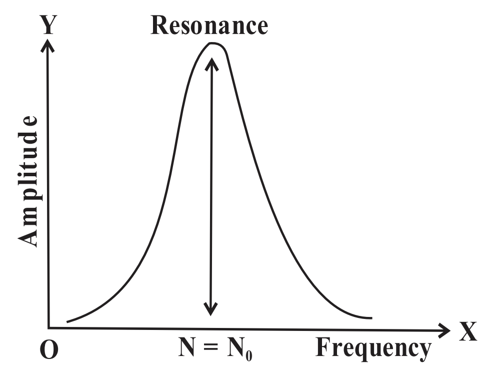
No	Free vibrations	Forced vibrations
i.	Free vibrations are produced by applying momentary force.	Forced vibrations are produced by applying continous external periodic force.
ii.	Body vibrates with natural frequency.	Body will not vibrate with natural frequency.
iii.	Natural frequency depends upon dimensions and material used.	Frequency of oscillation donot depend upon elastic constant or dimensions of vibrating body.
iv	Vibrations are continued for long time.	Vibrations will stop as soon as the external periodic force is removed.
v	Ex: Vibrations of air column in a pipe when sounded tuning fork is held over open end.	Ex:If a vibrating tuning fork is held with its stem in contact with a table top, the table top performs forced vibrations with the frequency equal to that of the fork.

Q.17 Explain resonance.

Ans:

i. **Resonance:** The phenomenon in which the body vibrates under action of external periodic force, whose frequency is equal to the natural frequency of the driven body, so that amplitude becomes maximum is called resonance.

ii. If the difference between natural frequency and frequency of external periodic force is large then the amplitude of forced vibrations is small. If this difference is gradually reduced, the amplitude of forced vibration goes on increasing and at certain stage amplitude becomes maximum.



Resonance curve (when driven system has a single natural frequency N_0)

6.7 Harmonics and Overtones

Q.18 What are harmonics and overtones?

Ans :

i. Harmonics :

a. The lowest harmonic is a fundamental frequency and integrat multiples of fundamental frequency are called harmonics.

b. Fundamental frequency is first harmonic. Second harmonic is twice fundamental frequency and so on

c. All harmonics may not be present in a given sound.

ii. **Overtones :**

a. The higher allowed frequencies are called the overtones.

b. Above the fundamental, the first allowed frequency is called the first overtone, the next higher frequency is the second over tone, and so on.

c. Fundamental frequency is not the overtone.

Q.19 Distinguish between harmonics and overtones.

Ans :

No	Harmonics	Overtones
i.	The integral multiple of fundamental frequency is called its harmonic.	Vibrations of higher frequencies which are actually present in addition to the fundamental frequency are called overtones.
ii.	Fundamental frequency itself is first harmonic.	Fundamental frequency is not the first overtone.
iii.	In some vibrations, some harmonics may be absent.	No question of any overtone being absent.
iv	In a pipe closed at one end, even harmonics are absent & all odd harmonics are present.	In a pipe closed at one end, 3n is the first overtone, 5n is the second overtone and so on, where n is the fundamental frequency

Organ pipe :

i. A device in which air column is vibrated to produce sound wave of desired frequency is called organ pipe.

ii. There are two types of organ pipe:

a. **Closed organ pipe :** In this type of organ pipe, one end is open and another end is kept closed.

b. **Open organ pipe :** In this type of organ pipe, both the ends are open.

Q.20 Explain the concept of end correction.

Ans :

i. When an air column vibrates either in a pipe closed at one end or open at both ends, boundary conditions demand that there is always an antinode at the open end(s) (since the particles of the medium are comparatively free) and a node at the closed end (since there is hardly any freedom for the particles to move).

ii. The antinode is not formed exactly at the open end but it is slightly beyond the open end as air is more free to vibrate there in comparison to the air inside the pipe.

iii. Also as air particles in the plane of open end of the pipe are not free to move in all directions, reflection takes place at the plane at small distance outside the pipe.

iv. <u>The distance between the open end of the pipe and the position of antinode is called the end correction.</u>

v. According to Reynold, to the first approximation, the end correction at an end is given by e = 0.3d, where d is the inner diameter of the pipe.

vi. Thus the length L of air column is different from the length l of the pipe.

vii. **For a pipe closed at one end :** The corrected length of air column L = length of air column in pipe l + end correction at the open end.

$\therefore \quad L = l + e$

viii. **For a pipe open at both ends :** The corrected length of air column L = length of air column in pipe l + end corrections at both the ends.

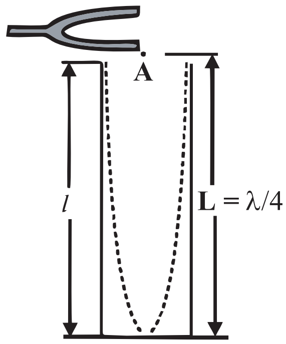
$\therefore \quad L = l + 2e$

★ **Q.21 Show that only odd harmonics are present in the vibrations of air column in a pipe closed at one end.**

Ans : For fundamental mode :

a. The first mode of vibrations of air column closed at one end is known as the fundamental mode

b. In first mode, one node and one antinode is formed.



Length of air column

$$L = \frac{\lambda}{4}$$

$$\lambda = 4L \quad ...(1)$$

where λ is the wavelength of fundamental mode of vibrations in air column.

c. If n is the fundamental frequency then

$v = n\lambda$

$$\therefore \quad n = \frac{v}{\lambda}$$

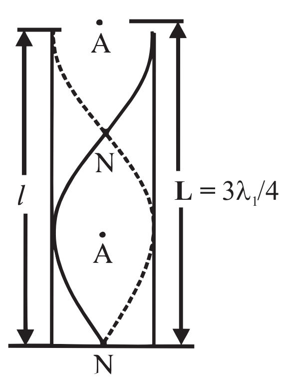
$$\therefore \quad \boxed{n = \frac{v}{4L}} \quad ...[\text{From equation (1)}]$$

$$\boxed{n = \frac{v}{4(l+e)}}$$

d. The fundamental frequency is also known as the first harmonic. It is the lowest frequency of vibration in air column in a pipe closed at one end.

ii. **For second mode or first overtone :**

a. In second mode two nodes and two antinodes are formed.



b. Here the air column is made to vibrate in such a way that it contains a node at the closed end, an antinode at the open end with one more node and antinode in between.

c. If n_1 is the frequency and λ_1 is the wavelength of wave in this mode of vibrations in air column, the length of the air column.

$$L = \frac{\lambda_1}{4} + \frac{\lambda_1}{2} = \frac{3\lambda_1}{4}$$

$$\therefore \quad \lambda_1 = \frac{4L}{3} = \frac{4(l+e)}{3} \quad ...(2)$$

d. The velocity in the second mode is given as $v = n_1 \lambda_1$

$$\therefore \quad n_1 = \frac{v}{\lambda_1} \quad ... [\text{ From equation (2)}]$$

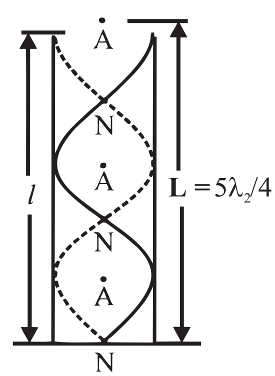
$$= \frac{3v}{4L} = \frac{3v}{4(l+e)}$$

$$\therefore \quad \boxed{n_1 = 3n}$$

e. This frequency is the third harmonic and the first overtone.

iii. **For third mode or second overtone :**

a. In third mode three nodes and three antinodes are formed.



b. Here the same air column is made to vibrate in such a way that it contains a node at the closed end, an antinode at the open end with two more nodes and antinodes in between.

c. If n_2 is the frequency and λ_2 is the wavelength of the wave in this mode of vibrations in air column, then length of air column

$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{2} + \frac{\lambda_2}{4} = \frac{5\lambda_2}{4}$$

$\therefore \quad \lambda_2 = \frac{4L}{5} = \frac{4(l+e)}{5} \quad ...(3)$

d. The velocity this mode is given as

$v = n_2\lambda_2$

$\therefore \quad n_2 = \frac{v}{\lambda_2} \quad ...[\text{ From equation(3)}]$

$= \frac{5v}{4L} = \frac{5v}{4(l+e)}$

$\therefore \quad n_2 = 5n$

e. <u>This frequency is the fifth harmonic and the second overtone.</u>

iv. Continuing in a similar way, for the p^{th} overtone we get the frequency n_p is given as

$n_p = (2p + 1)n$

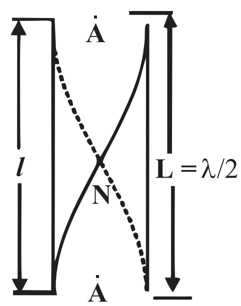
Thus for a pipe closed at one end only odd harmonics are present and even harmonics are absent.

