8 Electrostatics

Syllabus

8.1 Introduction

Note:

1. *Electric flux :*

It is the total number of lines of force passing normally through the given area.

i. *The total flux passes through the given surface is given by* $\phi_e = \vec{E}.\vec{S}$

$\therefore \quad \phi_e = ES\cos\theta$

where θ *is the angle made by the normal with the electric field.*

2. *Gauss Law :*

Statement : *The total normal electric flux* ϕ_e *over a closed surface is* $\frac{1}{\varepsilon_0}$ *times the total charge Q enclosed within the surface.*

$$\phi_e = \left(\frac{1}{\varepsilon_0}\right)Q$$

i. *Gauss Law is applicable for any distribution of charges and any type of closed surface, but it is easy to solve the problem of symmetric disribution of charge.*

ii. *The electric field* $\vec{E}$ *appearing in Gauss's theorem is due to all the charges, both inside and outside the closed surface. However, the charge q appearing in the therorem is only contained within the closed surface.*

iii. *The net flux through a closed surface due to a charge lying outside the closed surface is zero.*

iv. *If the net charge enclosed by a closed surface is zero (q = 0), then flux through it is also zero.*

$$\phi_e = \frac{q}{\varepsilon_0} = 0$$

v. *Gauss's theorm is based on the inverse square dependence on distance contained in the coulomb's law. In fact, it is applicable to any field obeying inverse square law.*

vi. *For a medium of absolute premitivity* ε *or dielecteric constant K, the Gauss's theorem can be expressed as*

$$\oint \vec{E}.\vec{dS} = \frac{q}{\varepsilon} = \frac{q}{k\varepsilon_0}$$

3. Gaussian Surface

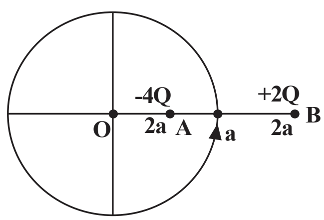
i. Any hypothetical closed surface enclosing a charge is called the Gaussian surface of that charge.

ii. It is chosen to evaluate the surface integral of the electric field produced by the charge enclosed by it, which, in turn, gives the total flux through the surface.

iii. Importance. By a clever choice of Gaussian surface, we can easily find the electric fields produced by certain symmetric charge configurations which are otherwise quite difficult to evaluate by the direct application of Coulomb's law and the principle of superposition.

★ **Q.1 Two charges of magnitudes –4Q and +2Q are located at points (2a, 0) and (5a, 0) respectively. What is the electric flux due to these charges through a sphere of radius 4a with its centre at the origin?**

Ans:



OA = 2a, OB = 5a, Radius, r = 4a

As Gaussian surface is sphere of radius 4a, it have only one charge (– 4Q) inside Gaussian surface.

By Gauss theorem, flux associated with closed surface is

$$\phi = \frac{q}{\varepsilon_0} = \frac{-4Q}{\varepsilon_0}$$

8.2 Application of Gauss's law

Q.2 Derive an expression for electric field intensity at any point due to charged spherical shell

Ans.

i. Consider a sphere of radius R Kept in a medium of permittivity $\varepsilon \left(\varepsilon = \varepsilon_0 k\right)$

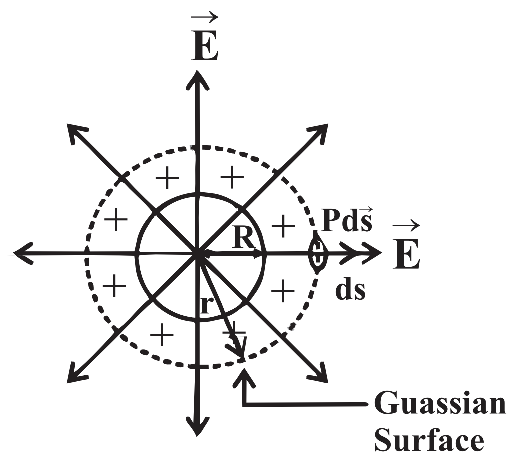
Let q be the total charges on sphere

Let σ be surface charge density

$\therefore \quad \sigma = \dfrac{q}{4\pi R^2}$

$\therefore \quad q = \sigma \times 4\pi R^2$

ii. Let P be any point at a distance r from the centre of sphere we suppose to find electric field at point P.



iii. **<u>Select Gaussian Surface</u>**

Consider imaginary concentric Gausssian sphere of radius r such that point P lies on the surface of sphere

Let ds be the small area around the point P on Gaussian surface

Due to symmetry and sphere being concentric, electric field is same over entire Gaussian sphere.

Electric field is acting outward and normal to the sphere.

$\therefore$ Angle between $\vec{E}$ and $\vec{ds}$ is 0.

iv. **<u>To find flux associated with surface</u>**

$$\phi = \oint \vec{E}.\vec{ds} = \oint E \, ds \cos\theta$$

$$= \oint E \, ds \cos 0 \qquad (\because \theta = 0)$$

$$\phi = E\oint ds$$

$$\phi = E \times 4\pi r^2 \qquad ....(1)$$

v. **Total charges inside Gaussian Surface = q** ....(2)

vi. **Applying Gauss Law.**

$$\phi = \oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0 K}$$

$\therefore$ $E \times 4\pi r^2 = \frac{q}{\varepsilon_0 K}$ (from 1 and 2)

$\therefore$ $\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}}$ ...(3)

This is expression for Electric field intensity at any point due to charged spherical shell.

From equation (3) we can conclude that a uniformly charged sphere is equivalent to point charge at its centre.

vii. **Electric field in terms of σ**

we know that, $q = \sigma \times 4\pi R^2$

Substituting in equation (3)

$\therefore$ $E = \frac{1}{4\pi\varepsilon_0 k} \cdot \frac{\sigma \times 4\pi R^2}{r^2}$

$\therefore$ $\boxed{E = \frac{\sigma R^2}{\varepsilon_0 k r^2}}$

viii. **Case-1:** **If point P lies on the surface of the sphere** , r = R

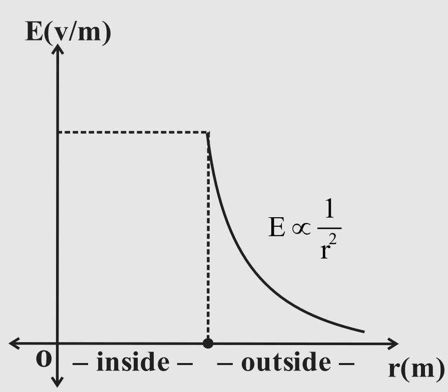
$\therefore$ $\boxed{E = \frac{1}{4\pi\varepsilon_0 k} \cdot \frac{q}{r^2} = \frac{\sigma}{\varepsilon_0 k}}$

So it is clear that electric field is maximum on the surface of sphere

Case-2: **If point P lies inside the sphere**

$\boxed{E = 0}$ $(\therefore \sigma = 0)$

Note



Q.3 Derive an expression for electric field intensity due to an infinitly long. straight charged wire / charged conducting cylinder

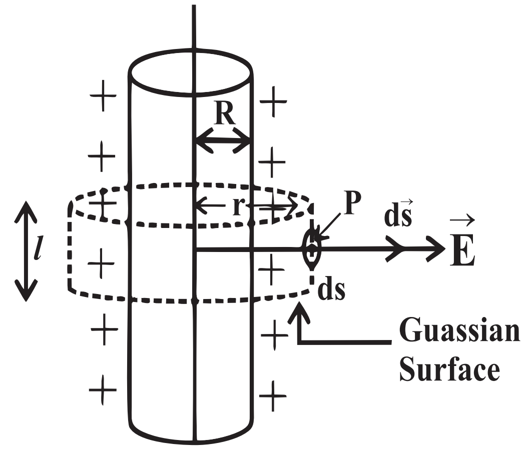
Ans:

i. Consider a uniformly charged wire of infinite length kept in a medium of permittivity $\varepsilon\ (\varepsilon = \varepsilon_0 k)$

Let λ be linear charge density

ii. Let P be any point at a distance r from axis of cylindrical wire.

We suppose to find electic field at point P.



iii. **Select Gaussian surface**

Gaussian surface is an imaginary coaxial cylinder of length l and radius r, such that point P lies on the curved surface of the cylinder.

Consider a very small area ds at point P on the Gaussian surface

By symmetry, the magnitude of electric field is same at all points on curved surface of the cylinder.

Electric field is directed outward and perpendicular to curved surface.

$\therefore$ Angle between $\vec{E}$ and $\vec{ds}$ on curved surface is 0.

Angle between $\vec{E}$ and $\vec{ds}$ of top and bottom cross section is 90°. Therefore no flux is linked with the cross section of cylinder

iv. **To find flux associated with Gaussian Surface**

Flux associated with Gaussian surface is only flux associated with curved surface of cylinder

$\therefore \quad \phi = \oint \vec{E}.\vec{ds} = \oint_{curved} E\, ds \cos\theta$

$= \oint_{curved} E\, ds \cos 0 = E \oint_{curved} ds$

$\phi = E \times 2\pi r l$ ...(1)

v. **Total charges inside Gaussian Surface**

$q = \lambda l$ ...(2)

vi. **According to Gauss law**

$$\phi = \oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0 k}$$

$\therefore \quad E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0 k}$ ...(from 1 and 2)

$$\therefore \quad \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 k r}}$$

This is expression for electric field at any point due to charged conducting wire.

Q.4 Obtain expression for electric field due to an infinite charged plane sheet

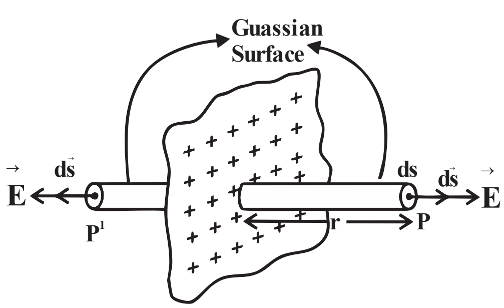
Ans:

i. Consider a uniformly charged plane sheet kept in a medium of permittivity $\varepsilon \left(\varepsilon = \varepsilon_0 k\right)$

Let σ be the surface charged density.

ii. Let P be any point at a distance r from the sheet.

We suppose to find electric field at point P.



iii. **Select Gaussian surface.**

Gaussian surface is an imaginary cylinder of length 2r such that point P lies on the cross section of the cylinder.

Axis of an imaginary cylinder is perpendicular to the sheet. Let sheet passes from middle of the length of the cylinder such that end points P and P' are equidistance from sheet

By symmetry electric field is perpendicular to plane sheet and directed outward having same magnitude at a given distance on either sides of sheet

Electric field is perpendicular to both end cross section and tangential to curved surface of cylinder

So flux is linked only with end cross section and not with curved surface

iv. **To find flux associated with Gaussian Surface**

$\phi = \oint \vec{E}.\vec{ds}$

$= \oint_{P} E\, ds \cos 0 + \oint_{P'} E\, ds \cos 0$

$= \oint_{P} E\, ds + \oint_{P'} E\, ds$

$= EA + EA = 2EA$ ...(1)

v. **Total Charge inside Gaussian Surface**

$q = \sigma A$ ...(2)

vi. **Apply Gauss Law**

$$\phi = \oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0 k}$$

$2EA = \frac{\sigma A}{\varepsilon_0 k}$ (from 1 and 2)

$$\therefore \quad \boxed{E = \frac{\sigma}{2\varepsilon_0 k}}$$

Type - I
Numerical based on Application of Gauss theorem

Formulae Used

1. For charged spherical shell

$E_{outside} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$

$E_{surface} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$

$E_{inside} = 0$

2. Long straight wire / cylinder

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda}{r}$$

3. Plane sheet $E = \frac{\sigma}{2\varepsilon_0}$

★ 1) A sphere of radius 10 cm carries a charges of 1 μC. Calculate the electric field
i. at a distance of 30 cm from the centre of the sphere
ii. at the surface of the sphere and
iii. at a distance of 5 cm from the centre of the sphere.

Data: $q = 1\,\mu C = 1\times10^{-6} C$

$R = 10\text{ cm} = 10\times10^{-2} = 10^{-1}\text{ m}$

To find: i. E_1 (when r = 30 cm)

ii. E_2 (when r = R)

iii. E_3 (when r = 5cm)

Formula: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

Solution:

i. For r = 30 cm = 30×10^{-2}m = 3×10^{-1}m

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$E_1 = \frac{9\times10^9\times10^{-6}}{(3\times10^{-1})^2} = \frac{9\times10^{9-6}}{9\times10^{-2}} = 10^5\text{ N/C}$$

ii. r = R = 10cm = 0.1m = 10^{-1}m

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$E_2 = \frac{9\times10^9\times10^{-6}}{(10^{-1})^2} = 9\times10^{9-6+2} = 9\times10^5\text{ N/C}$$

iii. r = 5cm = 0.05 m

∵ r < R, electric field inside the sphere is zero

∴ $E_3 = 0$

Ans : i. Electric field at a distance of 30cm from the centre of the sphere is 10^5 N/C.
ii. Electric field at surface is 9×10^5 N/C.
iii. Electric field at 5cm from centre is 0.

★ 2) The length of a straight thin wire is 2m. It is uniformly charged with a positive charges of 3 μC. Calculate
i. the charge density of the wire
ii. the electric intensity due to the wire at a point 1.5m away from the centre of the wire.

Data: $q = 3\,\mu C = 3\times10^{-6} C$, $l = 2$ m, r = 1.5m

To find: i. λ ii. E

Formulae: i. $\lambda = \frac{q}{l}$

ii. $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda}{r}$

Solution:

i. $\lambda = \frac{q}{l} = \frac{3\times10^{-6}}{2} = 1.5\times10^{-6}$ C/m

ii. $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda}{r}$

$$= \frac{9\times10^9\times2\times1.5\times10^{-6}}{1.5}$$

$= 18\times10^3 = 1.8\times10^4$ N/C

Ans : i. Linear charge density is 1.5×10^{-6}C/m.
ii. Electric field intensity due to wire is 1.8×10^4N/C

★ 3) The charge per unit area of a large flat sheet of charge is 3 μC/m². Calculate the electric field intensity at a point just near the surface of the sheet, measured form its midpoint.

Data: $\sigma = \frac{3\mu C}{m^2} = 3\times10^{-6}\text{ C m}^{-2}$

To find: E

Formula: $E = \frac{\sigma}{2\varepsilon_0}$

Solution: $E = \frac{\sigma}{2\varepsilon_0}$

$$= \frac{3\times10^{-6}}{2\times8.85\times10^{-12}} = \frac{3\times10^{-6}}{17.7\times10^{-12}}$$

$= 1.695 \times 10^5$ N/C

Ans : Electric field intensity is 1.695×10^5 N/C

Problem for Practice

1. A charge of 17.7×10^{-4} C is distributed uniformly over a large sheet of area 200 m^2. Calculate the electric field intensity at a distance of 20 cm from it in air.

Ans: 10^6 NC^{-1}.

2. A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly over its surface. What is the electric field (i) inside the sphere, (ii) just outside the sphere, (iii) at a point 18 cm from the centre of the sphere?

Ans: 0, $10^5 NC^{-1}$, 4.44×10^4 NC^{-1}

3. An infinite line charge produces a field of $9\times10^4 NC^{-1}$ at a distance of 4 cm. Calculate the linear charge density.

Ans: $2\times10^{-7} Cm^{-1}$

4. A charge of 1500 μC is distributed uniformly over a very large sheet of surface area 300 m^2. Calculate the electric field at the distance of 25 cm

Ans: 0.2824×10^6 V/m

5. The surface charge density of a conducting sphere is 8.85×10^{-10} C/m^2 and the electric field intensity at a distance of 4 m from the centre of the sphere is 10^{-2} V/m. Find the radius of the sphere, assuming the sphere to be in vacuum. **Ans: 4m**

6. A thin long cylinder of radius 1 cm carrying a charge 5μC/m is kept in water. Find the electric intensity at a point situated at a distance 10 cm from the axis of cylinder if it is immersed in water. ($k_{water} = 81$)

Ans: 1.111×10^4 N/C

8.3 Electric Potential and Potential Energy

Q.5 Explain concept of is electrostatic potential energy?