★ Q.22 Prove that all harmonics are present in the vibrations of the air column in a pipe open at both ends.

Ans :

i. For fundamental mode :

a. The fundamental mode one node and two Antinods is formed.



b. There are are two antinodes at two open ends and one node between them.

$\therefore$ Length of air column

$L = \frac{\lambda}{2}$

$\therefore \quad \lambda = 2L \quad ... (1)$

c. If n is a fundamental frequency then,

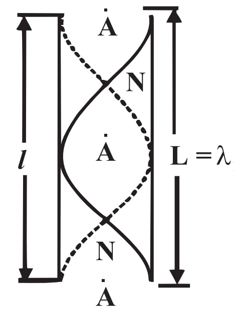
$\therefore \quad n = \frac{v}{\lambda} = \frac{v}{2L} \quad ...[\text{From equation(1)}]$

$= \frac{v}{2(l+2e)}$

<u>This is the fundamental frequency or the first harmonic.</u> It is the lowest frequency of vibration.

ii. For second mode or first overtone :

a. In second mode of vibration two nodes and three antinodes are formed.



b. Length of air column

$\therefore \quad L = \frac{\lambda_1}{2} + \frac{\lambda_1}{2} = \lambda_1$

d. If n_1 and λ_1 are frequency and wavelength of this mode of vibration of air column respectively, then

$v = n_1\lambda_1$

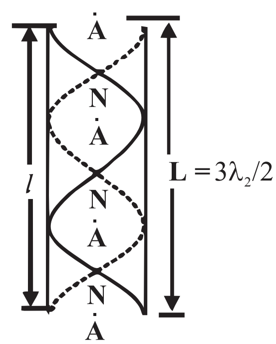
$\therefore \quad n_1 = \frac{v}{\lambda_1} = \frac{v}{L} = \frac{2v}{L}$

$\therefore \quad n_1 = 2n$

This is the frequency of second harmonic or first overtone.

iii. For third mode or second overtone :

a. In third mode four antinode and three nodes are formed.



b. Length of air column,

$L = \frac{3\lambda_2}{2}$

$\therefore \quad \lambda_2 = \frac{2L}{3} = \frac{2(l+2e)}{3} \quad ...(3)$

c. If n_2 and λ_2 are the frequency and wavelength of this mode of vibration of air column respectively, then $v = n_2\lambda_2$

$\therefore \quad n_2 = \frac{v}{\lambda_2} = \frac{3v}{2L}$

$\therefore \quad n_2 = 3n$

This is the frequency of third harmonic or second overtone.

iv. Continuing in this manner, the frequency n_p for p^{th} overtone is, $n_p = (p + 1)n$
where n is the fundamental frequency and $p = 0,1,2,3\ldots$

v. Thus all harmonics are present as overtones in the modes of vibration of air column open at both ends.

Q.23 Show that for pipe open at both ends, the end correction is $e = \frac{n_1 l_1 - n_2 l_2}{2(n_2 - n_1)}$

Ans :

i. For two pipes of same diameter but different lengths l_1 and l_2, is as follows.

ii. **For a pipe open at both ends:**

$v = 2n_1L_1 = 2n_2L_2$

$n_1L_1 = n_2L_2$

$n_1(l_1+2e) = n_2(l_2+2e)$

$n_1l_1+2en_1 = n_2l_2 + 2en_2$

$2en_2 - 2en_1 = n_1l_1 - n_2l_2$

$2e(n_2 - n_1) = n_1l_1 - n_2l_2$

$$e = \frac{n_1l_1 - n_2l_2}{2(n_2 - n_1)} \quad \text{OR} \quad e = \frac{n_2l_2 - n_1l_1}{2(n_1 - n_2)}$$

Q.24 Show that the end correction of a pipe closed at one end is given by $e = \frac{n_2l_2 - n_1l_1}{n_1 - n_2}$

Ans :

i. For two pipes of same diameter but different lengths l_1 and l_2, is as follows.

ii. **For a pipe closed at one end:**

$v = 4n_1L_1 = 4n_2L_2$

$n_1L_1 = n_2L_2$

$n_1(l_1 + e) = n_2(l_2+e)$

$n_1l_1+n_1e = n_2l_2 + n_2e$

$n_2e - n_1e = n_1l_1 - n_2l_2$

$e(n_2 - n_1) = n_1l_1 - n_2l_2$

$$e = \frac{n_1l_1 - n_2l_2}{(n_2 - n_1)} \quad \text{OR} \quad e = \frac{n_2l_2 - n_1l_1}{(n_1 - n_2)}$$

INTEXT QUESTION

Q. 62 A closed pipe and an open pipe have the same length. Show that no mode of the closed pipe has the same wavelength as any mode of the open pipe.

Ans :

i. For closed pipe, the frequency of pth mode is $n_p = (2p + 1)n^c$

$\therefore \quad n_p^c = (2p+1)\frac{v}{4L}\left(\because n^c = \frac{v}{4L}\right)$

$\therefore \quad \frac{\cancel{v}}{\lambda_p^c} = (2p+1)\frac{\cancel{v}}{4L}$

$\therefore \quad \lambda_p^c = \frac{4L}{(2p+1)}$ ...(1)

ii. For open pipe the frequency of mth mocle is

$n_m^o = (m+1)n^o$

$n_m^o = (m+1)\frac{v}{2L}\left(\because n^o = \frac{v}{2L}\right)$

$\therefore \quad \frac{\cancel{v}}{\lambda_m^o} = (m+1)\frac{\cancel{v}}{2L}$

$\therefore \quad \lambda_m^o = \frac{2L}{m+1}$ ...(2)

iii. If $\lambda_p^c = \lambda_m^o$, it would mean

$\frac{4L}{p+1} = \frac{2L}{m+1}$ (from 1 and 2)

$\therefore$ $2(m+1) = 2p + 1$ which is not possible
Hence the two pipes cannot have modes with the same frequency or wavelength.

Type - III
Numerical based on closed pipe and open pipe

Formula used

1. Fundamental frequency of closed pipe

$$n = \frac{v}{4L} = \frac{v}{4(l+e)}\ ;\ np = (2p + 1)\ n$$

2. Fundamental frequency of open pipe

$$n = \frac{v}{2L} = \frac{v}{2(l+2e)}\ ;\ np = (p + 1)\ n$$

★ 1) Find the fundamental, first overtone and second overtone frequencies of a pipe, open at both the ends, of length 25 cm if the speed of sound in air is 330 m/s.

Data: L = 25cm =0.25 m, v = 330m/s

To find: i. Fundamental freuency (n_0)
ii. First overtone (n_1)
iii. Second overtone (n_2)

Formulae: i. $n_0 = \frac{v}{2L}$ ii. $n_P = (p+1)\,n_0$

Solution:

i. $n_0 = \frac{v}{2L}$

$$n_0 = \frac{330}{2 \times 0.25} = 660 \text{ Hz}$$

ii. $n_P = (p+1)\,n_0 = n_1 = 2\,n_0$
$= 2 \times 660 = 1320$ Hz

ii. $n_P = (p+1)\,n_0 = n_2 = 3\,n_0$
$= 3 \times 660 = 1980$ Hz

Ans : The fundamental first overtone and second overtone frequencies are 660 Hz, 1320 Hz and 1980 Hz respectively.

★ 2) A pipe open at both the ends has a fundamental frequency of 600 Hz .The first overtone of a pipe closed at one end the same frequency as the first overtone of the open pipe. How long are the two pipes?

Data: $n_0 = 600$ Hz
$(n_0)_1 = (n_c)_1$
v = 330 m/s

To find: L_0 ; L_c

Formulae: i $(n_0)_p = (p + 1)\ n_0$
ii $(n_c)_p = (2p + 1)\ n_c$

Soluation:

i. For open pipe

$$n_0 = \frac{v}{2L_0}$$

$$\therefore \quad L_0 = \frac{v}{2n_0} = \frac{330}{2 \times 600}$$

$L_0 = 0.275$ m = 27.5 m

ii. According to given condition
$(n_c)_1 = (n_0)_1$
$3n_c = 2n_0$

$$3 \times \frac{v}{4L_c} = 2 \times \frac{v}{2L_0}$$

$$L_c = \frac{3}{4} \times L_0 = \frac{3}{4} \times 0.275$$

= 0.206 m = 20.6 cm

Ans : i. The length of open pipe is 27.5 cm
ii The length of one end closed pipe is 20.63 cm

★ 3) An air column is of length 17 cm long. Calculate the frequency of 5th overtone if the air column is
i. closed at one end and
ii. open at both ends.
(Velocity of sound in air = 340 ms^{-1})

Data: L = 17cm = 0.17 m
p = 5
v = 340 ms^{-1}

To find: i. $(n_c)_5$ ii. $(n_0)_5$

Formulae: i. For closed pipe

a. $n_0 = \frac{v}{4L}$ b. $n_p = (2p+1)n_0$

ii. For open pipe

a. $n_0 = \frac{v}{2L}$ b. $n_p = (p+1)n_0$

Solution: For closed pipe

i. $n_0 = \frac{v}{4L}$

$$n_0 = \frac{340}{4 \times 0.17} = \frac{340 \times 100}{4 \times 17} = 500\text{Hz}$$

$$n_p = (2p+1)n_0$$

$$n_5 = (2 \times 5 + 1) \times 500 = 5500\text{Hz}$$

ii. For open pipe

$$n_0 = \frac{v}{2L} = \frac{340}{2 \times 0.17} = 1000Hz$$

$$n_p = (p+1)n_0$$

$$n_5 = (5+1) \times 1000 = 6000Hz$$

Ans : i. For one end closed pipe.
a. Fundamental frequency is 500 Hz.
b. Fifth overtone is 5500 Hz
ii. For both end open
a. Fundamental frequency is 1000Hz.
b. Fifth overtone is 6000 Hz.

4) A pipe closed at one end can produce overtones at frequencies 640 Hz, 896 Hz and 1152 Hz. Calculate the fundamental frequency.

Data : Three successsive overtones
np = 640 Hz, nq = 896 Hz
nr = 1152 Hz

To Find : Fundamental frequency (n)

Formula : for open pipe

$$\text{Fundamental Frequency} = \frac{\text{Difference between two successive frequency}}{2}$$

Solution:

$$n = \frac{n_q - n_p}{2} = \frac{896 - 640}{2}$$

$$= \frac{256}{2} = 128Hz$$

Ans : Fundamental frequency is 128 Hz

5) A standing wave is produced in a tube open at both ends. The fundamental frequency is 300 Hz. What is the length of tube? (Speed of the sound = 340 ms^{-1})

Data: n_0 = 300 Hz, v = 340 ms^{-1}

To find: L

Formula: $n_0 = \frac{v}{2L}$

Solution: $n_0 = \frac{v}{2L}$

$$\therefore \quad 300 = \frac{340}{2L}$$

$$\therefore \quad L = \frac{340}{2 \times 300} = \frac{17}{30} = 0.57m$$

Ans : The length of the tube is 0.57m.

Problem for Practice

1. Calculate the length of an open organ pipe which resonates with a tuning fork of frequency 512 Hz, if velocity of sound in air is 340 m/s. (neglect the end correction)
Ans : 0.332 m

2. The fundamental frequency of a pipe closed at one end is in unison with the third overtone of an open pipe. Calculate the ratio of the lengths of their air columns.
Ans : $\frac{L_c}{L_0} = \frac{1}{8}$

3. Find the frequency of fifth overtone of an air column vibrating in a pipe closed at one, length of pipe is 42.10 cm and speed of sound in air at room temperature is 350 m/s. [Inner diameter of pipe is 3.5 cm]
Ans : 2230.59 Hz.

Vibration of string

Note :

i. *Velocity of transverse wave is string*

$$v = \sqrt{\frac{T}{m}}$$

where T : Tension in string
m : linear dinsity

ii. *Linear dinsity (m)*
It is defined as mass per unit length

$$m = \frac{\text{Mass of string}}{\text{length of string}}$$

$$m = \frac{M}{L}$$

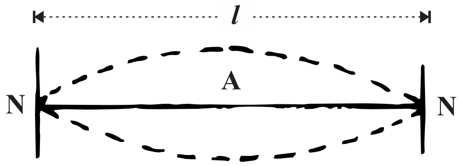
$$m = \frac{volume \times density}{L} = m = \frac{A \times \cancel{L} \times \rho}{\cancel{L}}$$

$$\therefore \quad m = A\rho = \pi r^2 \rho$$

Q.25 Explain the different modes of vibrating string. Obtain the expression for frequency in each case.

Ans:

i. **Fundamental mode :** The lowest frequency with which a string can vibrate is called the fundamental mode of vibrations of the string, and the corresponding frequency is called fundamental frequency of vibrations or the first harmonic. Let n be the fundamental frequency, and λ be the corresponding wavelength.



In this mode, only one loop is formed on the string. i.e two nodes and one antinode

$\therefore \quad \begin{pmatrix}\text{length of the}\\ \text{vibrating string}\end{pmatrix} = \begin{pmatrix}\text{length of}\\ \text{one loop}\end{pmatrix}$

$\therefore \quad l = \frac{\lambda}{2},$

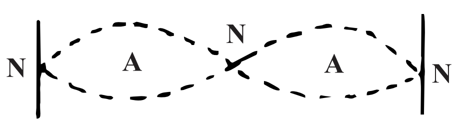
$\therefore \quad \lambda = 2l$

We have, $V = \sqrt{\frac{T}{m}}$ and $n = \frac{V}{\lambda}$

$\therefore \quad \boxed{n = \frac{1}{2l}\sqrt{\frac{T}{m}}}$ ... (1)

where, T - tension in the string
m-mass per unit length of the string.
This is the expression for fundamental frequency of vibrating string.

ii. **First overtone :** When two loops are formed.



means three nodes and two antinodes are formed

n_1 - frequency of vibration
λ_1 - corresponding wavelength.

We have, $\begin{pmatrix}\text{length of the}\\ \text{vibrating string}\end{pmatrix} = \begin{pmatrix}\text{length of}\\ \text{two loop}\end{pmatrix}$

$\therefore \quad l = 2.\frac{\lambda_1}{2}$

$\therefore \quad \lambda_1 = l$

Now, $V = \sqrt{\frac{T}{m}}$ and $n_1 = \frac{V}{\lambda_1}$

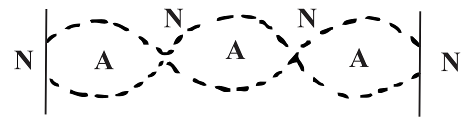
$\therefore \quad n_1 = \frac{1}{l}\sqrt{\frac{T}{m}}$

$\therefore \quad n_1 = 2.\frac{1}{2l}\sqrt{\frac{T}{m}},$

$\therefore \quad n_1 = 2n$

where, n_1 <u>frequency of first overtone or second harmonic.</u>

iii. **Second overtone :** In this case three loops are formed. Hence three antinode and four nodes are formed.



Let, n_2 - frequency of vibration
λ_2 - corresponding wavelength.

Now we have,

$\therefore \quad \begin{pmatrix}\text{length of the}\\ \text{vibrating string}\end{pmatrix} = \begin{pmatrix}\text{length of}\\ \text{two loop}\end{pmatrix}$

$l = 3.\frac{\lambda_2}{2}$

$\therefore \quad \lambda_2 = \frac{2l}{3}$

Now, $V = \sqrt{\frac{T}{m}}$ and $n_2 = \frac{V}{\lambda_2}$

$\therefore \quad n_2 = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3\frac{1}{2l}\sqrt{\frac{T}{m}}$

$\therefore \quad n_2 = 3n$

$\therefore$ <u>It is third harmonic, or second overtone.</u>

In this way, the string can vibrate in different modes with different frequencies.

iv. **For P^{th} mode of vibration :**

In this case P loops are formed. Therefore P antinodes and (P+1) nodes are formed.

Let n_P be the frequency of vibration and x_P be the corresponding wavelength.