Ans:

i. Electrostatic potential energy is the work done against the electrostatic forces to achieve a certain configuration of charges in a given system.

ii. As like charges repel and unlike charges attract each other, a charge always exerts a force on any other charge in its vicinity. Some work is always done to move a charge in the presence of another charge.

iii. Thus, potential energy arises from any collection of charges.

iv. Consider a positive charge Q fixed at some point in space, for bringing any other positive charge close to it, work is necessary. This work is equal to the change in the potential energy of their system.

Thus, work done againse a electrostaic force = Increase in the potential energy of the system.

∴ $\vec{F}.\vec{dr} = dU$,

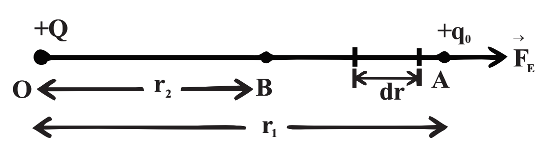
Where, dU is the increase in potential energy when the charge is displaced through $\vec{dr}$ and $\vec{F}$ is the force exerted on the charge.

Q.6 Derive an expression for electrostatic potential energy.

Ans:

i. Let us consider the electrostatic field due to a source charge +Q placed at the origin O.

ii. Let a small charge $+q_0$ be brought from point A to point B at respective distances r_1 and r_2 from O, agianst the repulsive forces on it.



iii. Work done against the electrostatic force $\vec{F_E}$, in displacing the charge q_0 through a small displacement $\vec{d_r}$ appears as an increase in the potential energy of the system.

$dU = \vec{F_E}.\vec{d}r = -F_E.dr$

Negative sign indicates displacement $\vec{d_r}$ is against the electrostatic force $\vec{F_E}$.

iv. For the displacement of the charge from the initial position A to the final postition B, the change in potential energy is

$$\Delta U = \int_{r_1}^{r_2} dU = \int_{r_1}^{r_2} -F_E . dr \qquad ...(1)$$

v. The electrostatic force between the two charges given by coulomb's law is

$$F_E = \frac{1}{4\pi\varepsilon_0} . \frac{Qq_0}{r^2} \qquad ...(2)$$

vi. Substituting equation (2) in equation (1),

$$\therefore \quad \Delta U = \int_{r_1}^{r_2} - \frac{1}{4\pi\varepsilon_0} . \frac{Qq_0}{r^2} dr$$

$$\Delta U = -\frac{1}{4\pi\varepsilon_0} . Qq_0 \int_{r_1}^{r_2} r^{-2} \, dr$$

$$\therefore \quad \Delta U = -\frac{1}{4\pi\varepsilon_0} . Qq_0 \left[\frac{r^{-1}}{-1}\right]_{r_1}^{r_2}$$

$$\Delta U = \frac{1}{4\pi\varepsilon_0} Qq_0 \left(\frac{1}{r}\right)_{r_1}^{r_2}$$

$$\boxed{\Delta U = \frac{1}{4\pi\varepsilon_0} Qq_0 \left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \qquad ...(3)$$

This is expression for change in P.E

vii It is convenient to choose infinity to be the point of zero potential energy as electrostatic force at $r = \infty$ is zero.

viii. P.E (U) of the system of two point charges q_1 and q_2 separated by r can be obtained as

$$\boxed{U(r) = \frac{1}{4\pi\varepsilon_0} . \frac{q_1 q_2}{r}} \qquad ...(4)$$

Q.7 Define one joule and one electron volt. State relation between joule and eV.

Ans:

i. **<u>One joule</u>** is the energy stored in moving a charge of 1 C through a potential difference of 1 volt.

ii. **<u>1 electron volt (eV)</u>** is the change in the kinetic energy of an electon while crossing two points maintained at a potential difference of 1 volt.

iii. joule is SI unit of energy,
$1 \text{ eV} = 1.6 \times 10^{-19}$ joule

Note :
Other related units are;
$1 \text{ meV} = 1.6 \times 10^{-22}$ J
$1 \text{ keV} = 1.6 \times 10^{-16}$ J and
$1 \text{ MeV} = 1.6 \times 10^{-13}$ J

Q.8 Develop the concepts of potential difference and electric potential. State and define their SI units.

Ans:

i. Consider a point charge +q located at a point O. Let A and B be two points in its electric field.

ii. When a test charge q_0 is moved from A to B, a work W_{AB} has to be done in moving against the repulsive force exerted by the charge +q. We then calculate the potential difference between points A and B by the equation:

$$V = V_B - V_A = \frac{W_{AB}}{q_0}$$

iii. *<u>So the potential difference between two points in an electric field may be defined as the amount of work done in moving a unit positive charge from one point to the other against the electrostatic forces.</u>*

iv. SI unit of potential difference is volt (V).

$$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

$1\text{V} = 1\text{NmC}^{-1} = 1 \text{ J C}^{-1}$

v. Hence *<u>the potential difference between two points in an electric field is said to be 1 volt if 1 joule of work has to be done in moving a positive charge of 1 coulomb from one point to the other against the electrostatic forces.</u>*

vi. **Electric potential** The electric potential at a point located far away from a charge is taken to be zero. In given figure, if the point A lies at infinity, the $V_A = 0$, so that

$$V = V_B = \frac{W}{q_0}$$

where, W is the amount of work done in moving the test charge q_0 from infinity to the point B and V_B referes to the potential at point B.

vii. *So the electric potential at a point in an electric field is the amount of work done in moving a unit positive charge from infinity to that point against the elctrostatic forces.*

$$\text{Electric potential} = \frac{\text{Work done}}{\text{Charge}}$$

viii. SI unit of electric potential is volt (V). *The electric potential at a point in an electric field is said to be 1 volt if one joule of work has to be done in moving a positive charge of 1 coulomb from infinity to that point against the electrostatic froce.*

Type-II
Numerical based on work done by charge

Formula Used

1. Electric potential at any point

$$V = \frac{W}{q_0}$$

2. When charge is moved from Point A to B PD is

$$V_B - V_A = \frac{W}{q_0}$$

★ **1) Potential at a point A in space is given as 4×10^5 V.**

i. Find the work done in bringing a charge of 3 μC from infinity to the point A.

ii. Does the anwer depend on the path along which the charge is brought?

Data: $V = 4\times10^5 V$, $q_0 = 3\ \mu C = 3\times10^{-6} C$

To find: W

Formula: $V = \frac{W}{q_0}$

Solution:

i. $W = q_0 V$

$\therefore$ $W = 3 \times 10^{-6} \times 4 \times 10^5 = 12\times10^{-1} = 1.2$ J

ii. As elecrostatic forces are conservative in nature, work done by these forces are path independent.

Ans : Work done in bringing a charge is 1.2 J and it does not depend on path.

★ **2) If 120 J of work is done in carrying a charge of 6 C from a place where the potential is 10 volt to another place where the potential is V, find V.**

Data: $W_{AB} = 120$ J, $q_0 = 6$ C, $V_A = 10$ V, $V_B = V$

To find: V

Formula: $V_B - V_A = \frac{W_{AB}}{q_0}$

Solution:

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

$$V - 10 = \frac{120}{6} = 20$$

$$V = 30\ V$$

Ans : Potential at another place is 30V

Problem for Practice

1. If 100 J of work has to be done in moving an electric charge of 4C from a place, where potential is –10 V to another place, where potential is V volt, find the value of V.

Ans:15 V

2. The work done in moving a charge of 2C between two points is 6 J. What is the potential difference between the two points?

Ans: 3 V

3. If 50 J of work must be done to move a charge of 2C from a point, where potential is +10V to another point where potential is V. Find V

Ans: +35V

4. On moving a charge of 2C from point A where potential is + 12V to a point B where potential is –12V. Find work done

Ans: – 48 J

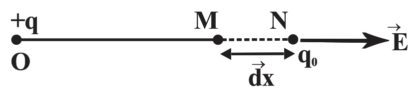
Q.9 Explain relation between electric field and electric potential.

Ans:

i. Consider the electric field produced by a charge +q kept at point O.

ii. A unit positive charge ($+q_0$) is present in vicinity is moved toward charge +q through small distance dx.

iii. As direction of electric field of charge +q is outward, displacement dx is in direction opposite to field .



$\therefore$ Work done by charge $+q_0$ is given by

iv. $dW = \vec{F}.\vec{dx} = -Fdx$

As $E = \frac{F}{q_0}$

$\therefore$ $F = q_0E$

we get,

$dW = -Eq_0\,dx$ ... (1)

v. P.D across dx is given by

$dV = \frac{dW}{q_0}$

$\therefore$ $dW = q_0 \times dV$ ...(2)

vi. From (1) and (2)

$\cancel{q_0} \times dV = -E\,\cancel{q_0}\,dx$

$\therefore$ $\boxed{E = -\frac{dV}{dx}}$

<u>Thus the electric field at a point in an electric field is the negative of potential gradient at that point.</u>

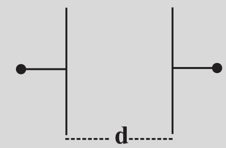
Type - III

Numerical based on relation between electric field and electric potential

Formula Used

1. $E = -\frac{dV}{dx}$

2. Electric field between two parallel plates seperated by distance d and potential v is



$E = \frac{V}{d} = \frac{\text{potential between plates}}{\text{distance between plates}}$

3. Potential gradient $= \frac{dv}{dx}$

1) Find the electric field between two metal plates 2 mm apart, connected to 10 V battery

Data: $d = 2\text{ mm} = 2 \times 10^{-3}\text{m}$, $V = 10\text{ V}$

To Find: E

Formula: $E = \frac{V}{d}$

Solution: $E = \frac{V}{d} = \frac{10}{2\times10^{-3}} = 5\times10^{3}\text{ V/m}$

Ans : Electric field betweeen two plates is $5\times10^{3}\text{V/m}$

★ 2) A small particle carrying a negative charge of 1.6×10^{-19} C is suspended in equilibrium between two horizontal metal plates 10 cm apart having a potential difference of 4000 V across them. Find the mass of the particle.

Data: $q = 1.6 \times 10^{-19}\text{C}$,

$dx = 10\text{ cm} = 0.1\text{ m}$, $dV = 4000\text{V}$

To find: m

Formula: i. $E = -\frac{dV}{dx}$ ii. weight = mg

Solution:

i. $E = -\frac{dV}{dx} = -\frac{4000}{10^{-1}} = -4\times10^{4}\text{Vm}^{-1}$

ii. As, the charged particle remain suspended, in equilibrium,

Weight of charge = Electrostatic force

$\therefore$ $F_G = F_E$

$\therefore$ $mg = qE$

$\therefore$ $m = \frac{qE}{g} = \frac{-1.6\times10^{-19}\times(-4\times10^{4})}{9.8}$

$\therefore \quad m = 6.531\times10^{-16}$kg

Ans : Mass of the particle is 6.531×10^{-16}kg

Problem for Practice

1. Find the electric field between two metal plates 3 mm apart, connected to 12 V battery.
Ans: 4×10^3 V/m

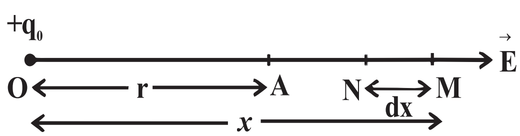
2. Calculate the voltage needed to balance an oil drop carrying 10 electrons when located between the plates of a capacitor which are 5 mm apart ($g = 10\ ms^{-2}$). The mass of oil drop is 3×10^{-16} kg
Ans: 9.47 V

3. A particle of mass 1.96×10^{-15}kg is kept is equillibrium between two horizontal metal plates having P.D of 400 v seperated apart by 0.02m, then find the charge of the particle
Ans: 6e

Q.10 Derive an expression for electric potential at any point in electric field due to a point charge.

Ans:

i. Consider a point charge +q located at point O. Let a positive unit charge (q_0) be brought from ∞ to point A .



ii. Let point 'M' is an intermediate point on this path such that OM = x.

iii. Magnitude of electrostatic force on a unit positive charge at M is

$$F = \frac{1}{4\pi\varepsilon_0}\times\frac{q_0}{x^2} \quad ...(1)$$

It is directed away from O, along OM.

iv. For infinitesimal displacement dx from M to N, the amount of work done is given by

$\therefore \quad dW = -Fdx \quad ...(2)$

The negative sign appears as the displacement is directed opposite to that of the force.

v. Total work done in displacing the unit positive charge from ∞ to point A is given by

$$W = \int_{\infty}^{r} -Fdx = \int_{\infty}^{r} -\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}dx$$

$$W = -\frac{1}{4\pi\varepsilon_0}q\,q_0\int_{\infty}^{r}\frac{1}{x^2}dx$$

$$W = -\frac{1}{4\pi\varepsilon_0}q\,q_0\int_{\infty}^{r}x^{-2}dx$$

$$= -\frac{1}{4\pi\varepsilon_0}.q\,q_0\left[\frac{x^{-2+1}}{-2+1}\right]_{\infty}^{r} \quad ...\left[\because \int x^n = \frac{x^{n+1}}{n+1}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}.q\,q_0\left[\frac{x^{-1}}{-1}\right]$$

$$W = \frac{1}{4\pi\varepsilon_0}.q\,q_0\left[\frac{1}{x}\right]_{\infty}^{r}$$

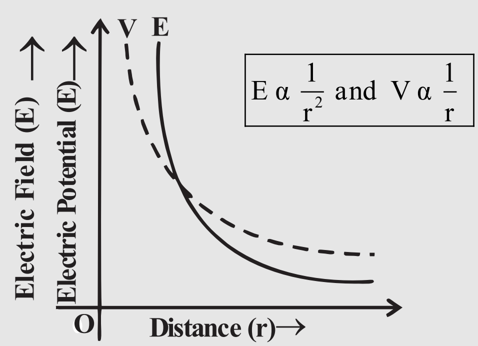
$$W = \frac{1}{4\pi\varepsilon_0}.q\,q_0\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

$$W = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r}$$

vi. Electrostatic potential at A due to charge q is

$$V = \frac{W}{q_0} = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$$

Note :



Type - IV
Numerical based on electric potential due to point charge

Formulae Used

1. $V = \frac{W}{q_0}$

2. $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$

★ **1) A wire is bent in a circle of radius 10 cm. It is given a charge of 250 μC which spreads on it uniformly. What is the electric potential at the centre?**

Data: $q = 250\ \mu C = 250 \times 10^{-6} C$

$r = 10\ cm = 10^{-1}\ m$

To find: V

Formula: $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Solution: $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{9\times10^{9}\times250\times10^{-6}}{10^{-1}}$

$= 2250\times10^{4} = 2.25\times10^{7}\ V$

Ans :	Electric potential at the centre of circle is $2.25\times10^{7}V$

★ **2) Two charges 5×10^{-8}C and -3×10^{-8} C are located 16 cm apart. At what point (s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.**

Data : $q_A = 5\times10^{-5}$ C

$q_B = -3\times10^{8}C$

$r_{AB} = 16\ cm = 16\times10^{-2}m = 0.16m$

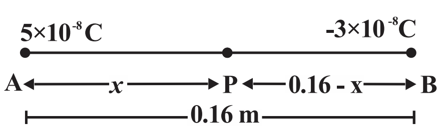
To Find : Point on line joining two charges where V = 0

Formula : $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$

Solution:

i. Consider two given charges situated at point A and B. Let the electric potential be zero at point P at a distance x from A.

Case I: Point P lies between A and B



At Point P,

$V_A + V_B = 0$

$$\frac{1}{4\pi\varepsilon_0}\times\frac{q_A}{r_A}+\frac{1}{4\pi\varepsilon_0}\times\frac{q_B}{r_B}=0$$

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{5\times10^{-8}}{x}+\frac{-3\times10^{-8}}{0.16-x}\right)=0$$

$\therefore\ \frac{5}{x}=\frac{3}{0.16-x}$

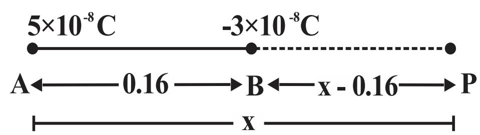
$\therefore\ 0.8 - 5x = 3x$

$\therefore\ 8x = 0.8$

$\therefore\ x = 0.1$ m

$\therefore$ Point lies at distance 0.1 m from charge q_A

Case II: Point P lies on extended segment of AB



At point P,

$V_A + V_B = 0$

$$\frac{1}{4\pi\varepsilon_0}\times\frac{q_A}{r_A}+\frac{1}{4\pi\varepsilon_0}\times\frac{q_B}{r_B}=0$$

$$\therefore\ \frac{1}{4\pi\varepsilon_0}\left(\frac{5\times10^{-8}}{x}+\frac{-3\times10^{-8}}{x-0.16}\right)=0$$

$\therefore\ \frac{5}{x}=\frac{3}{x-0.16}$

$\therefore\ 5x - 0.8 = 3x$

$\therefore\ 2x = 0.8$

$\therefore\ x = 0.4$ m

$\therefore$ Point lies at distance 0.4 m from charge 5×10^{-8}C

Ans :	0.1m and 0.4 m from point charge 5×10^{-8}C electric potential is zero.