Now,

$\begin{pmatrix}\text{length of the}\\ \text{vibrating string}\end{pmatrix} = \text{length of P loops}$

$\therefore \quad l = \frac{P\,\lambda_P}{2}$

$\therefore \quad \lambda_P = \frac{2l}{3}$

Now $V = \sqrt{\frac{T}{m}}$

$\therefore \quad n_P = \frac{v}{\lambda_P}$

$$n_P = \frac{P}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore \quad n_P = Pn$$

It is P^{th} harmonic and $(P-1)^{th}$ overtone.

Q.26 State the laws of vibrating string.

Ans: The fundamental frequency of a vibrating string when the string is plucked perpendicular to its length is given by,

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

where, T - tension in the string, and

m - mass per unit length of the string.

Laws of vibrating string :

i. $n \propto 1/l$, if T and m are constant.
The frequency of vibrating string is inversely proportional to its vibrating length, if its tension and mass per unit length are constant.

ii. $n \propto \sqrt{T}$, if l and m are constant.
The frequency of vibrating string is directly proportional to the square root of its tension, if its vibrating length and mass per unit length are constant.

iii. $n \propto 1/\sqrt{m}$, if l and T are constant.
The frequency of a vibrating string is inversely proportional to the square root of its mass per unit length, if its vibrating length and tension are constant.

iii. Now $m = \pi r^2 \rho$

$$\therefore \quad n \; \alpha \; \frac{1}{\sqrt{\pi r^2 \rho}}$$

$$\therefore \quad n \; \alpha \; \frac{1}{r\sqrt{\pi\rho}}$$

If r is constant $n \; \alpha \; \frac{1}{\sqrt{\rho}}$

If ρ is constant $n \; \alpha \; \frac{1}{r}$

Thus the fundamental frequency of vibration of stretchel string is inversely proportional to

i. radius of the string and

ii. Square root of density of the material of vibrating string.

Type - IV

Numerical based on vibration of string

Formulae used

1. $V = \sqrt{\frac{T}{m}}$

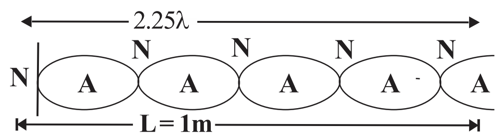
2. $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

3. $m = \frac{m}{l} = \pi r^2 \rho$

★ **1) A string 1 m long is fixed at one end. The other end is moved up and down with frequency 15 Hz. Due to this, a stationary wave with four complete loops, gets produced on the string. Find the speed of the progressive wave which produces the stationary wave. [Condition: The moving end is an antinode.]**

Data : L = 1m, n = 15 Hz,

$$P = 4 + \frac{1}{2} = \frac{9}{2}$$



To Find : v

Formula: i. $L = \frac{P\lambda}{2}$

ii. $v = n\lambda$

Solution:

i. $L = \frac{9}{4}\lambda$

$\therefore \quad \lambda = \frac{1 \times 4}{9} m = \frac{4}{9} m$

ii. $v = n\lambda$

$\therefore \quad v = n \times \lambda = 15 \times \frac{4}{9} = \frac{20}{3} = 6.67 ms^{-1}$

Ans : The velocity of the progressive wave is 6.67 ms^{-1}

★ **2) The string of a guitar is 80 cm long and has a fundamental frequency of 112 Hz. If**

a guitarist wishes to produce a frequency of 160 Hz, where should the person press the string?

Data: $L_1 = 80$ cm $= 0.8$ m, $n_1 = 112$ Hz, $n_2 = 160$ Hz,

To find: L_2

Formula: $n = \frac{1}{2L}\sqrt{\frac{T}{m}}$

Solution:

$$n_1 = \frac{1}{2L_1}\sqrt{\frac{T}{m}} \quad ....(1)$$

$$n_2 = \frac{1}{2L_2}\sqrt{\frac{T}{m}} \quad ....(2)$$

Dividing (1) by (2) we get,

$$\frac{n_1}{n_2} = \frac{L_2}{L_1}$$

$$\therefore \frac{112}{160} = \frac{L_2}{0.8}$$

$$\therefore L_2 = \frac{112 \times 0.8}{160} = \frac{89.6}{160} = \frac{896}{1600} = \frac{14}{25}$$

$= 0.56$ m $= 56$ cm

Ans : To produce 160 Hz, preson should press string at 56 cm.

★ **3) The velocity of a transverse wave on a string of length of 0.5 m is 225 m/s.**

i. What is the fundamental frequency of a standing wave on this string if both ends are kept fixed?

ii. While this string is vibrating in the fundamental harmonic, what is the wavelength of sound produced in air if the velocity of sound in air is 330 m/s?

Data: $L = 0.5$ m, $v = 225$ m/s, $v_s = 330$ m/s

To find: i. n ii. λ_s

Formulae: $v = n\lambda$

Solution:

i. As string is vibrating in fundamental mode

$\therefore L = \frac{\lambda}{2}$ i.e distance between two nodes

$\therefore \lambda = 2L$

Now, $v = n\lambda$

$$n = \frac{v}{\lambda} = \frac{v}{2L}$$

$$n = \frac{225}{2 \times 0.5} = 225\text{Hz}$$

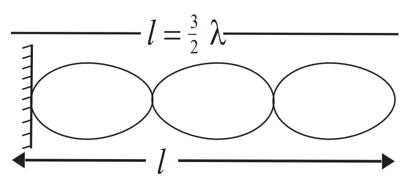
ii. For sound in air

$v_s = n\lambda_s$

$$\lambda = \frac{v_s}{n} = \frac{330}{225} = 1.497\text{m}$$

Ans : i. Fundamental frequency of string is 225 Hz
ii. Wavelength of sound produced is 1.467 m

4) A string 105 cm long is fixed at one end. Transverse vibrations of frequency 15 Hz are imposed at the free end. A stationary wave, produced in the string, consists of 3 loops. Calculate the speed of progressive waves which have produced the staionary waves in the string.



Data: $l = 105$ cm, $P = 3$

To Find : V

Formula: i. $l = \frac{P\lambda}{2}$ ii. $v = n\lambda$

Solution :

i. $l = 3\frac{\lambda}{2}$

$$\therefore \lambda = \frac{2}{3}l = \frac{2}{3} \times 105 = 70\text{cm} = 0.7\text{m}$$

ii. $v = n\lambda = 15 \times 0.7 = 10.5\text{ms}^{-1}$

Ans : The speed of the progressive waves is 10.5 ms^{-1}

★ **5) A violin string virates with fundamental frequency of 440 Hz. What are the frequency of first and second overtones?**

Data: $n = 440$ Hz, $n_1 = 2n$

To find: Frequency of first (n_1) and second (n_2)over tones.

Formula: $n_p = (p + 1)n$

Solution: $n_p = (p + 1)n_0$

$n_1 = 2 \times 440 = 880$ Hz

$n_2 = 3n$

$n_2 = 3 \times 440 = 1320$ Hz

Ans : The Frequencies of first and second overtones are 880Hz and 1320 Hz respectively.

Problem for Practice

1. A transverse wave is produced on a stretched string 0.9 m long and fixed at its ends. Find the speed of the transverse wave, when the string vibrates while emitting second overtone of frequency 324 Hz.

Ans:291.6 m/s

2. What should be tension applied to a wire of length 1m and mass 10 gram. if it has to vibrate with fundamental frequency of 50 Hz?

Ans : 100N

3. A metal wire of linear mass density of 9.8 g m^{-1} is stretched with a tension of 10kg (ωt) between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a parmanent magnet and it vibrates in resonance, when carrying an alternating current of frequency v. Find the frequency of the alternating source.

Ans : 50 Hz

4. A string vibrates with a frequency of 200 Hz. Its length is doubled and its tension is altered until it begins to vibrate with a frequency of 300 Hz. What is the ratio of new tension to the original tension?

Ans : 9:1

6.8 Sonometer

Q.27 Descrie Sonometer in detail.
OR
Explain construction and working of sonometer.

Ans : Construction :

i. A sonometer consists of a hollow rectangular wooden box called the sound box.

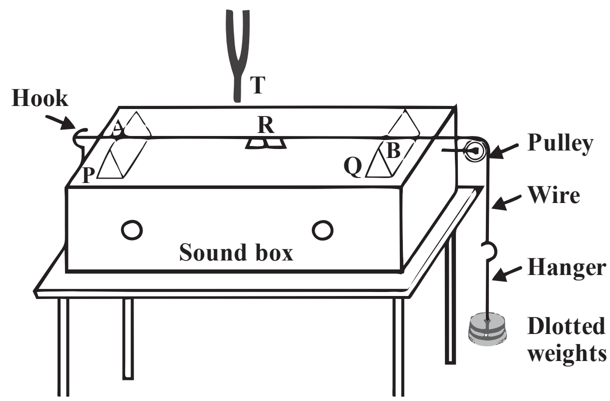
ii. The sound box is used to make a larger mass of air vibrate so that the sound produced by the vibrating string (metal wire in this case) gets amplified.

iii. There are two bridges P and Q along the width of the box which can be moved parallel to the length of box.

iv. A metal wire of uniform cross-section runs along the length of the box over the bridges. It is fixed at one end and its other end passes over a pulley.

v. A hanger with suitable slotted weights can be attached to the free end of wire. By changing the weights, the tension in the wire can be varied.

vi. The movable bridges allow us to change the vibrating length AB of the wire.



Working :

i. If the wire is plucked at a point midway between the bridges, transverse waves are produced in the wire.

ii. Stationary waves are produced between the two bridges due to reflection of transverse wave at the bridges and their superposition. Thus portion AB of the wire between the two bridges P and Q is the vibrating length.

iii. Wire can also be made to vibrate by holding a vibrating tuning fork near it.The frequency of vibration is then same as that of the tuning fork.

iv. If this frequency happens to be one of the natural frequencies of the wire, standing waves with large amplitude are set up in the wire since the two vibrate in resonance.

v. To identify the resonance, a small piece of paper, known as the rider R, is placed over the wire at a point in the middle of the length AB as determined by the position of the bridges P and Q.

vi. If the frequency of the tuning fork and of the fundamental mode of vibration of the wire match (this is achieved by adjusting the length AB of wire using the bridges P and Q), the paper rider happens to be at the antinode and flies off the wire.

Q.28 Law of length of vibratins string explain how can be verified using sonometer.

Ans : Verification of first law of a vibrating string:

i. By measuring length of wire and its mass, the mass per unit length (m) of wire is determined.

ii. Then the wire is stretched on the sonometer and the hanger is suspended from its free end. A suitable tension (T) is applied to the wire by placing slotted weights on the hanger.

iii. The length of wire (l_1) vibrating with the same frequency (n_1) as that of the tuning fork is determined as follows.

a. A light paper rider is placed on the wire midway between the bridges.

b. The tuning fork is set into vibrations by striking on a rubber pad. The stem of tuning fork is held in contact with the sonometer box.

c. By changing distance between the bridges without disturbing paper rider, frequency of vibrations of wire is changed.

d. When the frequency of vibrations of wire becomes exactly equal to the frequency of tuning fork, the wire vibrates with maximum amplitude and the paper rider is thrown off.

iv. In this way a set of tuning forks having different frequencies n_1, n_2, n_3, …… are used and corresponding vibrating lengths of wire are noted as l_1, l_2, l_3 …. by keeping the tension T constant .

v. We will observe that $n_1 l_1 = n_2 l_2 = n_3 l_3 = \ldots\ldots =$ constant, for constant value of tension (T) and mass per unit length (m).

$\therefore$ nl = constant

i.e., $n \propto \frac{1}{l}$ if T and m are constant.

Q.29 State law of tension of a vibrating string and explain how it can be varified using sonometer

Ans : Verification of second law of a vibrating string:

i. The vibrating length (l) of the given wire of mass per unit length (m) is kept constant for verification of second law.

ii. By changing the tension the same length is made to vibrate in unison with different tuning forks of various frequencies. If tensions T_1, T_2, T_3…..... correspond to frequencies n_1, n_2, n_3 ……etc. we will observe that.

$$\frac{n_1}{\sqrt{T_1}} = \frac{n_2}{\sqrt{T_2}} = \frac{n_3}{\sqrt{T_3}} \ldots\ldots = \text{constant}$$

or $\frac{n}{\sqrt{T}}$ = constant

iii $\therefore n \propto \sqrt{T}$ if l and m are constant. This is the second law of a vibrating string.

Q.30 Explain how law of linear density is varieties using sonometer.

Ans : of a vibrating string:

i. For verification of third law of a vibrating string, two wires having different masses per unit lengths m_1 and m_2 (linear densities) are used.

ii. The first wire is subjected to suitable tension and made to vibrate in unison with given tuning fork. The vibrating length is noted as (l_1).

iii. Using the same fork, the second wire is made to vibrate under the same tension and the vibrating length (l_2) is determined.

iv. Thus the frequency of vibration of the two wires is kept same under same applied tension T. It is found that,

$l_1\sqrt{m_1} = l_2\sqrt{m_2}$

$l\sqrt{m} = \text{constant}$

But by first law of a vibrating string, $n \propto \frac{1}{l}$

Therefore we get that, $n \propto \frac{1}{\sqrt{m}}$, if T and l are constant. This is the third law of vibrating string.

Type - V
Numerical based on Sonometer

Formulae used

1. $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

2. $n \propto \frac{1}{l}$ (law of length)

3. $n \propto \sqrt{T}$ (Law of tension)

4. $n \propto \frac{1}{\sqrt{m}}$ (law of linear density

5. $m = \frac{M}{l} = A\rho = \pi r^2\rho$

1) A sonometer wire of length 50 cm is streched by keeping weights equivalent of 3.5 kg. The fundamental frequency of vibration is 125 Hz. Determine the linear density of the wire.

Data: l = 50 cm = 0.5 m, M = 3.5 kg,
n = 125Hz, T = 3.5 kg - ωt
= 3.5 × 9.8 = 34.3N

To find: m

Formula: $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

T = Mg = 3.5 × 9.8 = 34.3 N

From Formula,

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Squaring both sides, we get

$$\therefore \quad n^2 = \frac{1}{4l^2}\frac{T}{m}$$

Solution : $m = \frac{T}{4n^2l^2}$

$$\therefore \quad m = \frac{34.3}{4\times(125)^2\times(0.5)^2} = \frac{34.3}{4\times(125)^2\times\left(\frac{1}{2}\right)^2}$$

$$= \frac{34.3\times\cancel{4}^{1}}{\cancel{4}_{1}\times125\times125} = 2.1952\times10^{-3}\ \text{kgm}^{-1}$$

Ans : The linear density of the wire is 2.1952×10^{-3} kgm^{-1}.

★ **2) Two wires of the same material and the same cross section are stretched on a sonometer in succession. Length of one wire is 60 cm and that of that of the other is 30 cm. An unknown load is applied to the first wire wire and second wire is loaded with 1.5 kg. If both the wires vibrate with the same fundamental frequencies, calculate the unknown load.**

Data:

	Wire - 1	**Wire - 2**
Density cross section area	$\rho_1 = \rho$ $A_1 = A$	$\rho_2 = \rho$ $A_2 = A$
linear density	$m_1 = m$	$m_2 = m$
length	l_1 = 60 cm	l_2 = 30 cm
Tension	$T_1 = T$	T_2 = 1.5 kg
Fundamental frequency	$n_1 = n$	$n_2 = n$

To find: T_1

Formula: $n = \frac{1}{2L}\sqrt{\frac{T}{m}}$

Solution:

$$n_1 = \frac{1}{2l_1}\sqrt{\frac{T_1}{m_1}} \quad ...(1)$$

$$n_2 = \frac{1}{2l_2}\sqrt{\frac{T_2}{m_2}} \quad ...(2)$$

Dividing (1) by (2),

$\therefore \quad \frac{n_1}{n_2} = \frac{l_2}{l_1}\sqrt{\frac{T_1 \times m_2}{T_2 \times m_1}}$

$\therefore \quad \frac{n}{n} = \frac{0.3}{0.6}\sqrt{\frac{T_1 \times m}{1.5 \times g \times m}}$

$\therefore \quad 1 = \frac{1}{2} = \sqrt{\frac{T_1}{1.5g}}$

$\therefore \quad 2 = \frac{1}{2} = \sqrt{\frac{T_1}{1.5 \times g}}$

$\therefore \quad 4 = \frac{T_1}{1.5 \times g}$

$\therefore \quad T_1 = 6 \times g$ N

Applied load = 6kg.