★ **3) One hundred twenty five small liquid drops, each carrying a charge of 0.5 μC and each of diameter 0.1 m form a bigger drop. Calculate the potential at the surface of the bigger drop.**

Data: $q = 0.5\ \mu C = 0.5\times10^{-6}$ C

$r = 0.05$ m, n = 125

To find: V of bigger drop

Formula: $V = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R}$

Solution:

i. Volume of big drop = Volume of 125 droples

$$\therefore \quad \frac{4}{3}\pi R^3 = 125 \times \frac{4}{3}\pi r^3$$

$R^3 = 125\ r^3$

$R = 5 \times r = 5 \times 0.05 = 0.25$ m

ii. Total charge on bigger drop

$Q = 125 \times q$

$= 125 \times 0.5 \times 10^{-6}$

$= 62.5 \times 10^{-6}$ C

iii. Potential at the surface of bigger drop

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R}$$

$$= \frac{9\times10^9\times62.5\times10^{-6}}{0.25}$$

$= 2250 \times 10^3 = 2.25\times10^6$ V

Ans :	Electric potential at the surface of bigger drop is 2.25×10^6V

Problem for Practice

1. Determine the electric potential at the surface of a gold nucleus. The radius is 6.6×10^{-15} m and the atomic number Z = 79. Given charge on a proton = 1.6×10^{-19}C.

Ans: 1.7×10^7 **V**

2. Twenty seven drops of same size are charged at 220 V each. They coalesce to form a bigger drop. Calculate the potential of the bigger drop.

Ans: 1980 V

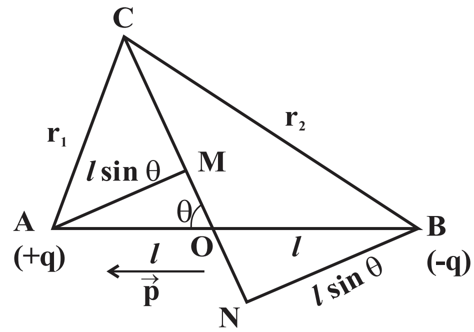
3. Two charges 3×10^{-8} C and -2×10^{-8}C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Ans: 9cm, 45 cm

Q.11 Derive an expression for electric potential at any point in electric field due to an electric dipole.

Ans:

i. Consider an electric dipole of length 2l. Let origin be at the centre of the dipole.



ii. Let C be any point near the electric dipole at a distance r from the centre O inclined at an angle θ with axis of the dipole.

Let r_1 and r_2 be the distances of point C from charges +q and –q, respectively.

iii. Potential at C due to charge +q at A is,

$$V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r_1}$$

Potential at C due to charge –q at B is,

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{-q}{r_2}$$

iv. The Potential at C due to the dipole is,

$$V_C = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \quad ....(1)$$

v. Draw AM $\perp$ OC,

In Δ AOM, $\cos\theta = \frac{OM}{OA}$

$\therefore$ OM = $l\cos\theta$

$\sin\theta = \frac{AM}{OA}$

$\therefore$ AM = $l\sin\theta$

CM = OC – OM = r – $l\cos\theta$

In Δ ACM, By Pythagoras Theorem

$AC^2 = CM^2 + AM^2$

$r_1^2 = (r - l\cos\theta)^2 + (l\sin\theta)^2$

$r_1^2 = r^2 - 2\,r\,l\cos\theta + l^2\cos^2\theta + l^2\sin^2\theta$

$\therefore$ $r_1^2 = r^2 + l^2 - 2\,r\,l\cos\theta$

Similarly

$r_2^2 = r^2 + l^2 + 2\,r\,l\cos\theta$

$$r_1^2 = r^2\left(1 + \frac{l^2}{r^2} - 2\frac{l}{r}\cos\theta\right)$$

$$r_2^2 = r^2\left(1+\frac{l^2}{r^2}+2\frac{l}{r}\cos\theta\right)$$

For a short dipole, $2l << r$ and

If $r >> l$; $\frac{l}{r}$ is small

$\therefore$ $\frac{l^2}{r^2}$ can be neglected

$$\therefore \quad r_1^2 = r^2\left(1-\frac{2l}{r}\cos\theta\right)$$

$$r_2^2 = r^2\left(1+\frac{2l}{r}\cos\theta\right)$$

$$\therefore \quad r_1 = r\left(1-\frac{2l}{r}\cos\theta\right)^{\frac{1}{2}}$$

$$r_2 = r\left(1+\frac{2l}{r}\cos\theta\right)^{\frac{1}{2}}$$

$$\therefore \quad \frac{1}{r_1} = \frac{1}{r}\left(1-\frac{2l}{r}\cos\theta\right)^{\frac{-1}{2}} \text{ and}$$

$$\frac{1}{r_2} = \frac{1}{r}\left(1+\frac{2l}{r}\cos\theta\right)^{\frac{-1}{2}} \quad \text{....(2)}$$

vi. Substituting (2) in Equation (1)

We get,

$V_C = V_1 + V_2$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1-\frac{2l}{r}\cos\theta\right)^{\frac{-1}{2}} - \frac{1}{r}\left(1+\frac{2l}{r}\cos\theta\right)^{\frac{-1}{2}}\right]$$

vii. Using binomial expansion,

$(1 + x)^n = 1 + nx$, $x << 1$ and retaining terms up to the first order of $\frac{1}{r}$ only, we get

$$V_C = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left[\left(1+\frac{l\cos\theta}{r}\right)-\left(1-\frac{l\cos\theta}{r}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left[1+\frac{l}{r}\cos\theta-1+\frac{l}{r}\cos\theta\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left(\frac{2l}{r}\cos\theta\right)$$

$$\therefore \quad \boxed{V_C = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}} \quad \text{....}(\because p = q \times 2l)$$

viii. **Case -1 :** At an axial point $\theta = 0$

$\cos 0 = 1$

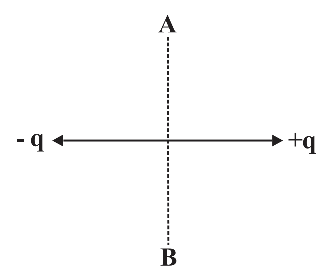
$$\therefore \quad \boxed{V_{axis} = \frac{1}{4\pi\varepsilon_0}.\frac{P}{r^2}}$$

Case -2 : At an equational point $\theta = 90°$

$\cos 90 = 1$

$$\therefore \quad \boxed{V_{eq} = 0}$$

★ **Q.12 A charge q is moved from a point A above a dipole of dipole moment P to a point B below the dipole in equatorial plane without acceleration. Find the work done in this process.**



Ans:

i. Displacement of charge is along equatorial line of dipole (AB) where potential is zero throughout.

ii. As work done, W = qV.

Work done in the process is zero.

Type - V

Numerical based on Potential due to Electric dipole

Formula Used

1. $V = \frac{1}{4\pi\varepsilon_0}.\frac{p\cos\theta}{r^2}$

2. At axial point

$V_{axis} = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^2}$

3. At Eqatorial Point

$V_{eq} = 0$

★ **1) A short electric dipole has dipole moment of 1 × 10^{-9} Cm. Determine the electric potential due to the dipole at a point distance 0.3 m from the centre of the dipole situated**

i. on the axial line

ii. on the equatorial line

iii. on a line making an angle of 60° with the dipole axis.

Data: $p = 1 \times 10^{-9}$ Cm, r = 0.3 m

To find: i. V_{axis} ii. V_{eq} iii. $V_{60°}$

Formula: $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

Solution:

i. For a point on the axis $\theta = 0$

$$V_{axis} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9\times1\times10^{-9}\times\cos0^0}{\left(3\times10^{-1}\right)^2}$$

$$= \frac{9}{9\times10^{-2}} = 100V$$

ii. For a point on equitorial line, $\theta = 90°$

$$V_{eq} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \qquad (\because \cos 90 = 0)$$

$= 0$ V

iii. For a point on a line making $\theta = 60°$

$$V_{60°} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9\times1\times10^{-9}\times\cos 60°}{\left(3\times10^{-1}\right)^2}$$

$$= \frac{9\times10^{9-9+2}\times 0.5}{9} = 50V$$

Ans : i. Electric potential on axial line100V
ii. Electric potential on equatorial line is 0V.
iii. Electric potential at an angle 60° with axial line is 50V.

Problem for Practice

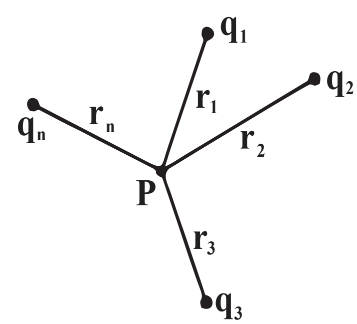
1. A short electric dipole has dipole moment of 4×10^{-9}Cm. Determine the electric potential due to the dipole at a point distant 0.3 m from the centre of the dipole situated (a) on the axial line (b) on equatorial line and (c) on a line making an angle of 60° with the dipole axis.

Ans : (a) 400 V (b) 0 (c) 200 V

Q.13 Derive an expression for electrostatic potential due to system of charges.

Ans:

i. Consider a system of charges q_1, q_2.........q_n at distances r_1, r_2, ...r_n respectively from point P .



ii. The potential V_1 at P due to the charge q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

Similarly the potentials V_2, V_3 ... V_n at P due to the individual charges q_2, q_3 ... q_n are given by,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2},$$

$$V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3},$$

$$V_n = \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n}$$

iii. By the superposition Principle, the potential V at P due to the system of charges is the algebraic sum of the potentials due to the individual charges.

∴ $V = V_1+V_2+.....+V_n$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+.....+\frac{q_n}{r_n}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

iv. For a continuous charge distribution, summation should be replaced by integration.

Type - VI
Numerical based on potential of charges

Formulae Used

1. $V = V_1 + V_2 + ... + V_n$

2. $V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$

1) Calculate the electric potential at the centre of a square of side $\sqrt{2}$ m, having charges 100 μC, –50 μC, 20 μC and –60 μC at the four corners of the square.

Data: $q_1 = 100\,\mu C = 100\times10^{-6} C$

$q_2 = -50\,\mu C = -50\times10^{-6} C$

$q_3 = 20\,\mu C = 20\times10^{-6} C$

$q_4 = -60\,\mu C = -60\times10^{-6} C$

$l = \sqrt{2}$ m (side of square)

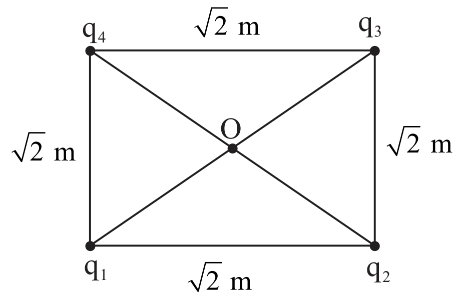
To Find : V at the centre of square

Formula : i. By principle of superposition

$V = V_1 + V_2 + V_3 + V_4$

ii. $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$

Solution:



i. Diagonal of the square $= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$

$= 2$ m

Distance of each charge from the centre of the square is

r = Half diagonal = 1 m

ii. Potential at the centre of the square is

$V = V_1 + V_2 + V_3 + V_4$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r}+\frac{q_2}{r}+\frac{q_3}{r}+\frac{q_4}{r}\right]$$

$$V = 9\times10^9\left[\frac{100\times10^{-6}}{1}-\frac{50\times10^{-6}}{1}+\frac{20\times10^{-6}}{1}-\frac{60\times10^{-6}}{1}\right]$$

$V = 9\times10^9\times10^{-6}\times[100 - 50 + 20 - 60]$

$= 9\times10^9\times10^{-6}\times10 = 9\times10^4 V.$

Ans : The electric potential at the centre of a square is 9×10^4 V

Problem for Practice

1. Calculate the potential at the centre of a square ABCD of each side $\sqrt{2}$ m due to charges 2, –2, –3 and 6 μC at four corners of it.

Ans: 2.5×10^4V

2. ABCD is a square of side 0.2 m. Charges of 2×10^{-9}, 4×10^{-9}, 8×10^{-9}C are placed at the corners A, B and C respectively. Calculate the work required to transfer a charge of 2×10^{-9}C from D to the centre O of the square.

Ans: 6.27×10^{-7} J

8.5 Equipotential Surfaces

Q.14 What is an equipotential surface? Give an example.

Ans:

i. Any surface that has same electric potential at every point on it is called an equipotential surface.

ii. The surface may be surface of a body or a surface in space.

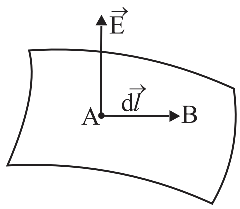
iii. For example, the surface of a charged conductor is an equipotential surface.

iv. By joining points of constant potential, we can draw equipotential surfaces throughout the region in which an electric field exists.

Q.15 State and prove the important properties of equipotential surfaces.

Ans:

i. **No work is done in moving a test charge over an equipotential surface.**



Let A and B be two points over an equipotential surface, as shown in figure.If the test charge q_0 is moved from A to B, the work done will be

W_{AB} = Charge × potential difference

$= q_0 \times (V_B - V_A)$

As the surface is equipotential, so

$V_B - V_A = 0$

Hence $W_{AB} = 0$

ii. Electric field is always normal to the equipotential surface at every point.

iii. Equipotential surfaces are closer together in the regions of strong field and farther apart in the regions of weak field. We know that electric field at any point is equal to the negative of potential gradient at that point.

i.e., $E = \frac{dV}{dr}$ or $dr = -\frac{dV}{E}$

For the same change in the value of dV i.e., when dV = constant, we have

$$dr \propto \frac{1}{E}$$

Thus the spacing between the equipotential surfaces will smaller in the regions, where the electric field is stronger and vice versa.

iv. No two equipotential surfaces can intersect each other. If they interesect, then there will be two values of electric potential at the point of intersection, which is impossible.

Q.16 Sketch and explain the equipotential surfaces for: (i) a point charge, (ii) two point charge +q and –q, separated by a small distance, (iii) two point charges +q and +q separated by a small distance and (iv) a uniform electric field.

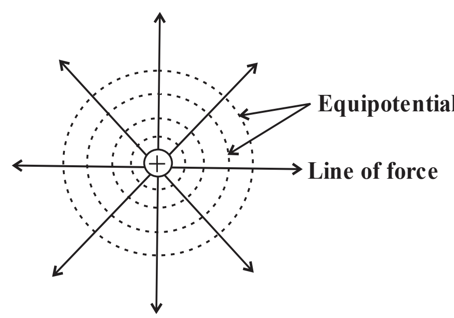
Ans: Equipotential surfaces of various charge systems. For the various charge systems, we represent equipotential surfaces by dashed curves and lines of forces by full line curves. Between any two adjacent equipotential surfaces, we assume a constant potential difference.

i. **Equipotential surfaces of a positive point charge.**

The electric potential due to a point charge q at distance r from it is given by

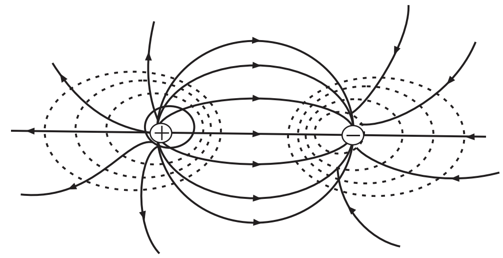
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

This shows that V is constant if r is constatnt. Thus, <u>the equipotential surfaces of a single point charge are concentric spherical shells with their centres at the point charge.</u> As the lines at the point radially outwards, so they are perpendicular to the equipotential surfaces at all points.



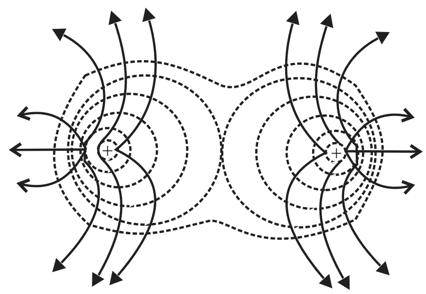
ii. **Equipotential surfaces of two equal and opposite point charges i.e Electric dipole.**

The equipotential surfaces of two equal and opposite charges, +q and –q, separated by a small distance. <u>They are close together in the region in between the two charges.</u>

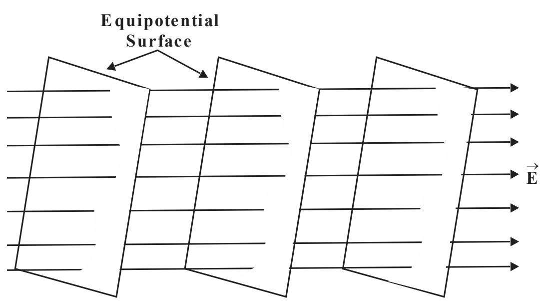


iii. **Equipotential surfaces of two equal**

positive charges. Figure shows the equipotential surfaces of two equal and positive charges, each equal to +q, separated by a small distance. <u>The equipotential surfaces are far apart in the regions in between the two charges, indicating a weak field in such regions.</u>



iv. **Equipotential surfaces for a uniform electric field.** figure shows the equipotential surfaces for a uniform electric field. <u>The lines of forces are parallel straight lines and equipotential surfaces are equidistant parallel planes perpendicular to the lines of force.</u>



Q.17 Give the importance of equipotential surfaces.

Ans: Importance of equipotential surfaces.

i. Like the lines of force, the equipotential surfaces give a visual picture of both the direction and the magnitude of field $\vec{E}$ in a region of space.

ii. If we draw equipotential surfaces at regular intervals of V, we find that equipotential surfaces are closer together in the regions of strong field and farther apart in the regions of weak field.

iii. $\vec{E}$ is normal to the equipotential surface at every point.

8.6 Electrical energy of two point charges and of a Dipole in an electrostatic field.

Q.18 Define electrostatic potential energy of a system of point charges.

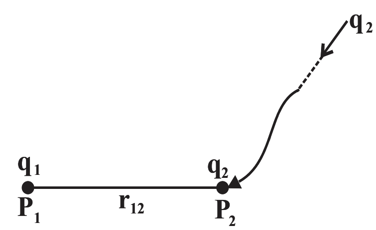
Ans: Electrostatic potential energy of a system of point charges is defined as the total amount of work done to assemble the system of charges by bringing them from infinity to their present locations.

Q.19 Deduce expressions for the potential energy of a system of two point charges and three point charges and hence gneralise the result for a system of N point charges.

Ans:

i. Potential energy of a system of two point charges.

a. Suppose a point charge q_1 is at rest at a point P_1 in space, as shown



b. It takes no work to bring the first charge q_1 because there is no field yet to work against.

$\therefore$ $W_1 = 0$

c. Electric potential due to charge q_1 at a point P_2 at distance r_{12} from P_1 will be

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{12}}$$

d. If charge q_2 is moved in from infinity to point P_2, the work required is

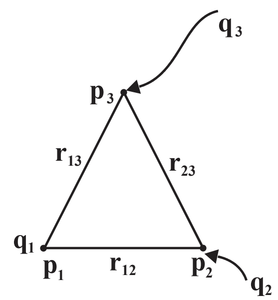
W_2 = Potential at point P_2 × charge

$$= V_1 \times q_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

As the work done is stored as the potential energy (U) of the system

$$\therefore \quad U = W_1 + W_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

ii. Potential energy of a system of three point charges. Now we bring in the chargen q_3 from infinity to the point P_3. Work has to be done against the forces exerted by q_1 and q_2.



Therefore

W_3 = Potential at point P_3 due to q_1 and q_2 × charge (q_3)

$$W_3 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right]\times q_3$$

$$W_3 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

Hence the elecrtostatic potential energy of the system is

U = Total work done to assemble the three charges

$= W_1 + W_2 + W_3$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

iii. Potential energy of a system of N point charges.

The expression for the potential energy of N point charges can be written as

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_i q_j}{r_{ij}}$$

Q.20 Write an expression for the potential energy of a single charge in an external field. Hence define electric potential.

Ans: Potential energy of a single charge.

i. We wish to find the potential energy of a charge q in an external electric field $\vec{E}$ at point P where the corresponding external potential is V.

ii. By definition, V at a point P is the amount of work done in bringing a unit positive charge from infinity to the point P.

iii. Thus, the work done in bringing a charge q from infinity to the point P will be qV,

∴ W = qV

iv. This work done is stored as the potential energy of the charge q.

v. If $\vec{r}$ is the position vector of point P relative to some origin, then

$$U(\vec{r}) = qV(\vec{r})$$

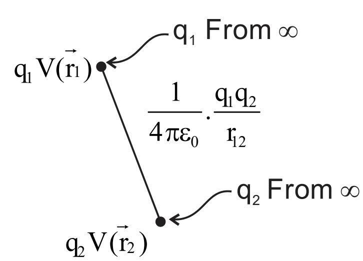
∴ P. E. of a charge in an external field

= Charge × external electric potential

Q.21 Write an expression for the potential energy of two point charges q_1 and q_2, separated by distance r in an electric field $\vec{E}$.

Ans: Potential energy of a system of two point charges in an external field.

i. Let $V(\vec{r_1})$ and $V(\vec{r_2})$ be the electric potentials of the field $\vec{E}$ at the points having position vectors $\vec{r_1}$ and $\vec{r_2}$ as shown in figure



ii. Work done in bringing q_1 from ∞ to $\vec{r_1}$ against the external field $W_1 = q_1 V(\vec{r_1})$

iii. Work done in bringing q_2 from ∞ to $\vec{r_2}$ against the external field $(W_2) = q_2 V(\vec{r_2})$

iv. Work done on q_2 against the force exerted by q_1

$$W_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}}$$

where r_{12} is the distance between q_1 and q_2.

v. Total potential energy of the system
= The work done in assembling the two charges

$\therefore$ $U = W_1 + W_2 + W_3$

or $U = q_1V(\vec{r_1}) + q_2V(\vec{r_2}) + \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r_{12}}$

Q.22 Define electron volt. Express it in joule.

Ans: Units of electrostatic potential energy.

i. Suppose an electron ($q = 1.6 \times 10^{-19}C$) is moved through a potential difference of 1 volt, then the change in its P.E. would be
$\Delta U = q\Delta V = 1.6\times10^{-19}C\times1V = 1.6\times10^{-19}J$

ii. This is a commonly used unit of energy in atomic physics and we call it **electron volt (eV)**.

iii. *Thus electron volt is the potential energy gained or lost by an electron in moving through a potential difference of 1 volt.*

iv. $1eV = 1.6 \times 10^{-19}J$

Type - VI

Numerical based on P.E. of System of Charge

Formulae Used :

1. Electric P.E. of system of two point charges

$$U = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r_{12}}$$

2. Electric P.E. of a System of Npoint charges

$$U = \frac{1}{4\pi\varepsilon_0}.\sum_{\text{all pairs}}\frac{q_jq_k}{r_{jk}}$$

★ **1) Two charges of magnitude 5 nC and –2 nC are placed at points (2 cm, 0, 0) and (20 cm, 0, 0) in a region of space, where there is no other external field. Find the electrostatic potential energy of the system.**

Data: $q_1 = 5\ nC = 5\times10^{-9}\ C$
$q_2 = -2\ nC = -2\times10^{-9}C$

To find: U

Formula: $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

Solution:

i. $r = (20 - 2)cm = 18\ cm = 18\times10^{-2}m$

ii. $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

$U = \frac{9\times10^9\times5\times10^{-9}\times(-2\times10^9)}{10\times10^{-2}}$
$= -5 \times 10^{-7}J = -0.5\ \mu J$

Ans : The electrostatic potential energy of the system is $-0.5\mu J$

★ **2) Two charged particles having equal charge of 3×10^{-5} C each are brought from infinity to a separation of 30 cm. Find the increase in electrostatic potential energy during the process.**

Data: $q_1 = q_2 = q = 3 \times 10^{-5}\ C$,
$r = 30cm = 0.3\ m$

To find: ΔU

Formula: $\Delta U = U_A - U_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

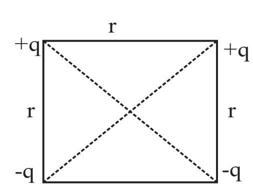
Solution: $\Delta U = U_A - U_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

$\Delta U = U_A - U_B = \frac{9\times10^9\times(3\times10^{-5})^2}{0.3}$

$= \frac{9\times9\times10^{9-10+1}}{3} = 9 \times 3 = 27\ J$

Ans : Increase in Potential energy of the system is 27J.

★ **3) Calculate the electrostatic potential energy of the system of chages shown in the figure.**

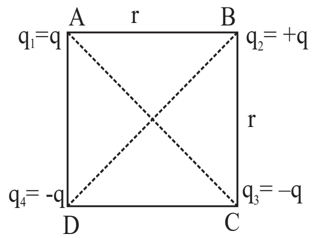


Data: $q_1 = q, q_2 = q, q_3 = -q, q_4 = -q$

To Find: Electrostatic P.E.(U)

Formula: $U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}$

Solution:



i. $r_{12} = r_{14} = r_{23} = r_{34} = r$

$r_{13} = r_{24} = \sqrt{2}\,r$ (By pythagoras theorem)

ii. Assuming potential at ∞ as zero, potential energy of system of charges,

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_4}{r_{14}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} + \frac{q_2q_4}{r_{24}} + \frac{q_3q_4}{r_{34}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{r} - \frac{q^2}{r} - \frac{q^2}{\sqrt{2}r} - \frac{q^2}{r} - \frac{q^2}{\sqrt{2}r} + \frac{q^2}{r}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-2q^2}{\sqrt{2}r}\right] = \frac{-\sqrt{2}q^2}{4\pi\varepsilon_0}$$

Ans : Electrostatic potential energy of given system of charges is $\frac{-\sqrt{2}q^2}{4\pi\varepsilon_0}$

★ 4)

I. Determine the electrostatic potential energy of a system consisting of two charges $-2\,\mu C$ and $+4\,\mu C$ (with no external field) placed at (–8 cm, 0, 0) and (+8 m, 0 , 0) respectively.

II. Suppose the same system of charges is now placed in an external electric field $E = A\frac{1}{r^2}$ where $A = 8\times10^5 cm^{-2}$, what would be the electrostatic potential energy of the configuration?

Data: $q_1 = -2\,\mu C = -2\times10^{-6} C$

$q_2 = +4\,\mu C = +4\times10^{-6} C$

$r_1 = 0.08$ m, $r_2 = 0.08$ m

$A = 8\times10^5\ cm^{-2}$

$E = A\frac{1}{r^2}$

To find: i. U ii. U'

Formulae: i. P.E of system of two charges

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

ii. $V = -\int E\,dr$

iii. In the presence of external field, for two charge system

$$U' = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} + q_1V(\vec{r_1}) + q_2V(\vec{r_2})$$

Solution:

i. **P.E of system of two charges**

Total distance, $r = r_1 + r_2 = 0.16$ m

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

$$= \frac{9\times10^9\times(-2\times10^{-6})\times4\times10^{-6}}{0.16}$$

$$= \frac{-9\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{4}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{16}}}\times10^{9-6-6+2}$$

$= -4.5\times10^{-1} = -0.45$ J

ii. $V = -\int E\,dr = -\int_{\infty}^{r}\frac{A}{r^2}dr$

$$V = -A\int_{\infty}^{r} r^{-2}dr$$

$$= -A\left[\frac{r^{-1}}{-1}\right]_{\infty}^{r}$$

$$= A\left[\frac{1}{r}\right]_{\infty}^{r} = A\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$V = \frac{A}{r}$$

ii. P.E of system of two charges in external electric field

$$U' = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} + q_1V(\vec{r_1}) + q_2V(\vec{r_2})$$

$$U' = U + \frac{q_1A}{r_1} + \frac{q_1A}{r_2}$$

$$= -0.45 + \frac{(-2\times10^{-6})\times8\times10^5}{0.08} + \frac{4\times10^{-6}\times8\times10^5}{0.08}$$

$= -0.45 - 2 \times 10^{5-6+2} + 4 \times 10^{5-6+2}$

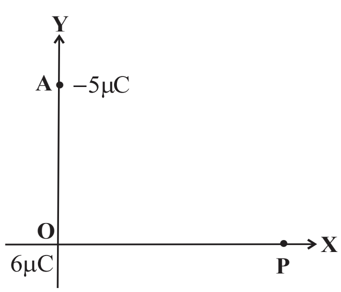
$= -0.45 - 20 + 40 = 19.55$ J

Ans : i. The potential energy of the system is – 0.45 J
ii. The potential energy of the system when it is placed in an external field is 19.55 J.

★ **5) A charge 6 μC is placed at the origin and another charge –5 μC is placed on the y axis at a position A (0, 6.0) m.**

i. Calculate the total electric potential at the point P whose coordinates are (8.0, 0) m.

ii. Calculate the work done to bring a proton from infinity to the point P? What is the significance of the negative sign?



Data : $q_1 = -5\,\mu C = 5\times10^{-6}$ C
$q_2 = 6\,\mu C = 6\times10^{-6}$ C
$q_0 = 1.6\times10^{-19}$ C (change of proton)

To Find : i. V at point P
ii. Work done to bring proton from ∞ to point p

Formula: i. $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$ ii. $V = \frac{W}{q_0}$

Solution:

i. let AP = $r_1 = \sqrt{6^2 + 8^2} = 10$m
OP = r_2 = 8m

ii. Potential at point P
$V = V_1 + V_2$

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}.\frac{q_2}{r_2}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

$$= 9\times10^9\times10^{-6}\left(\frac{-5}{10} + \frac{6}{8}\right)$$

$= 2.25\times10^3$V

iii. Work done in bringing a proton from infinity,
$W = -q_0 \times V$ ....(As PE at ∞ is zero)
$= -1.6 \times 10^{-19}\times2.25 \times 10^3$
$= -3.6 \times 10^{-16}$ J

Significance: If a positive charge is moved against the electric field, change in its potential energy is positive.
Since work done = – (Change in PE by conservative force)
Work done in this case is negative.

Ans : i. Electric potential is 2.25×10^3V.
ii. Work done is -3.6×10^{-16}J.

Problem for Practice

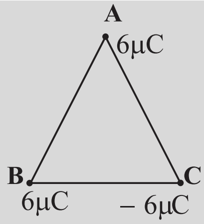
1. Two point charges +10 μC and –10 μC are separated by a distance of 2.0 cm in air. Calculate the potential energy of the system, assuming the zero of the potential energy to be at infinity.

Ans: –45 J

2. Two point charges 20×10^{-6}C and -4×10^{-6}C are separated by a distance of 50 cm in air. (i) Find the point on the line joining the charges, where the electric potential is zero. (ii) Also find the electrostatic potential energy of the system.

Ans: 41 cm from the charge of 20 ×10⁻⁶ C,–1.44J

3. Find the amount of work done in arranging the three point charges, on the vertices of an equilateral triangle ABC, of side 10 cm, as shown in the adjacent figure.



Ans: –3.24J

4. Three point charges +q,+2q and Q are placed at the three vertices of an equilateral triangle. Find the value of charge Q (in terms of q), so that electric potential energy of the system is zero.

Ans : Q = –2q/3

8.7 Conductors and Insulators, Free charges and Bond Charges inside a conductor

Q.23 Distinguish between conductors and insulators.

Ans:

No	Conductors	Insulators
i.	Conductors are materials or substances which allow electricity to flow through them.	Insulators are materials or substances which resist electricity to flow through them.
ii.	Then contain a large number of free charge carriers (free electrons).	They do not free charges carries.
iii.	Electric charge exist on the surface of conductors.	Electric charges are absent in Insulators.
iv	Resistance is low.	Resistance is high.