Ans : The unknown load is 6 kg.

★ **3) A wire has linear density 4.0 × 10⁻³ kg/m. It is streched between two rigid supports with a tension of 360 N.The wire resonates at a frequency of 420 Hz and 490 Hz in two succes sive modes. Gind the length of the wire.**

Data: $m = 4.0 \times 10^{-3}$ kg/m, T = 360 N, $n_p = 420$ Hz, $n_{p+1} = 490$ Hz

To find: l

Formula: $n_p = \frac{P}{2l}\sqrt{\frac{T}{m}}$

Solution:

i. $420 = \frac{P}{2l}\sqrt{\frac{T}{m}}$ . ...(1)

$490 = \frac{P+1}{2l}\sqrt{\frac{T}{m}}$ ...(2)

Dividing (1) by (2), we get

$\frac{42}{49} = \frac{P}{p+1}$

$\therefore \quad 49\,p - 42\,p = 42$

$\therefore \quad p = \frac{42}{7} = 6$

ii. $n_p = \frac{p}{2l}\sqrt{\frac{T}{m}}$

$420 = \frac{6}{2L}\sqrt{\frac{360}{4 \times 10^{-3}}}$

$L = \frac{6}{2 \times 420}\sqrt{\frac{36 \times 10^4}{4}}$

$= \frac{6 \times 6 \times 100}{2 \times 420 \times 2} = \frac{900}{420} = 2.14\text{m}$

Ans : The length of the string 2.14 m .

★ **4) Two wires of the same material and same cros section are stretched on a sonometre. One wire is loaded with 1.5 kg and another is loaded with 6 kg. The vibrating length of first wire is 60cm and its fundamental frequency of vibration is the same as that of the second wire. Calculate vibrating length of the other wire.**

Data:

		Wire - 1	Wire - 2
i.	Density	$\rho_1 = \rho$	$\rho_2 = \rho$
ii.	Area	$A_1 = A$	$A_2 = A$
iii.	Tension	$T_1 = 1.5$ kg-ωt	$T_2 = 6$ kg - ωt
iv.	Fundamental frequency	$n_1 = n$	$n_2 = n$
v.	Length	$L_1 = 60$ cm	L_2
vi.	linear density	$m_1 = m$	$m_2 = m$

To find: L_2

Formula: $n = \frac{1}{2L}\sqrt{\frac{T}{m}}$

Solution:

$n_1 = n = \frac{1}{2L_1}\sqrt{\frac{T_1}{m_1}}$ ...(1)

$n_2 = n = \frac{1}{2L_2}\sqrt{\frac{T_2}{m_2}}$ ...(2)

$\because$ From (1) and (2)

$$\therefore \quad \frac{1}{2L_1}\sqrt{\frac{T_1}{m_1}} = \frac{1}{2L_2}\sqrt{\frac{T_2}{m_2}}$$

$$\therefore \quad \frac{1}{4L_1^2}\frac{T_1}{m_1} = \frac{1}{4L_2^2}\frac{T_2}{m_2}$$

$$\therefore \quad L_2^2 = \frac{T_2}{T_1}\times L_1^2 = L^2 = \sqrt{\frac{T_2}{T_1}\times(L_1)^2}$$

$$= \sqrt{1.5g\times\frac{6g}{(0.6)^2}} = 2\times 0.6 = 1.2 \text{ m}$$

Ans : The vibrating length of the wire is 1.2 m.

Problem For Practice

1. A 36 cm long sonometer wire vibrates with frequency of 280 Hz in fundamental mode, when it is under tension of 24.5 N. Calculate linear density of the material of wire

Ans : 0.168 kg/m

2. The length of a sonometer wire is 0.75 m and density 9×10^3 kg m^{-3}. If can bear a strees of 8.1×10^8 N.m^2 withoput exceeding the elastic limit. What is the fundamental frequency that can be produced in the wire ?

Ans : 200 Hz

3. A sonometer wire is under a tension of 40 N and the length between the bridges is 50 cm. A metre long wire of the sonometer has a mass of 1.0 g. Determine its foundamental frequency.

Ans : 200 Hz

4. A sonometer wire has a length of 114 cm between its two fixed ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1:3:4?

Ans : At a distance of 72 cm, 96 cm from one end

6.9 Beats

Q.31 Define the following terms:
i. Beat, ii. Waxing, iii. Beat frequency iv. One beat, v. Period of a beat vi. Waning

Ans:

i. **Beat :**
The production of alternate waxing (maximum sound) and waning (minimum sound) due to superposition of two sound waves of same amplitude but slight difference in frequency are propagating along same path and direction known as phenomena of beats.

ii. **Waxing :**
The maximum of sound is called waxing (i.e., resultant amplitude should be maximum).

iii. **Beat frequency :**
The number of beats heard per second is called the frequency of beats or beat frequency.

iv. **One beat :**
One waxing and one waning form one beat.

v. **Period of beats :**
The time interval between two successive waxings (or wanings) is called the period of beats.

vi. **Waning :**
The minimum of sound is called waning (i.e., resulting amplitude should be minimum).

Q.32 Obtain an expression for the beat frequency by analytical method by using:
i. waxing ii. waning.

Ans:

i. Let a be the amplitude and n_1,n_2 be the frequencies of the two waves such that $n_1 > n_2$.
The displacement of a particle due to the two waves arriving at that point are :

$y_1 = a \cdot \sin2\pi n_1 t$ ..... (1)

$y_2 = a \cdot \sin2\pi n_2 t$ ..... (2)

ii. The resultant displacement is given by the principle of superposition of waves.

$y = y_1 + y_2$

$= a\cdot\sin2\pi n_1 t + a\cdot\sin2\pi n_2 t$

Using the trigonometric relation,

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$y = 2a \sin\left[2\pi\left(\frac{n_1+n_2}{2}\right)t\right] \cos\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right]$$

$\therefore$ $$y = R\sin\left[2\pi\left(\frac{n_1+n_2}{2}\right)t\right] \quad .....(iii)$$

where, $R = 2a\cos\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right]$

iii. Equation (iii) shows that the resultant motion is also a SHM with frequency $\left(\frac{n_1+n_2}{2}\right)$ and an amplitude R.

iv. If we consider the superposition of the two waves in the medium, we find that at a given instant there are points with maximum displacement (waxing) and minimum displacement (waning) alternately occurring in the medium. Also, at a given point the amplitude varies with time, passing successively through a maximum of +2a, zero, and then to a minimum of –2a.

Waxing : The resultant intensity of sound will be maximum when the amplitude is given by

$R = \pm 2a$

i.e., $a.\cos\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right] = \pm 2a$

i.e., $\cos\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right] = \pm 1$

i.e., $\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right] = 0, \pi, 2\pi, 3\pi.......$

i.e., when t = 0,

$$\frac{1}{n_1-n_2}, \frac{2}{n_1-n_2}, \frac{3}{n_1-n_2}, ...$$

Now,

a. Period of beats = time interval between two successive waxings.

$$= \frac{2}{n_1-n_2} - \frac{1}{n_1-n_2} = \frac{1}{n_1-n_2}$$

b. Beat frequency = $\frac{1}{\text{period of beats}}$

$$= \frac{1}{1/(n_1-n_2)}$$

Thus, beat frequency = $(n_1 - n_2)$

Waning : The resultant intensity of sound will be minimum, when the amplitude is zero.

$\therefore$ R = 0

i.e., $2a\cos\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right] = 0$

i.e., $\cos\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right] = 0$

i.e., $2\pi\left(\frac{n_1-n_2}{2}\right)t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, ...$

$\therefore$ $$t = \frac{1}{2(n_1-n_2)}, \frac{3}{2(n_1-n_2)}, \frac{5}{2(n_1-n_2)}, ...$$

v. Now, period of beats = time intervals between two successive wanings.

$$= \frac{5}{2(n_1-n_2)} - \frac{3}{2(n_1-n_2)} = \frac{1}{(n_1-n_2)}$$

vi. Beat frequency = $\frac{1}{\text{period of beats}}$

Thus, beat frequency = $(n_1 - n_2)$

Thus, beat frequency is the difference between the frequencies of the two sound waves.

Remark :

The intensity of sound at the point of interference is, $I_{max} \propto 4a^2$ and $I_{min} = 0$

If the amplitudes a_1 and a_2 are not equal then $I_{max} \propto (a_1+a_2)^2$ and $I_{min} \propto (a_1-a_2)^2$

Q.33 Explain the application of beats to determine unknown frequency.