Q.24 State the properties of conductor in electrostatic conditions.

Ans:

i. In the interior of a conductor, net electrostatic field is zero.

ii. Potential is constant within and on the surface of a conductor.

iii. In static situation, the interior of a conductor can have no charge.

iv. Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point.

v. Surface charge density of a conductor could be different at different points.

Q.25 Explain electrostatic shielding with examples.

Ans:

i. To protect a delicate instrument from the disturbing effects of other charged bodies near it, it is placed inside a hollow conductor where E = 0. This is called electrostatic shielding.

ii. Thin metal foils are used in making the shields.
Examples:
  a. During lightning and thunder storm it is always advisable to stay inside the car than near a tree in open ground, since the car acts as a shield.
  b. Faraday Cages: It is an enclosure which is used to block the external electric fields inconductive materials.
  c. Electro-magnetic sheilding: MRI scanning rooms are built in such a manner that they prevent the mixing of the external radio frequency signals with the MRI machine.

★ Q.26 The safest way to protect yourself from lightening is to be inside a car. Justify.

Ans:

i. When person is sitting in car with a metal body is an almost ideal for a day cage.

ii. When a car is stuck by lightning, the charge flows on the outside surface of the car to the ground.

iii. Also electric field inside car is zero.
So the safest way to protect yourself from lightenings is to be inside a car.

Q.27 What are free charges and bound charges.

Ans: Free charges:

i. In metallic conductors, the electrons in the outermost shells of the atoms are loosely bound to the nucleus and hence can easily get detached and move freely inside the metal.

ii. When an external electric field is applied, they drift in a direction opposite to the direction of the applied electric field. These charges are called free charges.

Bound charges :

i. The nucleus, which consist of the positive ions and the electrons of the inner shells, remain held in their fixed positions. These immobile charges are called bound charges.

8.8 Dielectric and Electric Polarisation

Q.28 What are dielectrics ?

Ans:

i. Dielectrics are non- conducting substances.

ii. Dielectric substances do not contain any free charges or electrons. i.e. cannot transmit electric charge through them.

iii. Example : Mica, ceramic, plastic, rubber, glasses, asbestose , mineral oil, silicon, magnesia etc.

Q.29 Explain the terms :

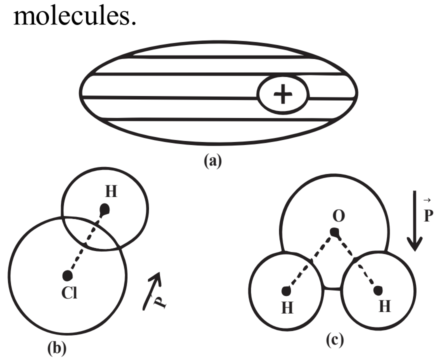
A Polar molecules

B Non - polar molecules

Ans:

A. Polar molecules :

i. The molecules in which centre of mass of positive nucleii and revolving electrons do not coincide are known as polar molecules.



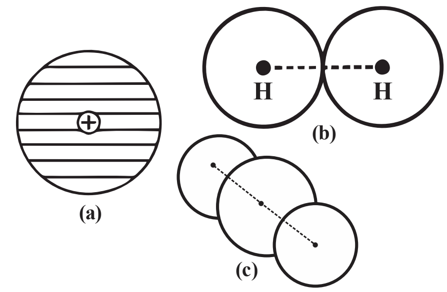
Examples : HCl, H_2O, N_2O etc.

ii. It have a permanent electric dipole moment.

iii. Polar substances behave as a tiny electric dipole.

B) Non - polar molecules :

i. The molecules in which centre of gravity of positive nucleii and revolving electrons coincide are known as nonpolar molecules.



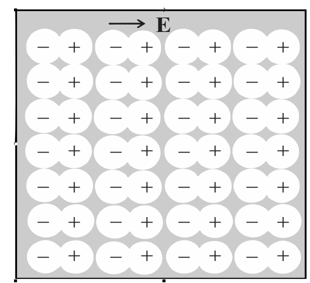
Examples : O_2, H_2, CO_2, polyethelene, polystyrene etc.

ii. It do not have permanent electric dipole moment.

Q.30 Explain polarization of dielectrics slab in external uniform electric field.

Ans:

i. Consider a thin slab of dielectric of permitivity ε and placed in external uniform field.



ii. In presence of electric field, molecules of dielectrics are oriented such that negative charge leaving at left and positive charge leaving on right surface of dielectric.

iii. In the dielectric slab induced charges are equal and opposite. Hence, dielectric slab as a whole is electrically neutral.

iv. The charges so obtained on the surface of dielectric slab are called polarization charges.

v. The polarized dielectric is equivalent to two charged surface with induced charges.

vi. The polarization charges weaken external field because they produce electric field opposite to external electric field.

Q.31 Explain polarization of polar dielectric in an external electric field:

Ans:

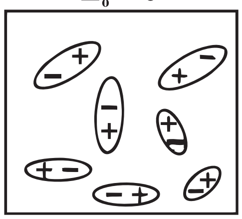
i. The molecules of a polar dielectric have tiny permanent dipole moments.

ii. Due to thermal agitation in the material in the absence of any external electric field, these dipole moments are randomly oriented and the total dipole moment is zero.

iii. When an external electric field is applied the dipole moments of different molecules tend to align with the field.

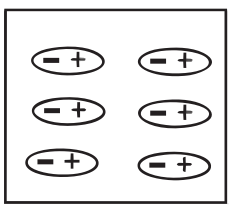
iv. As a result the dielectric develops a net dipole moment in the direction opposite of the external field. Hence the dielectric is polarized.

$\vec{E}_o = 0$



A polar dielectric in absence of electric field.

$\vec{E}_o \neq 0$



$\vec{E}_o \longrightarrow$

A polar dielectric in presence of an external field.

v. The extent of polarization depends on the relative values of the two opposing energies.
   a. The applied external electric field which tends to align the dipole with the field.
   b. Thermal energy tending to randomise the alignment of the dipole.

Q.32 Explain polarization of non-polar dielectric in an external electric field

Ans:

I. In the presence of an external electric field E_o, the centres of the positive charge in each molecule of a non-polar dielectric is pulled in the direction of E_o, while the centres of the negative charges are displaced in the opposite direction.

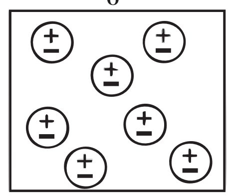
ii. Therefore, the two centres are separated and the molecule gets distorted.

iii. The displacement of the charges stops when the force exerted on them by the external field is balanced by the restoring force between the charges in the molecule.

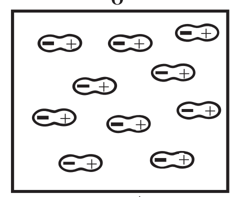
iv. Each molecule becomes a tiny dipole having a dipole moment.

v. The induced dipole moments of different molecules add up giving a net dipole moment to the dielectric in the presence of the external field.

$\vec{E}_O = 0$



$\vec{E}_O \neq 0$



$\vec{E}_O \longrightarrow$

Note :

Polarization

i. *It is the amount of induced surface charge per unit area or the surface density of polarization charges appearing at right angles to applied external electric field.*

ii. *Polarization is given by,*

$$P = \frac{q_p}{A} = \sigma_P$$

where,

q_p = *polarization charges*

σ_p = *charges density of polarization charges*

A = *area of cross - section of dielectric*

iii. *It is a vector quantity and directed from negative induced charges to positive induced charges.*

iv. *Assuming continuously polarized dielectric the value of polarization P at any point is defined as net dipole moment (Ql) of small volume Δv as $\Delta v \to 0$*

$$P = \lim_{\Delta v \to 0} \frac{Ql}{\Delta v}$$

where,

Q = nq = *charge of all dipoles*

N = *number of dipoles per unit volume*

Ql = *net dipole moment*

Q.33 Define electric polarization ? Obtain relation between polarisation density and susceptibility.

Ans:

i. Electric polarization (P) is defined as dipole moment per unit volume.

ii. For small electric field, polarisation is directly proportional to electric field

$$\vec{P} \propto \vec{E}$$

$$\vec{P} = \chi e \vec{E}$$

where, χ_e is electric susceptibility of dielectric material.

8.9 Capacitors and Capacitance Combination in Series and Parallel

Q.34 What is capacitor?

Ans:

i. *Capacitor is a device used to store electric charge and electric energy.*

ii. Capacitor is a system consisting of two conductors having equal and opposite charges separated by an insulator or dielectric.

Q.35 What is meant by the capacity of a conductor?

Ans:

i. Capacity of a conductor, is the ability to hold the charge.

ii. The capacity depends on
   a. the shape and size of the conductor,
   b. the nature of the surroundings, and
   c. the presence of other charged objects near the conductor.

iii. When some charge is added to a conductor, its potential rises .
The charge on the conductor is proportional to its potential.

$Q \propto V$

∴ $\boxed{Q = CV}$

The proportionality constant C is called the capacity or capacitance of the conductor.

iv. Capacity of conductor is given by,

∴ $C = \dfrac{Q}{V}$

If V = 1volt, then C = Q

i.e. *the capacity of a conductor is the quantity of charge required to raise its potential by one unit.*

Q.36 State and define SI unit of capacity.

Ans:

i. The SI unit of capacity is called the **farad**.

ii. Capacity of conductor is given by,

∴ $C = \dfrac{Q}{V}$

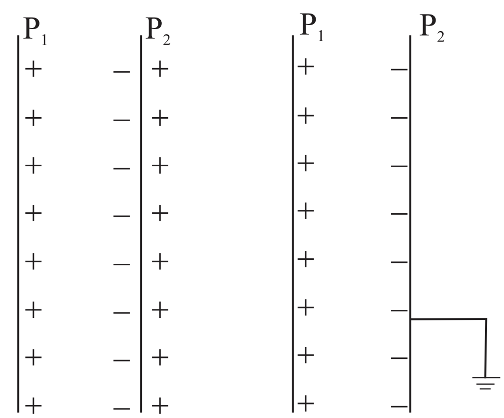
∴ $1 \text{ Farad (F)} = \dfrac{1 \text{ coulomb(C)}}{1 \text{ volt (V)}}$

Thus, a conductor has capacitance one farad, if one coulomb of charge raises its potential by one volt.

iii. Dimension : $[M^{-1}L^{-2}T^{4}A^{2}]$

Q.37 How capacity of a conductor can be increased? Explain.

Ans:



i. A parallel plate capacitor consist of two metal plate P_1 and P_2, each of area A and separated by small distance d.

ii. Let, +Q be the charge given to the plate P_1 when it is fully charge and V is potential, then its capacity.

$C_1 = \dfrac{Q}{V}$ ...(1)

iii. The another identical plate P_2 is kept parallel and near to plate P_1.The charge – Q is induced on the inner surface and +Q on outer surface of plate P_2 its far side.

iv. If plate P_2 is earthed, the charge +Q escapes to earth and only the charge –Q remains on plate P_2.

v. The induced – Q charge lowers potential of plate P_1

vi. If $-V_1$ is the potential due to the – Q charge on plate P_2

The net electric potential of the system is $(V - V_1)$.

The new capacity is,

$$C_2 = \frac{Q}{V - V_1},$$

vii. As $C_2 > C_1$ i.e. the capacity of metal plate P_1 is increase by keeping another identical earth connected metal plate P_2 neat it.

Note :

The three main types of condensers depending on their shape are -

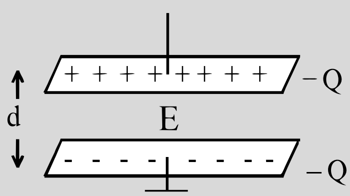
1. *Parallel plate condenser.*
2. *Spherical condenser.*
3. *Cylindrical condenser.*

1. *Parallel plate condenser :*

i. *It consists two parallel metal plates each of area A separated by a finite distance (d) containing dielectric between them.*

ii. *One of the plate is positive charged and other is connected to earth.*

$$c = \frac{\varepsilon_0 kA}{d}$$



2. *Spherical condenser :*

i. *It consists of two concentric spheres of radii a and b (b > a).*

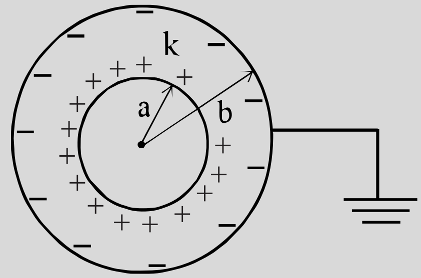
ii. *The outer surface of inner sphere is positively charged.*

iii *Inner surface of outer sphere acquires a negative charge by induction.*

iv. *Outer surface of outer sphere is earth connected.*

v. *A medium of permittivity ε is present in the space between two spheres.*

$$c = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$



3) *Cylindrical condenser :*

i. *Cylindrical condenser consists of two coaxial cylinders of radius a and b respectively (b > a).*

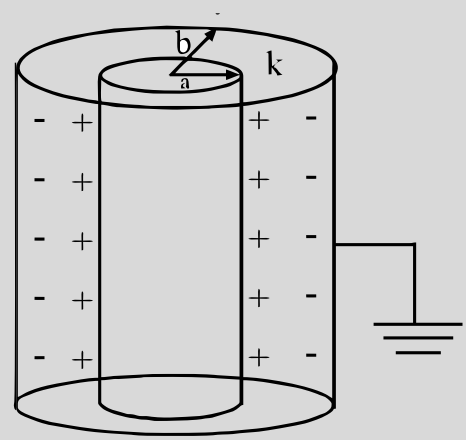
ii. *Outer surface of inner cylinder is positively charged.*

iii. *Inner surface of outer cylinder acquires negative charge.*

iv. *The outer surface of outer cylinder is earth connected.*

v. *A medium of permitivities ε is present between two cylinders.*

$$c = \frac{2\pi\varepsilon_0 l}{\log_e\left(\frac{b}{a}\right)}$$



Spherical *conductor* :

i. *Capacity of a spherical conductor with radiusR and having charge Q on its surface, is given by, $C = 4\pi\varepsilon_0 R$*

ii. *Capacity of earth conductor (R = 6400 km) is nearly 711 μF.*

Q.38 If the difference between the radii of the two spheres of a spherical capacitor is increased state whether the capacitance will increase or decrease.

Ans : For a spherical capacitor $C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

Where, (b – a) is the difference between radii of spheres.

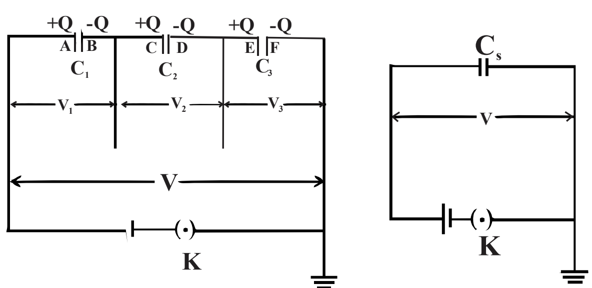
As (b – a) increases, due to inverse relation with capacitance (C), capacitance of capacitor will decrease.

Q.39 Derive an expression for effective capacitance of three parallel plate capacitors connected in series.

Ans:

i. In the series combination number of capacitors are connected one after the another.

ii. Let, C_1, C_2, C_3 be the three capacitors made of parallel metal plates A and B, E and D, E and F respectively and connected in series across a battery through plug key as shown in figure.



ii. When key k is closed plate A is positively charged.

iv. The left plate of capacitor C_1 has a charge +Q. An equal but opposite charge –Q is induced on the right of this plate. The charge –Q induces a charge +Q on the left of plate C_2 and so on.

v. In this combination, charge on each capacitor is same but potential difference across each is different.

Let, V_1, V_2, V_3 be the p.d. across C_1,C_2,C_3 respectively

∴ From figure

$$V = V_1 + V_2 + V_3 \quad ...(1)$$

Now

$$V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}, V = \frac{Q}{C_s}$$

Where, C_S is effective capacitance of series combination.

∴ Equation (i) becomes,

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{Q}{C_s} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \quad ...(2)$$

OR $C_s = \dfrac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_1C_3}$

i.e. the reciprocal of equivalent capacitance of series combination capacitor is equal to the sum of reciprocals of their individual capacitances.

Note:

i. *If two capacitors of capacity C_1 and C_2 are connected in series, then*

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

∴ $$C_s = \frac{C_1C_2}{C_1 + C_2} \quad ...(3)$$

ii. *If n capacitors of capacitance C_1, C_2, C_3,....C_n are connected in series then the effective capacitance is given by,*

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} \quad ...(4)$$

∴ $$C_s = \frac{C_1C_2C_3....C_n}{C_2C_3...C_n + C_1C_3...C_n + ... + C_1C_2...C_{n-1}}$$

iii. *For n - identical condensers*

i.e. $C_1 = C_2 = ... = C_n = C$

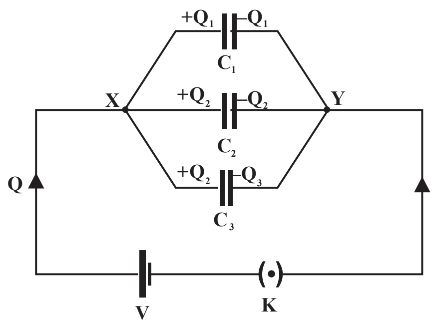
∴ $$C_s = \frac{C}{n}$$

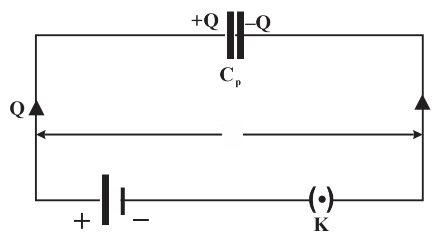
Q.40 Derive an expression for effective capacitance of three parallel plate capacitors connected in parallel.

Ans:

i. In the parallel combination number of capacitors are connected between two common points.

ii. Let, C_1, C_2, C_3 be the three capacitors made of parallel metal plates A and B, C and D, E and F respectively connected between junction X and Y and across a battery through plug key as shown in figure.





iii. When key K is closed, charge Q flows to junction X and split into $+Q_1 + Q_2, + Q_3$ on first plate of C_1, C_2 and C_3 respectively, which induced charge $- Q_1 - Q_2 - Q_3$ on second plate respectively as shown in figure.

iv. In this combination, charge on each capacitor is different but potential difference across each is same.

v. According to conservation of charge
$Q = Q_1 + Q_2 + Q_3$ ...(i)
From figure,
$Q_1 = C_1V, Q_2 = C_2V\ Q_3 = C_3V, Q = C_PV$
where, C_P is effective capacitance of parallel combination

∴ Equation (i) become,
$C_PV = C_1V + C_2V + C_3V$
$C_PV = (C_1 + C_2 + C_3)\ V$

∴ $C_P = C_1 + C_2 + C_3$
i.e. the equivalent capacitance of parallel combination capacitor is equal to the sum of their individual capacitance

Note:

i. *If two condensers of capacity C_1 and C_2 are connected in parallel then*
$C_P = C_1 + C_2$

ii. *If n condensers of capacity C_1, C_2,C_3..... C_n are connected in parallel then,*
$C_p = C_1 + C_2 + C_3 + ...+ C_n$

iii. *For n-identical condensers.*
i.e. $C_1 = C_2 = ... = C_n = C$

∴ $C_p = nC$

Type - VII
Numerical based on Capacitance, Capacitance in Series and Parallel

Formula Used

1. Q = CV

2. Capacitance in series

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2}\ ,\ C_S = \frac{C_1C_2}{C_1 + C_2}$$

3. Capacitance in Parallel

$$C_P = C_1 + C_2$$

★ **1) When 10^8 electrons are transferred from one conductor to another, a potential difference of 10 V appears between the conductors. Find the capacitance between the two conductors.**

Data: $n = 10^8$, V = 10 V

To find: C

Formulae: i. Q = ne ii. $C = \frac{Q}{V}$

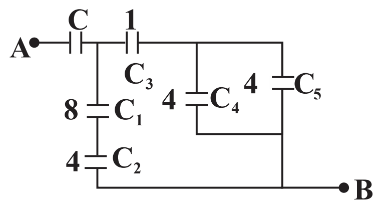
Solution:

i. $Q = ne = 10^8 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-11}C$

ii. $C = \frac{Q}{V} = \frac{1.6 \times 10^{-11}}{10} = 1.6 \times 10^{-12}\ F$

Ans : The capacitance between the two conductors is 1.6×10^{-12} F

★ **2) From the figure given below find the value of the capacitance C if the equivalent capacitance between A and B is to be 1 μF. All other capacitors are in microfarad.**



Solution:

$C_1 = 8\,\mu F$, $C_2 = 4\,\mu F$, $C_3 = 1\,\mu F$,
$C_4 = 4\,\mu F$, $C_5 = 4\,\mu F$

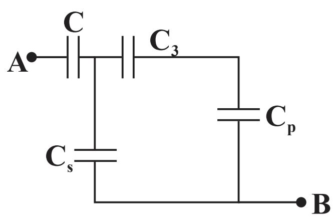
i. Capacitance C_4 and C_5 are in parallel.

$\therefore$ $C_P = C_4 + C_5 = 4 + 4 = 8\,\mu F$

and Capacitance C_1 and C_2 are in series.

$$\therefore \quad C_S = \frac{C_1 C_2}{C_1 + C_2} = \frac{8 \times 4}{8+4} = \frac{32}{12} = \frac{8}{3}\ \mu F$$

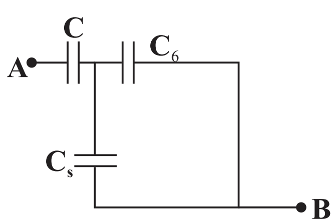
Redrawing the circuit



ii. Now C_3 and C_P are in series such that equivalent capacitance,

$$C_6 = \frac{C_3 C_P}{C_3 + C_P} = \frac{1 \times 8}{1+8} = \frac{8}{9}\ \mu F$$

Redrawing the circuit



iii. C_6 and C_S are in parallel such that equivalent capacitance,

$$C_7 = C_6 + C_S = \frac{8}{9} + \frac{8}{3} = \frac{32}{9}\ \mu F$$

iv. Now C_7 and C are in series forming total Capacitance C_{eq}

$$\therefore \quad C_{eq} = \frac{C_7 \times C}{C_7 + C}$$

$$1 = \frac{\frac{32}{9} \times C}{\frac{32}{9} + C} \qquad (\because C_{eq} = 1\mu F)$$

$$\therefore \quad 9\left(\frac{32}{9} + C\right) = 32\ C$$

$\therefore$ $32 = 32\ C - 9\ C$

$\therefore$ $23C = 32$

$\therefore$ $C = 1.39\,\mu F$

Ans: value of C is $1.39\mu F$

3) Two capacitors of capacitance of $6\,\mu F$ and $12\,\mu F$ are connencted in series with a battery. The votage across the $6\,\mu F$ capacitor is 2V. Compute the total battery voltage.

Data : $C_1 = 6\,\mu F$, $C_2 = 12\,\mu F$, $V_1 = 2V$

To Find : V

Formula : i. $C_s = \dfrac{C_1 C_2}{C_1 + C_2}$ ii. $Q = CV$

iii. $V = V_1 + V_2$

Solution:

i. Charge on Capacitor (C_1) is

$Q = C_1 V_1$

$\therefore$ $Q = 6\,\mu F \times 2V$

$Q = 12\,\mu C$

ii. As capacitors C_1 and C_2 are connected in series

$\therefore$ Charge on both capacitors is same

$\therefore$ Charge on capacitor C_2 is $12\,\mu C$

iii. P.D. across capacitor C_2

$$V_2 = \frac{Q}{C_2} = \frac{12\mu C}{12\mu F} = 1\ \text{Volt}$$

iv. Total potential by battery

$V = V_1 + V_2 = 2 + 1 = 3V$

Ans: Total Battery voltage is 3V

4) Two capacitors of capacitances $3\,\mu F$ and $6\,\mu F$, are charged to potentials of 2V and 5V respectively. These two charged

capacitors are connected in series. Find the potential across each of the two capacitors now.

Data : $C_1 = 3\,\mu F$, $C_2 = 6\,\mu F$

$V_1 = 2V$, $V_2 = 5V$

To Find : P.D across both capacitors when connected in series

Formula : $Q = CV$

Solution:

i. Charge on capacitors C_1

$Q_1 = C_1V_1 = 3\times2 = 6\,\mu C$

Charge on capacitors C_2

$Q_2 = C_2V_2 = 6\times5 = 30\,\mu C$

ii. Then both capacitors are connected in series charges are conserved

∴ Total Charge = $Q_1 + Q_2 = 6 + 30 = 36\,\mu C$

iii. Potential across $3\,\mu F$ capacitor

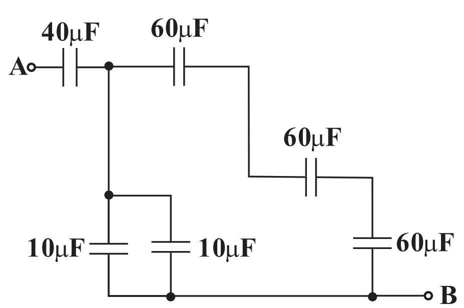
$$= \frac{Q}{C_1} = \frac{36\,\mu C}{3\,\mu F} = 12\text{ V}$$

iv. Potential across $6\,\mu F$ capacitor

$$= \frac{Q}{C_2} = \frac{36\,\mu C}{6\,\mu F} = 6\text{ V.}$$

Ans : Potential across 3μF Capacitor is 12V and across 6μF capacitor is 6V.

5) **Find the equivalent capacitance of the combination of capacitors between the points A and B as shown in figure. Also calculate the total charge that flows in the circuit when a 100 V battery is connected between the point A and B.**



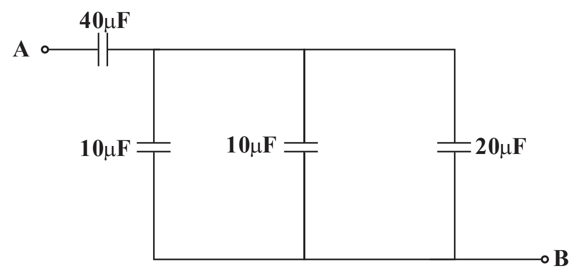
Solution:

i. Three cpacitors of $60\,\mu F$ each are connected in series. Their equivalent capacitance C_1 is given by

$$\frac{1}{C_1} = \frac{1}{60} + \frac{1}{60} + \frac{1}{60} = \frac{3}{60} = \frac{1}{20}$$

$C_1 = 20\,\mu F$

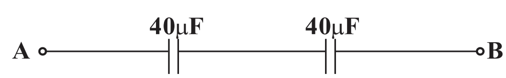
Redrawing the circuit



ii. Now, the three capacitors of $10\,\mu F$, $10\,\mu F$ and $20\,\mu F$ are in parallel. Their equivalent capacitance is

$C_2 = 10+10+20 = 40\,\mu F$

Now, the circuit reduces to the equivalent circuit shown in figure. We have two capacitor of $40\,\mu F$ each connected in series.



iii. The equivalent capacitance between A and B is

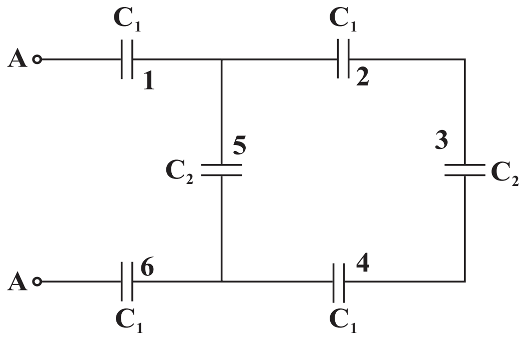
$$C = \frac{40\times40}{40+40} = 20\ \mu F$$

iv. When battery of 100 V is connected across A and B

∴ Charge, $q = CV = 20\,\mu F \times 100V = 2000\,\mu C$

Ans : Equivalent capacitance is 20μF and charge through battery is 2000μc

6) **If C_1 = 3pF and C_2 = 2pF, Calculate the equivalent capacitance of the given network between points A and B.**



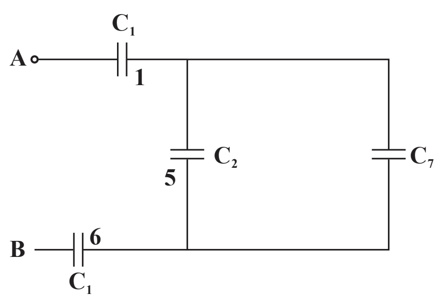
Solution:

i. Capacitors 2, 3 and 4 from a series combination. Their total capacitance C' is given by

$$\frac{1}{C'}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_1}=\frac{1}{3}+\frac{1}{2}+\frac{1}{3}=\frac{7}{6}$$

$\therefore$ $C_7=\frac{6}{7}$ pF

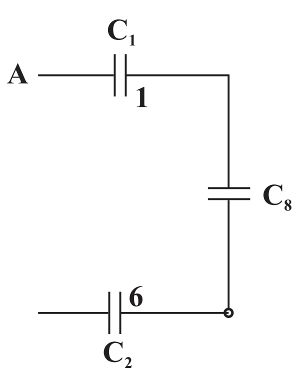
The equivalent Capacitance reduces to



ii. The capacitance C_7 forms a parallel combination with capacitor 5, so their equivalent capacitane is

$$C_8=C_7+C_2=\frac{6}{7}+2=\frac{20}{7}\text{pF}$$

The equivalent capacitance reduces to



iii. The capacitance C_8 forms a series combination with capacitors 1 and 6. The equivalent capacitance C of the entire network is given by

$$\frac{1}{C}=\frac{1}{C_8}+\frac{1}{C_1}+\frac{1}{C_1}=\frac{7}{20}+\frac{1}{3}+\frac{1}{3}=\frac{61}{60}$$

$\therefore$ $C=\frac{60}{61}$ pF.

Ans : The equivallent capacitance between A and B is $\frac{60}{61}$ pF

Problem for Practice

1. Two capacitors have a capacitance of 5 μF when connected in parallel and 1.2 μF when connected in sereis. Calculate their cpacitances.

Ans: 2 μF, 3 μF

2. Two capacitors of equal capacitance when connected in series have net capacitance C_1, and when connected in parallel have net capacitance C_2. What is the value of C_1/C_2?

Ans: C_1/C_2 = 1/4

3. The equivalent capacitance of the combination between A and B in figure is 4 μF.

i. Calculate capacitance of the capacitor C.

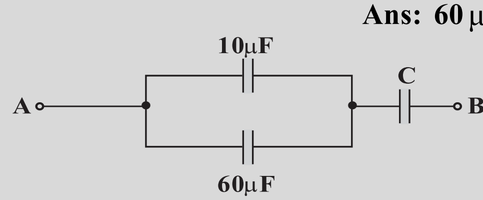
ii. Calculate charge on each capacitor if a 12 V battery is connected across terminals A and B.

iii. What will be the potential drop across each capacitor?

Ans: 5 μF, 48 μC , 2.4V and 9.6V

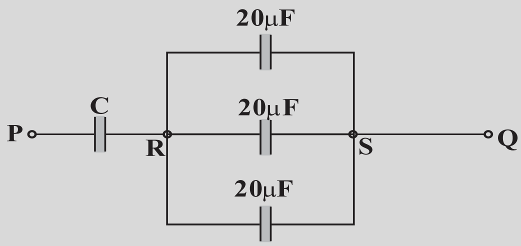
4. Calculate the capacitance of the capacitor in figure if the equivalent capacitance of the combination between A and B is 15 μF.

Ans: 60 μF



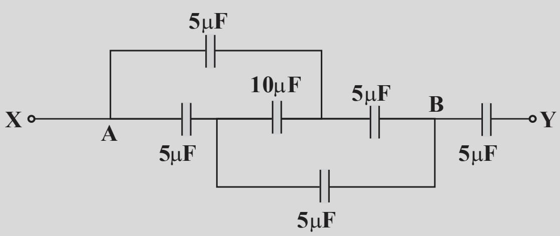
5. Calculate the capacitance of the capacitor C in figure. The equivalent capacitance of the combination between P and Q is $30\,\mu F$.