Ans :

i. Unknown frequency of a sound note can be determined by using the phenomenon of beats.

ii Initially the sound notes of known and unknown frequency are heard simultaneously.

iii. The known frequency from a source of adjustable frequency is adjusted in such a way that the beat frequency reduces to zero.

iv. At this stage frequencies of both the sound notes become equal. Hence unknown frequency can be determined.

Type VI
Numerical based on Beats

Formulae used

If $n_1 > n_2$

Beat frequency = $n_1 = n_2$

1) Two sound waves having wavelength 81 cm and 82.5 cm produce 8 beats per second. Calculate the speed of sound in air.

Data: $\lambda_1 = 81cm$

$\lambda_2 = 82.5cm$

As $\lambda_2 > \lambda_1$ $\quad \therefore n_1 > n_2$

$\therefore$ $n_1 - n_2 = 8$ Hz

To find: v

Formula: $v = n\lambda$

$\therefore$ $n = \frac{v}{\lambda}$

Solution:

i. $n_1 - n_2 = 8$ Hz

$$= \frac{v}{\lambda_1} - \frac{v}{\lambda_2} = 8 \quad \left(\because n = \frac{v}{\lambda}\right)$$

$$v\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = 8$$

$$v\left[\frac{1}{81} - \frac{1}{82.5}\right] = 8$$

$$v\left[\frac{82.5 - 81}{81 \times 82.5}\right] = 8$$

$$v \times \frac{\cancel{1.5}^{1}}{81 \times \cancel{82.5}_{55}} = 8$$

v = 8 × 81 × 55 = 35640 cm/s

= 356.4 m/s

Ans : The speed of sound in air is 356.4m / s.

2) Wavelength of two sound waves in air are $\frac{66}{153}$ m and $\frac{66}{151}$ m . If these produce 5 beats per second with third note of fixed frequency, find the velocity of sound in air and frequency of third note.

Data : $\lambda_1 = \frac{66}{153}m$

$\lambda_2 = \frac{66}{151}m$

Beat frequency = 5 with third note of frequency N

To find : i. v, ii. N

Formula : $v = n\lambda$

Solution :

As $\lambda_1 - \lambda_2$

$n_1 > n_2$

Let N be the frequency of third note

$n_1 > N > n_2$

$n_1 - N = 5$ ...(1)

$N - n_2 = 5$ ...(2)

Assing equation (1) and (2)

$n_1 - n_2 = 10$ ... (3)

$$\frac{v}{\lambda_1} - \frac{v}{\lambda_2} = 10 \quad ...\left(\because n = \frac{v}{\lambda}\right)$$

$$v\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = 10$$

$$v\left[\frac{153}{66} - \frac{151}{66}\right] = 10$$

$$v \times \frac{\cancel{2}^{1}}{\cancel{66}_{33}} = 10$$

v = 330m/s

ii. we know that

$$n_1 = \frac{v}{\lambda_1}$$

$$n_1 = \frac{330}{66} \times 153 = 5 \times 153 = 765 \text{ Hz}$$

Substituting n1 = 765 Hz in eq (1)

iii. $n_1 - N = 5$

765 – N = 5

N = 760 Hz

Ans :	Velocity of sound in air is 330 m/s and frequency of third note is 760 Hz

★ 2) Two tuning foks having frequencies 320 Hz and 340 Hz are sounded together to produce sound waves.The velocity of sound in air is 326.4 ms^{-1}. Find the difference in wavelength of these waves.

Data: $n_1 = 320$ Hz, $n_2 = 340$ Hz,
$v = 326.4\ ms^{-1}$

To find: $|\lambda_1 - \lambda_2|$

Formula: $\lambda = \frac{v}{n}$

Soluation:

$\because$ $n_1 < n_2$

$\therefore$ $\lambda_1 > \lambda_2$

Difference in wavelength,

$\therefore$ $\lambda_1 - \lambda_2 = \frac{v}{n_1} - \frac{v}{n_2}$

$\therefore$ $\lambda_1 - \lambda_2 = v\left[\frac{1}{n_1} - \frac{1}{n_2}\right]$

$\therefore$ $\lambda_1 - \lambda_2 = 326.4\left[\frac{1}{320} - \frac{1}{340}\right]$

$\therefore$ $\lambda_1 - \lambda_2 = 326.4\left[\frac{340-320}{320\times 340}\right]$

$= \frac{6528}{320 \times 340}$

$= \frac{192}{320\times 10} = \frac{6}{100} = 0.06$ m

Ans : The difference in wavelength is 0.06m.

4) **A tuning fork P produces 6 beats per second with a tuning fork Q of frequency 288 Hz. when prongs of P are loaded with a little wax, the number of beats heard per second reduces to find frequency of P**

Solution :

i. Before putting wax

$n_Q = 288$ Hz

$B = 6$

$n_p = n_Q \pm B$

$n_p = 288 \pm 6$

$n_p = 294$ Hz OR np = 282 Hz

ii. On loading wax

a. Frequency of P decreases and

b. Number of beats decreases from 6 to 4

This condition is satisfied only when frequency of P is 294 Hz

$\therefore$ $n_p = 294$Hz

Ans : Frequency of tuning fork P is 294 Hz

5) **A sonometer wire of length 0.5m is stretched by a weight of 5 kg. The fundamental frequency of vibration is 100 Hz. Calculate linear density of wire.**

Data: $L = 0.5$ m, $M = 5$kg, $n_0 = 100$Hz

To find: Linear density of wire (m)

Formulae: i. $T = Mg$

ii. $n_0 = \frac{1}{2L}\sqrt{\frac{T}{m}}$

Solution: $T = Mg$

$T = 5 \times 9.8 = 49$ N

From formula (ii),

$100 = \frac{1}{2\times 0.5}\sqrt{\frac{49}{m}}$

Squaring both side, we get

$10^4 = \frac{49}{m}$

$\therefore$ $m = \frac{49}{10^4} = 4.9 \times 10^{-3}$ kg/m

Ans : The linear density of the wire is 4.9×10^{-3} kg / m

6) **A set of 8 tuning forks is arranged in a series of increasing order of frequencies. Each fork gives 4 beats per second with the next one and the Frequencyof last fork is twice that of the first. Calculate the frequencies of the first and the last fork.**

Solution:

Let $n_1, n_2, n_3, \ldots\ldots\ldots, n_8$ be the frequency of Eight tuning fork arranged in increasing order.

$\therefore$ $n_2 > n_1$

$\therefore$ $n_2 - n_1 = 4$

$\therefore$ $n_2 = n_1 + 4$

$\therefore$ $n_2 = n_1 + 4(2 - 1)$

Similarly

$n_3 = n_1 + 4(3 - 1)$

:

:

:

$n_8 = n_1 + 4(8 - 1)$

$= n_1 + 4 \times 7$

$n_2 = n_1 + 28$ ....(1)

According to Given condition

$\therefore$ $n_8 = 2n_1$

$n_1 + 28 = 2n_1$

- $n_1 = 28$ Hz

From 1,

$n_8 = n_1 + 28 = 28 + 28 = 56$Hz

Ans : The frequencies of first and eight fork are 28 Hz and 56 Hz respectively.

7) 32 tuning forks are arranged in descending order of frequencies. If any two consecutive tuning forks are sounded together the number of beats heared is eight per second. The frequency of the first tuning fork is octave that of the last fork. Calculate the frequency of the first, last and the 21st fork.

Solution :

Let n_1, n_2, n_3,..........., n_{21}.........., n_{32} be the frequency of 32 tuning fork arranged in decreasing order.