Ans: 60 μF

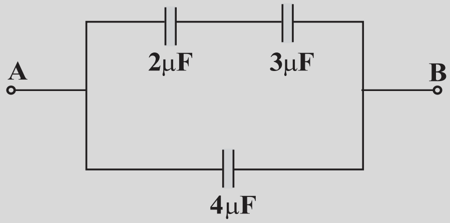


6. Find the resultant capacitance between the points X and Y of the combination of capacitors shown in figure

Ans: 2.5 μF



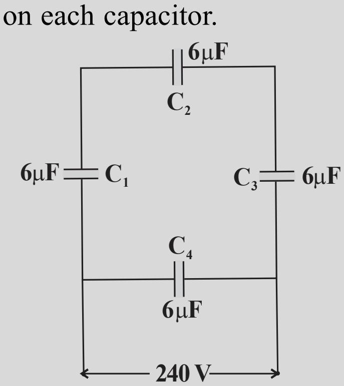
7.



Three condensers are connected as shown in figure. Calculate the effective capacitance between A and B.

Ans: $\frac{26}{5}\mu F$

8. A network of four capacitors of $6\,\mu F$ each is connected to a 240 V supply. Determine the charge on each capacitor.



Ans: $Q_1=Q_2=Q_3=4.8\times10^{-4}C$, $Q_4= 1.44\times10^{-3}C$

9. Three capacitors of capacities $8\,\mu F$, $8\,\mu F$ and $4\,\mu F$ are connected in a series and a potential difference at 120 volt is maintained across the combination. Calculate the charge on capacitor of capacity $4\,\mu F$.

Ans: 290µC

10. A parallel plate air condenser has a capacity of $20\,\mu F$. What will be the new capacity if :

a. the distance between the two plates is double?

b. a marble slab of dielectric constant 8 is introduced between the two plates?

Ans: 10µC, 160µC

8.10 Capacitance of a Parallel plate without and with dielectric medium between the plates

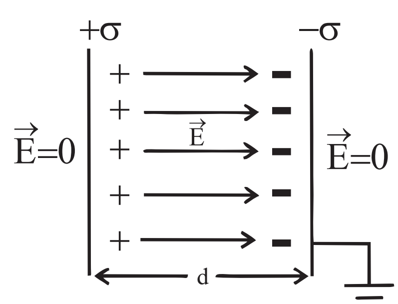
Q.41 Obtain an expression for capacitance of a parallel plate capacitor without a dielectric.

Ans:

i. Consider parallel plate capacitor consists of two thin conducting plate kept parallel to each other

let A be the area of the plates and d be the distance between two plates.

ii. Let medium between two plates be air. One of the plates is insulated and the other is earthed as shown in figure below.



iii. When a charge +Q is given to the insulated plate, then a charge –Q is induced on the inner face of earthed plate and +Q is induced on its farther face. But as this face is earthed the charge +Q being free, flows to earth.

iv. In the outer regions the electric fields due to the two charged plates cancel out, making net field is zero.

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

v. In the inner regions between the two capacitor plates the electric fields due to the two charged plates add up. The net field is thus

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0} \quad ...(1)$$

The direction of E is from positive to negative plate.

vi. Let V be the potential difference between the two plates. Then electric field between the plates is give by

$$E = \frac{V}{d}$$

$\therefore$ V= Ed ....(2)

vii Substituting equation (1) in equation (2),

$$V = \frac{Q}{A\varepsilon_0} d$$

viii. Thus , capacitance of the parallel plate capacitor is given by,

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)} = \frac{A\varepsilon_0}{d}$$

Q.43 Obtain an expression for capacitance of a parallel plate capacitor with a dielectric.

Ans:

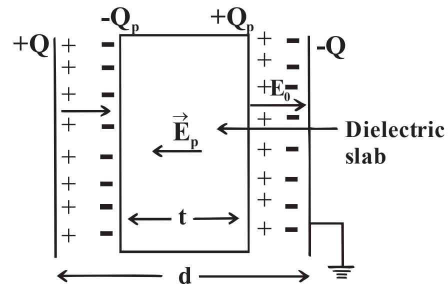
i. Consider a parallel plate capacitor with the two plates each of area A separated by a distance d. The capacitance of the capacitor is given by

$$C_0 = \frac{A\varepsilon_0}{d}$$

ii. Let E_0 be the electric field intensity between the plates before the introduction of the dielectric slab. Then the potential difference between the plates is given by $V_0 = E_0 d$,

Where $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$,and σ is the surface charge density on the plates.

iii. Let a dielectric slab of thickness t (t < d) be introduced between the plates of the capacitor



iv. The field E_0 polarizes the dielectric, inducing charge $-Q_p$ on the left side and $+Q_p$ on the right side of the dielectric.

v. These induced charges set up a field E_p inside the dielectric in the opposite direction of E_0. The induced field is given by

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{Q_p}{A\varepsilon_0} \quad .... \left(\because \sigma_p = \frac{Q_p}{A}\right)$$

vi. The net field (E) inside the dielectric reduces to $E_0 - E_p$.

$$\therefore \quad E = E_0 - E_p = \frac{E_0}{k} \quad .... \left(\because \frac{E_0}{E_0 - E_p} = k\right)$$

where k is a constant called the dielectric constant.

vii. The field E_p exists over a distance t and E_0 over the remaining distance (d – t) between the capacitor plates. Hence the potential difference between the capacitor plates is

$V = E_0 (d - t) + E (t)$

$$= E_0(d-t) + \frac{E_0}{k}(t) \quad .... \left(\because E_0 = \frac{E_0}{k}\right)$$

$$= E_0\left[(d-t) + \frac{t}{k}\right]$$

$$= \frac{Q}{A\varepsilon_0}\left[d - t + \frac{t}{k}\right]$$

viii. The capacitance of the capacitor on the introduction of dielectric slab becomes

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A\varepsilon_0}\left(d - t + \frac{d}{k}\right)} = \frac{A\varepsilon_0}{\left(d - t + \frac{t}{k}\right)}$$

Special Case :

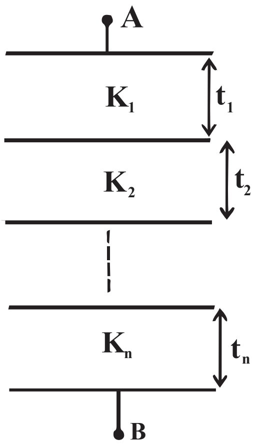
1. If the dielectric fills up the entire space then t = d

$\therefore \quad C = \dfrac{A\varepsilon_0 k}{d} = kC_0$

$\therefore$ **capacitance of a parallel plate capacitor increases k times** i.e. $k = \dfrac{C}{C_0}$

2. If the capacitor is filled with n dielectric slabs of thickness $t_1, t_2 ...... t_n$ then this arrangement is equivalent to n capacitors connected in series

$$\therefore \quad C = \frac{A\varepsilon_0}{\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + ... \frac{t_n}{k_n}\right)}$$



3. If the arrangement consists of n capacitors in parallel with plate areas $A_1, A_2, ... A_n$ and plate separation d

$$C = \frac{\varepsilon_0}{d}(A_1k_1 + A_2k_2 + ...A_nk_n)$$

if $A_1 = A_2 ...A_n = \dfrac{A}{n}$ then

$$C = \frac{A\varepsilon_0}{dn}(k_1 + k_2 + ... k_n)$$

4. If the capacitor is filled with a conducting slab ($k = \infty$) then

$$C = \left(\frac{d}{d - t}\right)C_0 \qquad \therefore C > C_0$$

The capacitance thus increases by a factor $\left(\dfrac{d}{d - t}\right)$

★ Q.44 A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor?

Ans:

i. Capacitance of a capacitor with dielectric of thickness 't' is

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}}$$

ii. Instead of dielectric if metal plate is introduced between the capacitor then $k = \infty$ (for metals) we get,

$$C = \frac{\varepsilon_0 A}{d - t}$$

$$\therefore \quad C = \frac{\left(\varepsilon_0 \frac{A}{d}\right)}{d - \frac{t}{d}} = \frac{dC_0}{d - t}$$

Where $C_0 = \dfrac{\varepsilon_0 A}{d}$, capacitance of capacitor without dielectric

$$\therefore \quad C = \left(d - \frac{t}{d}\right)C_0$$

From this we can conclude, capacitance increase by a factor $\left(d - \dfrac{t}{d}\right)$

Type - VIII
Numerical based on Capacitance of parallel plate capacitors

Formula used

1. Capacitance with dielectric of thickness 't'

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}}$$

2. Capacitance without dielectric

$$C = \frac{\varepsilon_0 Ak}{d}$$

3. Capacitance without dielectric

$$C_0 = \frac{\varepsilon_0 A}{d}$$

4. $C = kC_0$

★ **1) A parallel plate capacitor has an area of 4 cm^2 and a plate separation of 2 mm**

i. Calculate its capacitance

ii. What is its capacitance if the space between the plates is filled completely with a dielectric having dielectric constant of constant 6.7

Data: $A = 4\ cm^2 = 4\times10^{-4}\ m^2$

$d = 2mm = 2\times10^{-3}\ m$, $k = 6.7$,

To find: i. C_0 ii. C

Formulae: i. $C_0 = \frac{A\varepsilon_0}{d}$

ii. $C = \frac{\varepsilon_0 kA}{d}$ $\therefore C = kC_0$

Solution:

i. $$C_0 = \frac{A\varepsilon_0}{d} = \frac{\overset{2}{\cancel{4}}\times10^{-4}\times8.85\times10^{-12}}{\underset{1}{\cancel{2}}\times10^{-3}}$$

$$= 1.77\times10^{-12}\ F$$

ii. $C = kC_0 = 6.7\times1.77\times10^{-12}$

$= 1.186\times10^{-11}\ F$

Ans : i. Capacitance of the capacitor is 1.77×10^{-12} F.
ii. Capacitance of capacitor when dielectric is filled completely is 1.186×10^{-11} F

★ **2) In a parallel plate capacitor with air between the plates, each plate has an area of $6\times10^{-3} m^2$ and the separation between the plates is 2 mm.**

i. Calculate the capacitance of the capacitor,

ii. If this capacitor is connected to 100V supply, what would be the charge on each plate?

iii. How would charge on the plates be affected if a 2 mm thick mica sheet of k = 6 is inserted between the plates while the voltage supply remains connected?

Data: $A = 6\times10^{-3}\ m^2$, $d = 2mm = 2\times10^{-3} m$,

$V = 100V$

To find: i. C ii. Q iii. Q'

Formulae: i. $C = \frac{\varepsilon_0 A}{d}$ ii. $Q = CV$

iii. $Q' = kQ$

Solution:

i. $$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times6\times10^{-3}}{2\times10^{-3}}$$

$= 2.655\times10^{-11} F$

ii. $Q = CV = 2.655\times10^{-11}\times100$

$= 2.655\times10^{-9} C$

iii. $Q' = kQ = 6\times2.655\times10^{-9}$

$= 1.593\times10^{-8} C$

Ans : i. Capacitance is 2.655×10^{-11} F.
ii. Charge on plates is 2.655×10^{-9} C
ii. Charge on plates when dielectric is inserted in it is 1.593×10^{-8} C

★ **3) In a capacitor of capacitance 20 μF, the distance between the plates is 2 mm. If a dielectric slab of width 1 mm and dielectric constant 2 is inserted between the plates, what is the new capacitance?**

Data: $C_0 = 20\ \mu F = 20\times10^{-6}\ F$, $k = 2$

$d = 2mm = 2\times10^{-3}\ m$

$t = 1mm = 1\times10^{-3}\ m$

To find: Capacitance when slab is inserted between plates. (C)

Formulae: i. $C_0 = \frac{\varepsilon_0 A}{d}$ ii. $C = \frac{A\varepsilon_0}{d - t + \frac{t}{k}}$

Dividing formula (ii) by (i), we get

$$\frac{C}{C_0}=\frac{d}{d-t+\frac{t}{k}}$$

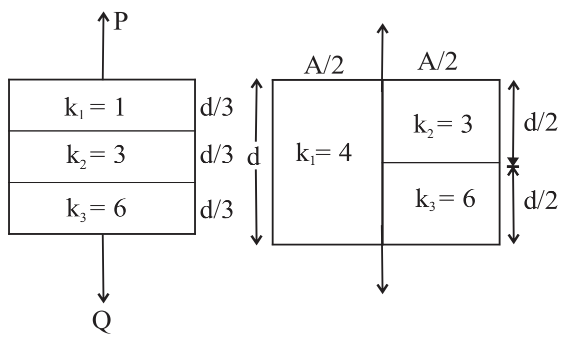
Solution: $C=\left[\frac{d}{d-t+\frac{t}{k}}\right]C_0$

$\therefore$ $$C=\left(\frac{2\times10^{-3}}{\left(2-1+\frac{1}{2}\right)\times10^{-3}}\right)\times 20$$

$\therefore$ $C=\frac{80}{3}=26.67\ \mu F$

Ans :	New capacitance after inserting dielectric is 26.67μF

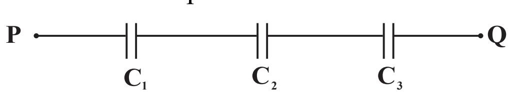
★ **4) Find the equivalent capacitance between P and Q. Given, area of each plate = A and separation between plates = d.**



Solution:

i. Three tiers of dielectric slabs are equivalent to 3 different capacitors connected in series.

$\therefore$



where,

$$C_1=\frac{Ak_1\varepsilon_0}{d/3}=\frac{A\varepsilon_0}{d/3}$$

$$C_2=\frac{Ak_2\varepsilon_0}{d/3}=\frac{3A\varepsilon_0}{d/3},$$

$$C_3=\frac{Ak_3\varepsilon_0}{d/3}=\frac{6A\varepsilon_0}{d/3}$$

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

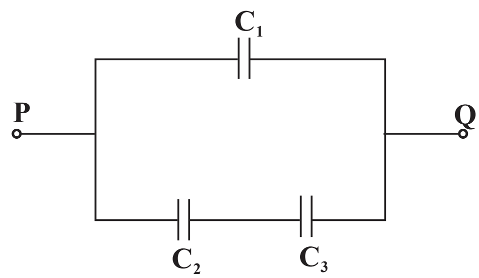
$\therefore$ $$\frac{1}{C}=\frac{d/3}{A\varepsilon_0}+\frac{d/3}{3A\varepsilon_0}+\frac{d/3}{6A\varepsilon_0}$$

$$=\frac{d}{3A\varepsilon_0}\left[1+\frac{1}{3}+\frac{1}{6}\right]$$

$$=\frac{d}{3A\varepsilon_0}\times\frac{9}{6}$$

$\therefore$ $\frac{1}{C}=\frac{d}{2A\varepsilon_0}$, $C=\frac{2A\varepsilon_0}{d}$

ii. Second arrangement is equivalent to first capacitor in parallel with series combination of second and third capacitor.



$$C_1=\frac{\varepsilon_0 k_1\frac{A}{2}}{d}=\frac{\varepsilon_0 k_1 A}{2d}=\frac{4\varepsilon_0 A}{2d}=\frac{2\varepsilon_0 A}{d}$$

$$C_2=\frac{\varepsilon_0 k_2\frac{A}{2}}{\frac{d}{2}}=\frac{\varepsilon_0 k_2 A}{d}=\frac{3\varepsilon_0 A}{d}$$

$$C_3=\frac{\varepsilon_0 k_3\frac{A}{2}}{\frac{d}{2}}=\frac{\varepsilon_0 k_3 A}{d}=\frac{6\varepsilon_0 A}{d}$$

Step 1: Now C_2 and C_3 are in series

$$C'=\frac{C_2\times C_3}{C_2+C_3}=\frac{\frac{3\varepsilon_0 A}{d}\times\frac{6\varepsilon_0 A}{d}}{\frac{3\varepsilon_0 A}{d}+\frac{6\varepsilon_0 A}{d}}$$

$$=\frac{\frac{18\varepsilon_0^2A^2}{d}}{\frac{9\varepsilon_0 A}{d}}=\frac{18\varepsilon_0^2A^2}{d}\times\frac{d}{9\varepsilon_0 A}$$

$$= C' = \frac{2\varepsilon_0 A}{d}$$

Step 2: Now C' and C_1 are in parallel equivalent capacitance between P and Q is
$C_{eq} = C' + C_1$

$$= \frac{2\varepsilon_0 A}{d} + \frac{2\varepsilon_0 A}{d} = \frac{4\varepsilon_0 A}{d}$$

Problems for Practice

1. In a parallel plate capacitor, the capacitance increases from $4\,\mu F$ to $80\,\mu F$, on introducing a dielectric medium between the plates. What is the dielectric constant of the medium?

Ans: 20

2. A parallel plate capacitor with air between the plates has a capacitance of $8\,\mu F$. The separation between the plates is now reduced by half and the space between them is filled with a medium of dielectric constant 5. Calculate the value of capacitance of the capacitor in the second case.

Ans: 80 pF

3. A parallel plate air condenser has an area 2×10^{-4} m^2 and separation between the two plate is 1 mm. Find its capacity.
[$\varepsilon_0 = 8.85\times10^{-12} C^2/Nm^2$]

Ans : 1.77 pF

4. An electric field $E_0 = 3\times10^4$ Vm^{-1} is established between the plates, 0.05 m apart, of a parallel plate capacitor. After removing the charging battery, an uncharged metal plate of thickness t = 0.01 m is inserted between the capacitor plates. Find the p.d. across the capacitor (i) before , (ii) after the introduction of the plate. (iii) What would be the p.d. if a dielectric slab (k = 2) were introduced in place of metal plate?

Ans :(i) 1500V (ii) 1200V (iii) 1350V

5. An ebonite plate (k = 3), 6 mm thick , is introduced between the parallel plates of a capacitor of plate area 2×10^{-2}m^2 and plate separation 0.01 m. Find the capacitance.

Ans: 29.5pF

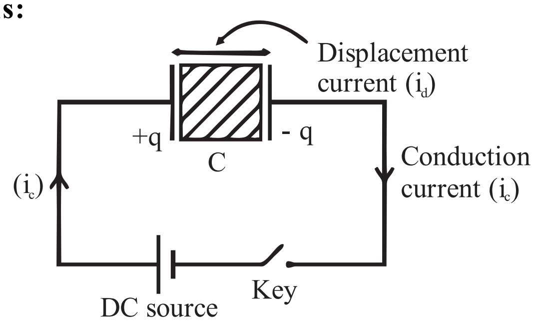
6. A parallel plate capacitor is charged to a certain potential difference. When a 3.0 mm thick slab is slipped between the capacitor plates, then to maintain the same p.d. between the plates, the plate separation is to be increased by 2.4 mm. Find the dielectric constant of the slab.

Ans: 5

8.11 Displacement Current

Q.45 Explain origin of Displacement current.

Ans:



i. electric current in a DC circuit constitutes a flow of free electrons.

ii. In a circuit as shown in Fig a parallel plate capacitor with a dielectric is connected across a DC source.

iii. In the conducting part of the circuit free electrons are responsible for the flow of current. But in the region between the plates of the capacitor, there are no free electrons available for conduction in the dielectric.

iv. As the circuit is closed, the current flows through the circuit and grows to its maximum value (i_C) in a finite time (time constant of the circuit). The conduction current, i_C is found to be same everywhere in the circuit except inside the capacitor.

v. As the current passes through the leads of the capacitor, the electric field between the plates increases and this in turn causes polarisation of the dielectric.

vi. Thus, there is a current in the dielectric due to the movement of the bound charges. The current due to bound charges is called displacement current (i_d) or charge-separation current.

Q.46 Obtain relation between conduction current and displacement current.

Ans:

i. Charge produced on the plates of a capacittor is

$q = cv$

$= \frac{Ak\varepsilon_0}{d} \times E d$

$q = Ak\varepsilon_0 E$

Differentiating the above equation, we get

$$\frac{dq}{dt} = Ak\varepsilon_0 \frac{dE}{dt} \quad ...(1)$$

Where dq/dt is the conduction current (i_C) in the conducting part of the circuit.

This gives, $i_C = Ak\varepsilon_0 \frac{dE}{dt}$

$\Rightarrow \frac{dE}{dt} = \frac{i_C}{Ak\varepsilon_0}$

$\therefore \frac{dE}{dt} \alpha\, i_C$ (If A is constant)

Thus, the rate of change of electric field (dE/dt) across the capacitor is directly proportional to the current (i_C) flowing in the conducting part of the circuit.

ii. The quantity on R.H.S of equation (1) has dimensions of current and indicates the displacement of bound charges in the dielectric of the capacitor under the influence of the electric field. This current, called displacement current (i_d),

$\therefore i_d = Ak\varepsilon_0 \left(\frac{dE}{dt}\right)$

for k = 1, $i_d = A\varepsilon_0 \left(\frac{dE}{dt}\right)$

This means, displacement current (i_d) exists at any point in space where, time varying electric field (E) exists (i.e., $dE/dt \neq 0$).

8.12 Energy Stored in Capacitor

Q.47 Derive an expression for the energy stored in charged capacitor. Explain its different forms.

Ans:

i. Some work is done on charging the capacitor and it stored in the form of electrostatic energy

ii. Let, Q be the charge when capacitor is fully charged,

The p.d. between the two plates is,

$$V = \frac{Q}{C} \quad ...(1)$$

iii. Let, q be the charge, and v be the potential at an intermediate stage of charging of a capacitor, then

$$v = \frac{q}{C} \quad ...(2)$$

iv. If a small charge dq is added, further then work done is given by dw = vdq

From equation (2),

$$\therefore dw = \frac{q}{C} dq \quad ... (3)$$

v. The total work done in charging the capacitor from 0 to Q is,

$W = \int dw$

$= \int_0^Q \frac{q}{C}\, dq$

$= \frac{1}{C} \int_0^Q q\, dq$

$= \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2}\frac{Q^2}{C}$

This work is stored in the form of electrostatics energy.

vi. The energy stored in the capacitor is,

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C}} \quad ... (4)$$

vii. As, Q = CV,

$$\therefore \boxed{U = \frac{1}{2}CV^2} \quad ... (5)$$

viii. As, $C = \frac{Q}{V}$,

$$\therefore \boxed{U = \frac{1}{2}QV} \quad ... (6)$$

equation (4), (5) and (6) are various forms of energy stored in capacitor.

★ **Q.48 Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2, so that the energy stored in these two cases becomes the same.**

Ans: Let V_S and V_P be potential applied across series and parallel combinations respectively

Also, C_S and C_P be the capacitance in series and parallel combination respectively.

Now,

$$\therefore \quad C_s = \frac{C_1C_2}{C_1+C_2} \text{ and } C_p = C_1 + C_2$$

As energy stored in both case is same

$\therefore \quad U_S = U_P$

$$\frac{1}{2}C_S V_S^2 = \frac{1}{2}C_P V_P^2$$

$$\left(\frac{V_P}{V_S}\right)^2 = \frac{C_S}{C_P} = \frac{C_1+C_2}{C_1C_2} \times \frac{1}{C_1+C_2} = \frac{1}{2}$$

....(Given C_1:C_2 = 1 : 2)

$$\therefore \quad \frac{V_P}{V_S} = \frac{1}{\sqrt{2}}$$

★ **Q.49 A capacitor has some dielectric between its plates and the capacitor is connected to a DC source. The battery is now disconnected and then the dielctric is removed. State whether the capacitance, the energy stored in it, the electric field, charge stored and voltage will increase, decrease or remain constant.**

Ans:

i. Capacitance of parallel plate capacitor with dielectric k and connected to battery is

$$C = \frac{\varepsilon_0 kA}{d} \quad \text{... (1)}$$

ii. When battery is discounnected and dielectric is removed

a. New capacitance

$$C' = \frac{\varepsilon_0 A}{d} = \frac{1}{k}C$$

Capacitance decreases by the factor k

b. New charge :

As battery is disconnected after charging capacitor

So charge remains constant.

c. New potential

When Q is constant

$$V \alpha \frac{1}{C}$$

$$\therefore \quad \frac{V'}{V} = \frac{C}{C'} = k$$

$\therefore \quad V' = kV$

So voltage (potential) increases by factor k

d. New electric field

As $E = \frac{V}{d}$

At constant d

$E \alpha V$

$$\therefore \quad \frac{E'}{E} = \frac{V'}{V} = k$$

$\therefore \quad E' = kE$

So electric field increases by k factor

e. New energy stored

$$U = \frac{1}{2}QV$$

At constant Q,

$U \alpha V$

$$\therefore \quad \frac{U'}{U} = \frac{V'}{V} = k$$

$\therefore \quad U' = ku$

So energy stored is increased by the k factor.

Type - IX
Numerical based on energy stored in capacitors

Formula used

1. $$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Qv$$

★ **1) A parallel plate air capacitor has a capacitance of 3×10^{-9} farad. A slab of dielectric constant 3 and thickness 3 cm completely fills the space between the**

plates. The potential difference between the plates is maintained constant at 400 volt. What is the change in the energy of capacitor after introducing slab?

Data: $C_0 = 3\times10^{-9}$ F, V = 400V
d = 3 cm, k = 3

To find: Change in energy of capacitor (ΔU)

Formulae: i. $C = kC_0$ ii. $U = \frac{1}{2}CV^2$

Solution:

i. Initial energy stored by capacitor,

$$U_i = \frac{1}{2}CV^2$$

$$U_i = \frac{1}{2}\times3\times10^{-9}\times(400)^2 = \frac{1}{2}\times3\times10^{-9}\times16\times10^4$$

$= 24\times10^{-9+4} = 24\times10^{-5}$ J

ii. When the slab of k = 3 is introduced between the plates of the capacitor, the capacitance of the capacitor , $C = kC_0$

∴ $C = 3\times3\times10^{-9} = 9\times10^{-9}$ F

iii. Final energy stored in capacitor

$$U_f = \frac{1}{2}CV^2$$

$$U_f = \frac{1}{2}\times9\times10^{-9}\times(400)^2$$

$$= \frac{1}{2}\times9\times10^{-9}\times16\times10^4 = 72\times10^{-5}\text{ J}$$

iv. Change in energy

$\Delta U = U_f - U_i$
$= (72\times10^{-5}) - (24\times10^{-5})$
$= (72 - 24)\times10^{-5} = 48\times10^{-5}$J

Ans : The change in energy of the capacitor when dielectric slab is introduced is $48\times10-5$ J.

★ **2) A $6\,\mu F$ capacitor is charged by a 300 V supply. It is then disconnected from the supply and is connected to another uncharged $3\,\mu F$ capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?**

Data: $C_1 = 6\,\mu F$, V = 300 V, $C_2 = 3\,\mu F$

To Find: loss of energy (ΔU)

Formula : $U = \frac{1}{2}QV$

Solution :

i. Initial charge on capacitor C_1
$Q = C_1V = 6\times10^{-6}\times300 = 1800\times10^{-6}$C
$= 18\times10^{-4}$ C

ii. Energy stored in Capacitor C_1

$$U_1 = \frac{1}{2}QV = \frac{1}{2}\times18\times10^{-4}\times300$$

$= 2700\times10^{-4} = 27\times10^{-2}$ J

iii. Now Capacitor C_1 is disconnected and connected to uncharged capacitor C_2.
The system of two capacitors will acquire common potential V' and charge Q'
From charge conservation,
Q = Q'

∴ $VC_1 = V'(C_1 + C_2)$

∴ $$V' = \frac{VC_1}{(C_1+C_2)} = \frac{300\times6\times10^{-6}}{(6+3)\times10^{-6}} = 200\text{ V}$$

iv. Energy stored in system,

$$U_2 = \frac{1}{2}QV' = \frac{1}{2}\times18\times10^{-4}\times200$$

$= 1800\times10^{-4} = 18\times10^{-2}$ J

v. Loss in energy $= U_1 - U_2$
$= 27\times10^{-2} - 18\times10^{2} = 9\times10^{-2}$ J

Ans : Loss of energy is 9×10^{-2} J

Problem for Practice

1. Energy stored in a charged capacitor of capacity 25pF is 4 J. Find the charge on its plate.

Ans:14.14 μC

2. A condenser of capacity $100\,\mu F$ is charged to a potential of 1 kV. Calculate the energy stored in the condenser.

Ans : 50J

3. A capacitor carries a charge of $8\,\mu F$ at a potential 400V. How much electrostatic energy stored in the capacitor?

Ans: 64×10^{-2} J

4. Electrostatic energy of 3.5×10^{-4} J is stored in capacitor at 700 V. What is the charge on the capacitor?

Ans : 10^{-6}C = 1 μC

5. A 10 μF capacitor is connected with 100V battery. What would be the electrostatic energy stored?

Ans: 5×10^{-2}J

8.13 Van de Graaff Generator

Q.50 State principle of Van de Graff generator.

Ans: **Principle:** This generator is based on

i. the phenomenon of corona discharge (action of sharp points),

ii. the property that charge given to a hollow conductor is transferred to its outer surface and is distributed uniformly over it,

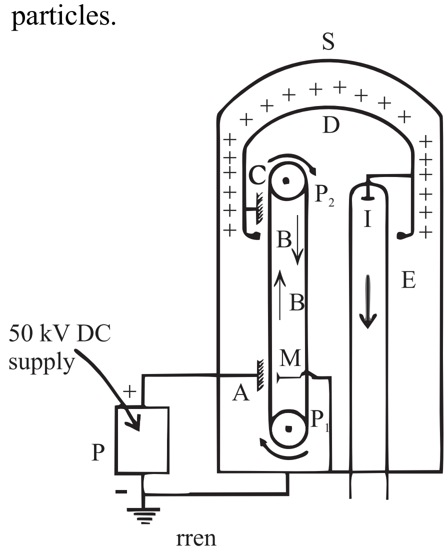
iii. if a charge is continuously supplied to an insulated metallic conductor, the potential of the conductor goes on increasing.

Q.51 Explain construction and working of Van-de Graff Generator.

Ans:

Van-de Graff Generator :

It is an electrostatic generator, that can produce high potential of the order of millions of volt. It is used to accelerate charged particles such as protons, deuterons, α - particles.



Construction:

i. If consist of hollow metal polished sphere S is supported by two vertical insulating columns. E is an evacuated accelerating tube having an electrode I at its upper end, connected to the dome - shaped conductor.

ii. Two pullies P_1, P_2 on which a endless close belt of insulating material is continuously made to run with the help of electric motor.

iii. The spray brush A, consisting of a large number of pointed wires, is connected to the positive terminal of a high voltage DC power supply.

iv. From this brush positive charge can be sprayed on the belt which can be collected by another similar brush C.

v. To prevent the leakage of charge from the dome, the pulley and belt arrangement, the dome and a part of the evacuated tube are enclosed inside a large steel vessel S, filled with nitrogen at high pressure.

vi. A small quantity of freon gas is mixed with nitrogen to ensure better insulation between the vessel S and its contents. A metal plate M held opposite to the brush A on the other side of the belt is connected to the vessel S, which is earthed.

Working :

i. The electric motor connected to the pulley P_1 is switched on, which begins to rotate setting the conveyor belt into motion.

ii. The DC supply is then switched on. From the pointed ends of the spray brush A, positive charge is continuously sprayed on the belt B.

iii. The belt carries this charge in the upward direction, which is collected by the collector brush C and sent to the dome shaped conductor.

iv. As the dome is hollow, the charge is distributed over the outer surface of the dome. Its potential rises to a very high value due to the continuous accumulation of charges on it. The potential of the electrode I also rises to this high value.

v. The positive ions such as protons or deuterons from a small vessel (not shown in the figure) containing ionised hydrogen or deuterium are then introduced in the upper part of the evacuated accelerator tube.

vi. These ions, repelled by the electrode I, are accelerated in the downward direction due to the very high fall of potential along the tube, these ions acquire very high energy.

vii. These high energy charged particles are then directed so as to strike a desired target.

Q.52 State uses of Van de Graaff generator.

Ans: **Uses:** The main use of Van de Graff generator is to produce very high energy charged particles having energies of the order of 10 MeV. Such high energy particles are used

i. to carry out the disintegration of nuclei of different elements,

ii. to produce radioactive isotopes,

iii. to study the nuclear structure,

iv. to study the different types of nuclear reactions,

v. accelerating electrons to sterlize food and to process materials.

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