$\therefore$ $n_1 > n_2$

$\therefore$ $n_1 - n_2 = 8$

$\therefore$ $n_2 = n_1 - 8$

$\therefore$ $n_2 = n_1 - 8 \times 1$

$\therefore$ $n_2 = n_1 - 8(2 - 1)$

Similarly

$n_3 = n_1 - 8(3 - 1)$

:

:

:

$n_{21} = n_1 - 8(21 - 1)$

$n_{21} = n_1 - 8 \times 20 = n_1 - 160$ ....(1)

:

:

:

$n_{32} = n_1 - 8(32 - 1)$

$n_{32} = n_1 - 8 \times 31 = n_1 - 248$ ....(2)

According to Given condition

$\therefore$ $n_1 = 2n_{32}$

$\therefore$ $n_1 = 2(n_1 - 248)$ ....(From 2)

$\therefore$ $n_1 = 2n_1 - 496$

$\therefore$ $n_1 = 496$ Hz

From 1,

$n_{21} = n_1 - 160 = 496 - 160 = 336$ Hz

From 2,

$n_{32} = n_1 - 248 = 496 - 248 = 248$ Hz

Ans:	Frequency of 1st tuning fork A is 496 Hz
	Frequency of 21st tuning fork A is 336 Hz
	Frequency of last tuning fork A is 248 Hz

8) A sonometer wire is stretched by tension of 40 N.It vibrates in unison with a tuning fork of frequency 384 Hz. How many numbers of beats get produced in two seconds if the tension in the wire is decreased by 1.24 N?

Solution:

Let T_1 be the initial tension in string and n_1 be the corresponding frequency of string which is unison with tuning fork

$T_1 = 40$ N, $n_1 = 384$ Hz

Let T_2 be the final tension in String and n_2 be the corresponding frequency.

$T_2 = 40 - 1.24 = 38 - 76$ N

Now

$$n_1 = \frac{1}{2l}\sqrt{\frac{T_1}{m}} \quad ....(1)$$

$$n_2 = \frac{1}{2l}\sqrt{\frac{T_2}{m}} \quad ....(2)$$

Dividing 1 by 2

$$\therefore \quad \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\therefore \quad \frac{384}{n_2} = \sqrt{\frac{40}{38.76}}$$

$$\frac{384}{n_2} = \sqrt{1.031}$$

$$\frac{384}{n_2} = 1.015$$

$$n_2 = \frac{384}{1.015} = 378.32 \approx 378$$

Beat frequency = $n_1 - n_2 = 384 - 378 = 6$

Number of beats heard in 2 second = 6 × 2
= 12

Ans : Number of beat produced in two seconds is 12 Hz.

9) A sonometer wire 60cm long produces a resonance with a tuning fork. When its length is decreased by 5cm, to beats per seconds are heard. Find the frequency of tuning fork.

Solution:

initial length of sonometer wire (l_1) = 60cm

Let n_1 be the corresponding frequency of wire which is resonance with tuning fork

Final length of sonometer wire (l_2)

= 60 – 5 = 55 cm

Let n_2 be the corresponding frequency of wire.

Now $l_1 > l_2$

$\therefore$ $n_2 > n_1$ .... $\left(\because n \propto \frac{1}{l}\right)$

$\therefore$ $n_2 - n_1 = 10$

By using law of length,

$$n \propto \frac{1}{l}$$

$$\therefore \frac{n_2}{n_1} = \frac{l_1}{l_2} = \frac{60}{55} = \frac{12}{11}$$

$$\therefore \frac{n_2 - n_1}{n_1} = \frac{12-11}{11}$$

$$\frac{10}{n_1} = \frac{1}{11}$$

$\therefore$ $n_1 = 110$ Hz

Ans : The frequency of tuning fork is 110 Hz.

10) When a sonometer wire and tuning fork are sounded together they produces 5 beat per second when length of wire is either 90 cm or 95 cm. Determine frequency of the fork

Solution:

$l_1 = 90$cm

Let n_1 be the corresponding frequency of wire

$l_2 = 95$ cm

Let n_2 be the corresponding frequency of wire

$l_2 > l_1$

$\therefore$ $n_1 > n_2$ .... $\left(\because n \propto \frac{1}{l}\right)$

Let N be the frequency of tuning fork

$\therefore$ $n_1 > N > n_2$

$n_1 - N = 5$ ....(1)

$N - n_2 = 5$ ....(2)

Adding (1) and (2)

$n_1 - n_2 = 10$ ....(3)

Using law of length,

$$n \propto \frac{1}{l}$$

$$\therefore \frac{n_1}{n_2} = \frac{l_2}{l_1} = \frac{\cancel{95}^{19}}{\cancel{90}_{18}} = \frac{19}{18}$$

$$\frac{n_1 - n_2}{n_2} = \frac{19-18}{18}$$

$$\frac{10}{n_2} = \frac{1}{18}$$

$\therefore$ $n_2 = 180$ Hz

From 2,

$N - n_2 = 5$

$N - 180 = 5$

$N = 185$ Hz

Ans : Frequency of tuning fork is 185 Hz.

11) A sonometer wire is in unision with a tuning fork when streched by a weight of specific gravity 'nine' on completely immersing the weight in water, wire produces 4 beats per second with the fork. Calculate the frequency of the fork

Solution:

Let T_1 be the tension in string when weight is suspended in air

Let n_1 be the corresponding frequency of wire which is unison with tuning fork

Let T_2 be the tension in string when weight is immersed in water

Let n_2 be the corresponding frequency of wire.

$$\text{Specific gravity} = \frac{\text{Wt in air}}{\text{loss of weight in water}}$$

$\therefore \quad 9 = \frac{T_1}{T_1 - T_2}$

$\therefore \quad 9T_1 - 9T_2 = T_1$

$8T_1 = 9\ T_2$

$\frac{T_1}{T_2} = \frac{9}{8}$ ....(1)

As $T_1 > T_2$

$\therefore \quad n_1 > n_2$ ....$\left[\because n \ \alpha \sqrt{T}\right]$

$\therefore \quad n_1 - n_2 = 4$ ....(2)

Using law of tension

$n \ \alpha \sqrt{T}$

$\frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{9}{8}}$ ....(From 1)

$\therefore \quad \frac{n_1}{n_2} = \frac{3}{2\sqrt{2}} = \frac{3}{2 \times 1.414}$

$\frac{n_1}{n_2} = \frac{3}{2.828}$

$\frac{n_1 - n_2}{n_2} = \frac{3 - 2.828}{2.828}$

$\frac{4}{n_2} = \frac{0.172}{2.828}$

$n_2 = \frac{2.828 \times 4}{0.172}$

$n_2 = 65.76$ Hz

From 2,

$n_1 - n_2 = 4$

$n_1 - 65.76 = 4$

$n_1 = 69.46$ Hz

Ans : The frequency of tuning fork is 69.76 Hz.

Problem For Practice

1. The displacement of particle performing S.H.M. is given by

$x = \left[5 \sin \pi t + 12 \sin\left(\pi t + \frac{\pi}{2}\right)\right]$ cm Determine the amplitude, period and initial phase of the motion.

Ans:

3. Two second notes have wavelengths $\frac{83}{170}$ m and $\frac{83}{172}$ m in air. These notes when sounded together produce 8 beats per second. Calculate the velocity of sound in air and frequencies of two notes.

Ans :

5. A set of 12 tuning forks is arranged in the increasing order of frequencies. Each tuning fork produce 'Y' beats per second with previous one. The last tuning fork is an octave of the first. The fifth tuning fork has the frequency 90 Hz. Find Y and hence find the frequency of the last fork.

Ans :

6. Wavelength of two notes in the air are $\frac{70}{153}$ m and $\frac{70}{157}$ m. Each of these notes produces 8 beats per second with a tuning fork of fixed frequency. Find the velocity of sound in the air and frequency of the tuning fork.

Ans :

7. A sonometer wire is in unision with a tuning fork of frequency 125 Hz when it is stretched by a weight. When the weight is completely immersed in water, 8 beats are heard per second. Find the specific gravity of the material of the weight.

Ans :

8. In a set, 21 tuning forks are arranged in a series of decreasing frequencies. Each tunning fork produces 4 beats per second with the preceding fork. If the first fork is an octave of the last fork, find the frequencies of the first and tenth forks.

Ans :

9. A set of 48 tuning forks is arranges in a series of descending frequencies such that each fork gives 4 beats per second with preceding one. The frequency of first is 1.5 time the frequency of the last fork. find the frequency of the first and 42^{nd} tuning fork.

Ans :

10. The wavelength of two sound waves in air $\frac{81}{173}$ m and $\frac{81}{170}$ m. They produce 10 beats per second. Calculate the velocity of sound in air.

Ans :

11. A stretched sonometer wire is in unison with a tuning fork. When the length of the wire is increased by 5% the number of beats heard per second is 10. Find the frequency of the tuning fork.

Ans :

❑ ❑ ❑